From Exact to Anytime Solutions for Marginal MAP

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Introduction

- Marginal MAP
  - Mode of probability distribution after marginalizing subset of variables
  - Complexity Class: NP^{PP} Complete
    - MPE (NP-Complete) : optimizing over max variables
    - PR (#P-Complete) : evaluating full instantiation

- Application to Probabilistic Planning
  - Marginal MAP query returns optimal probabilistic conformant plan*

* "Applying Search Based Probabilistic Inference Algorithms to Probabilistic Conformant Planning: Preliminary Results", 2016 ISAIM

Earlier Works on Marginal MAP Inference

Earlier Approaches

[Park & Darwiche 2003]	→	[Yuan& Hansen 2009]
• Exact Solution • Depth First Branch and Bound with Dynamic Variable Ordering • Join-tree upper bound Relax ordering **Systematic Search Algorithm**	→	• Exact Solution • Depth First Branch and Bound with Static Variable Ordering • Incremental Join-tree upper bound **Reduced heuristic computation time**

[Marinescue, Dechter, Ihler 2014]	→	[Marinescue, Dechter, Ihler 2014]
• Exact Solution • AND/OR Branch and Bound • WMB + Cost shifting schemes **Stronger Heuristic** **Compacter AND/OR Search Space**	→	• Exact Solution • AND/OR Best First • AND/OR Recursive Best First **Best First Based Search Strategy** **Avoid Solving Summation Problems**

- [Liu, Iher 2013] Variational algorithms
- [Maua, De Campos 2012] Factor-set elimination algorithm

Motivation

- Best First Schemes avoid evaluating summation sub problems, but they requires enormous amount of memory → Turn to anytime approach

Probabilistic Graphical Models

- A graphical model (**X**, **D**, **F**)
  - $X = \{X_1, \ldots, X_n\}$ variables
  - $D = \{D_1, \ldots, D_n\}$ domains
  - $F = \{f_1, \ldots, f_m\}$ functions
- Operators
  - Combination (product)
  - Elimination (max/sum)
- Tasks
  - Probability of Evidence (PR)

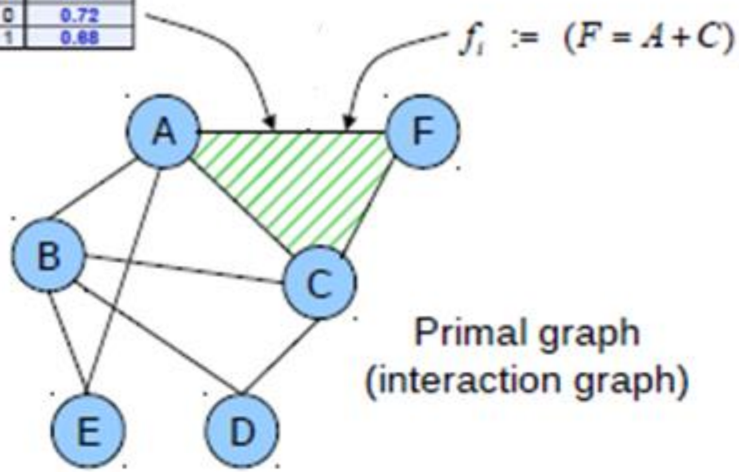
    $$Pr(e) = \sum_{X_s} \prod_j f_j(X_s, e)$$
  - Most Probable Explanation (MPE)

    $$\mathbf{x}_{MPE} = argmax_{\mathbf{x}} \prod_j f_j(\mathbf{x})$$
  - Marginal MAP (Maximum A Posteriori)

    $$\mathbf{x}_{MMAP} = argmax_{\mathbf{x}_m \in X_M} \sum_{\mathbf{x}_s \in X_s} \prod_j f_j(\mathbf{x_m}, \mathbf{x_s})$$

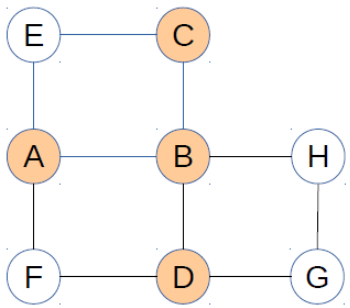
CPT

A	C	F	P(F|A,C)
0	0	0	0.14
0	0	1	0.96
0	1	0	0.40
0	1	1	0.60
1	0	0	0.35
1	0	1	0.65
1	1	0	0.72
1	1	1	0.68



All these tasks are NP-hard
Exploit problem structure (primal graph)

AND/OR Search Space for MMAP

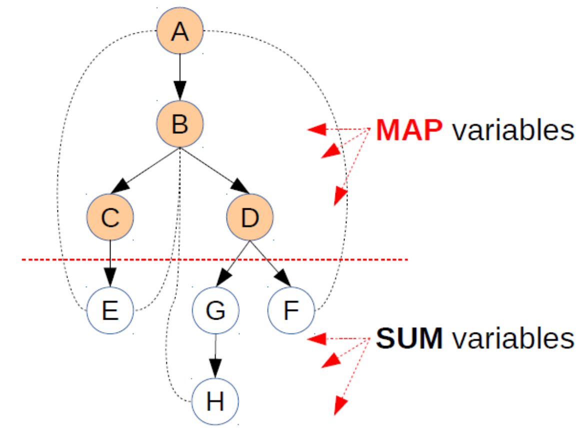


primal graph

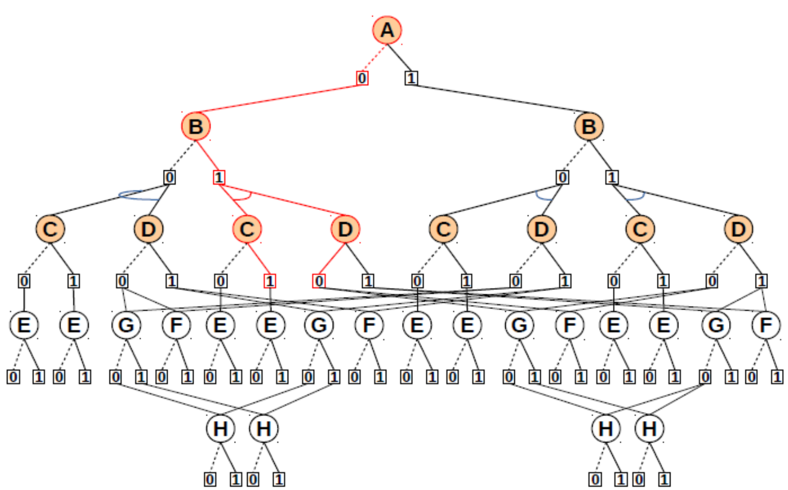
$$X_M = \{A, B, C, D\}$$

$$X_S = \{E, F, G, H\}$$

constrained variable ordering



constrained pseudo tree



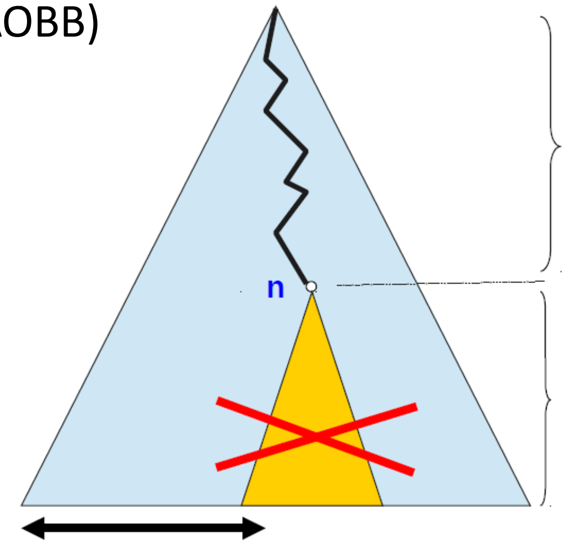
constrained pseudo tree as backbone

merge identical sub-problems (conditional independence)

Anytime AND/OR Search for MMAP

Anytime AOBB (BRAOBB)

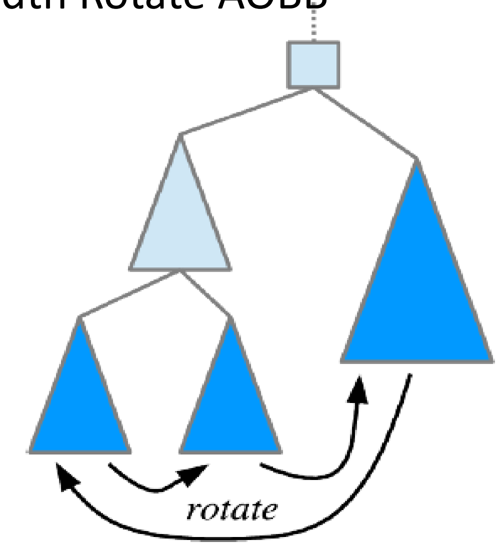
Depth First Branch and Bound (AOBB)



(UB) Upper Bound = best solution so far

Prune node n if current best solution is better than optimistic evaluation at n

Breadth Rotate AOBB

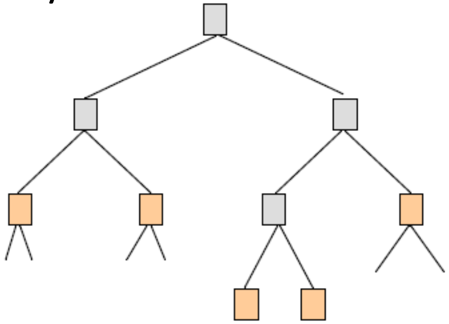


Problem decomposition rejects anytime performance of AOBB
Rotate through sub-problems

Anytime AND/OR Search for MMAP

Weighted Best First Search

AND/OR Best First



Expand Nodes with best heuristic evaluation value f(n)

Weighted Best First

Initialize w
While w >= 1
Inflate heuristic by w
AOBF (sub-optimal solution within w)
optionally Revise traversed search space
reduce w

- Weighted Restarting AOBF (WAOBF)
- Weighted Restarting RBFAOO (WRBFAOO)
- Weighted Repairing AOBF (WRAOBF)

Experiment Setup

Benchmark Instances

Domain	# instances
GRID	75
PEDIGREE	50
PROMEDAS	50

Problem instances are modified from PASCAL2 Probabilistic Inference Challenge Data Set (http://www.cs.huji.ac.il/project/PASCAL/)

Algorithm Parameters

Algorithm	Parameters	Memory
Weighted Mini Bucket Heuristic	i-bound from 2 to 20	-
BRAOBB	Rotation Limit 1000	Max 24 GB
WAOBF/ WRAOBF /WRBFAOO	Starting Weight 64	Max 24 GB, Cache 4 GB

Performance Measures

- Responsiveness, Quality Score

Performance Regimes

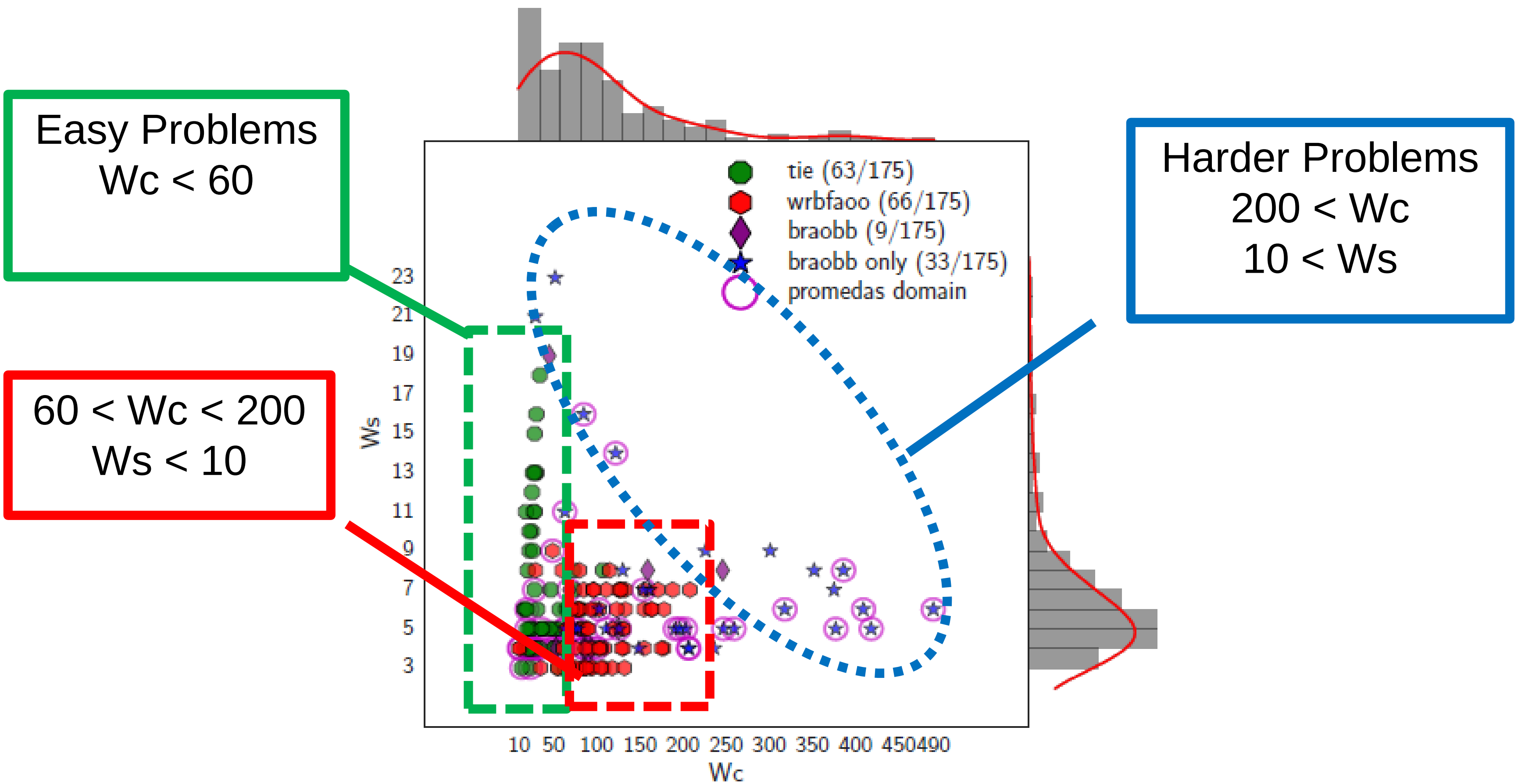
		Overall		Pedigree		Promedas	
AND/OR Search for MMAP		Resp.	Quality	Resp.	Quality	Resp.	Quality
Exact	AOBB	89%	339%	84%	342%	86%	405%
	AOBF	50%	208%	42%	158%	42%	258%
	RBFAOO	58%	90%	42%	95%	42%	132%
Anytime	WAOBF	82%	365%	88%	442%	54%	266%
	WRBFAOO	86%	394%	**90%**	**440%**	**60%**	**305%**
	WRAOBF	82%	339%	88%	364%	54%	261%
	BRAOBB	86%	365%	58%	259%	94%	473%

- Summarized from 1 hour time bound,
- Responsiveness: WMB-MM(18), Quality Score: WMB-MM(12) heuristic

- **WRBFAOO is the overall best** performed algorithm
- BRAOBB is the second best performer, but the **best at PROMEDAS DOMAIN**

WRBFAOO vs. BRAOBB

- Closer look at individual problem instances



- Each point (Wc, Ws) represents difficulty of problem
- Time/ Memory Complexity is Exponential in W

Conclusion

- Improvement from Exact to Anytime
  - Anytime Best-First approach
    - Recovers responsiveness close to Depth-First schemes
    - Provide high quality solutions

- Future Work
  - Better Search Strategy
    - Memory issue with hard problems (Ws > 10, Wc > 200)
  - Integrate approximation for summation problems
    - From exact to approximation