# Heuristic AND/OR Search for Solving Influence Diagrams

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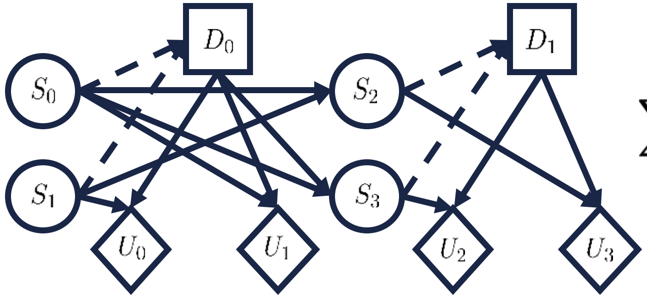
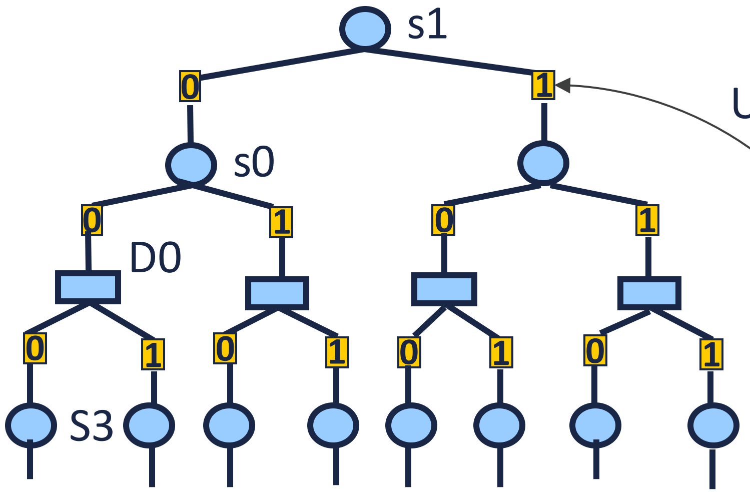
# Heuristic AND/OR Search with Decomposition Bounds

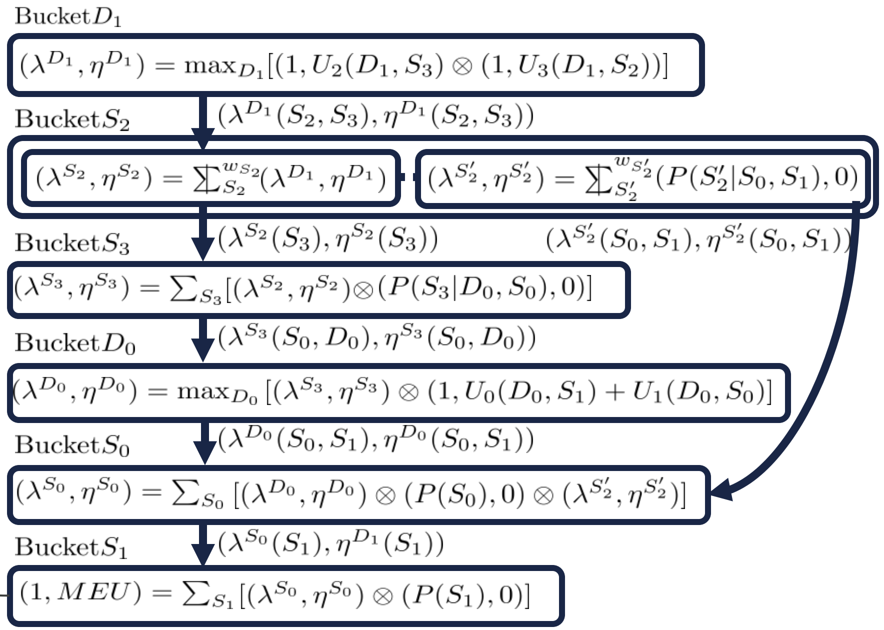
**We present initial result of AND/OR search for influence diagrams guided by weighted mini-bucket heuristics**




$$\sum_{S_0,S_1} \max_{D_0} \sum_{S_2,S_3} \max_{D_1} [\prod_{P_i \in \mathbf{P}} P_i][\sum_{U_i \in \mathbf{U}} U_i][\prod_{\Delta_i \in \mathbf{\Delta}} \Delta_i]$$




Upper bounds



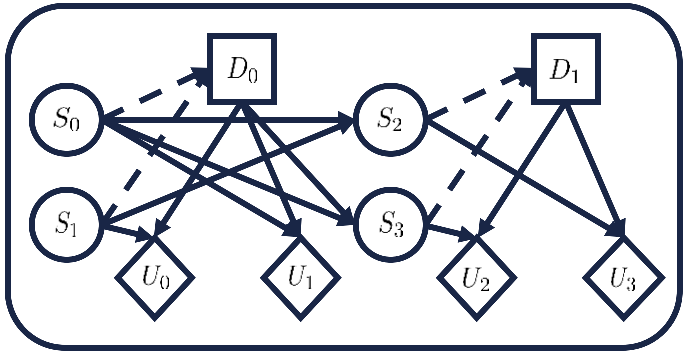

[Marinescu 2010]

AND/OR Search Graph for Influence Diagrams

[Lee, et.al 2019]

Weighted Mini-bucket elimination bound for Influence Diagrams

## Influence Diagram [Howard, Matheson 1981]




[Jensen, et al 1994, Maua, et. al 2012]

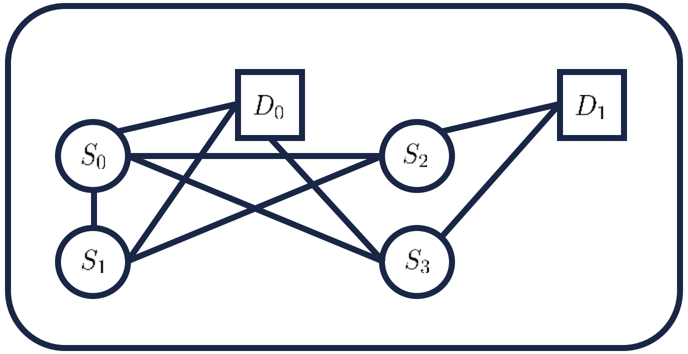
## Valuation Algebra for Influence Diagrams

$$\mathbf{\Psi} := \{(P_i, 0) | P_i \in \mathbf{P}\} \cup \{(1, U_i) | U_i \in \mathbf{U}\}$$

$$\Psi_1 \otimes \Psi_2 := (P_1 P_2, P_1 V_2 + P_2 V_1)$$

$$\textstyle\sum_{\mathbf{Y}}^{\mathbf{w}} \Psi := (\sum_{\mathbf{Y}}^{\mathbf{w}} P, \sum_{\mathbf{Y}}^{\mathbf{w}} V)$$

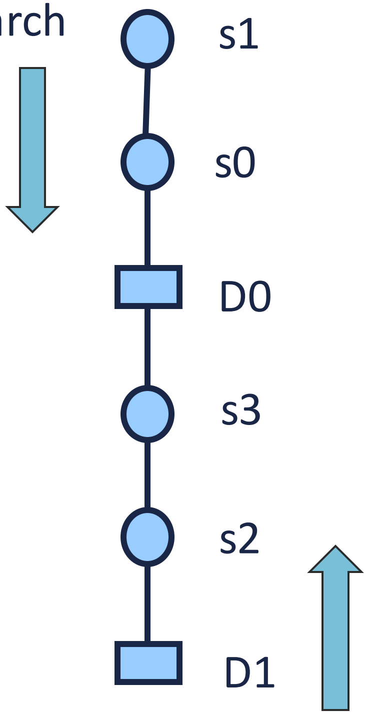
## Primal Graph




[Freuder, Quinn 1985]

## Pseudo-tree

Search

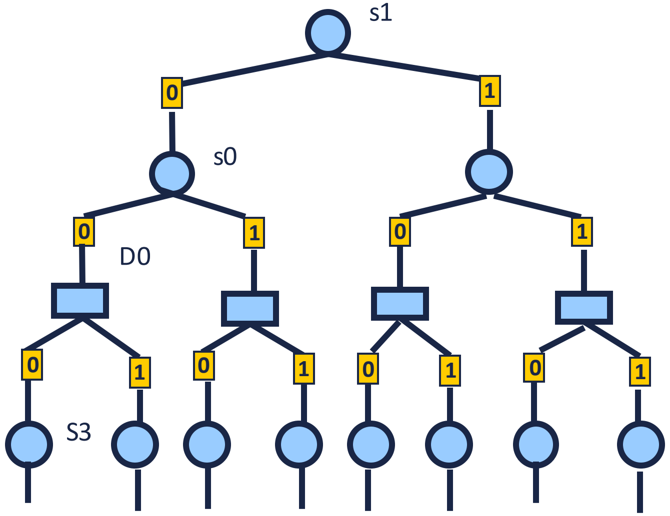



Inference [Dechter 1999]

By bucket elimination

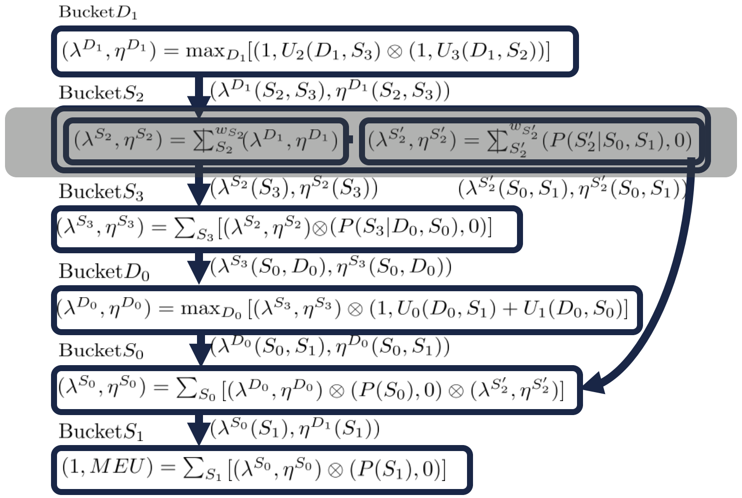
[Dechter, et al 2007, Marinescu, et al2010]

## AND/OR Search Space




[Dechter, et al 2003, Liu, et al 2011, Lee, et al 2019]

## Decomposition bounds from bucket tree



# Experiment Results

| Instances | $n, f, w, i$ | WMBE | OPT | AO | AOBB+MBE | AOBB+WMBE |
|---|---|---|---|---|---|---|
| SA1-T5 | 130,180,20,10 | 98.4 | 96.6 | 81 | 60 | **13** |
| SA1-T10 | 250,350,20,10 | 197.2 | 183.5 | 180 | 164 | **31** |
| SA2-T5 | 190,265,30,15 | 147.5 | 139.7 | 998(m) | 2218(m) | **2062** |
| SA2-T10 | 365,515,30,15 | 295.7 | NA | 1037(m) | 2518(m) | 3597(m) |

- System Admin MDP 10,15 servers up to 5, 10-time horizons
- n: number of variables, f: number of functions, w: constrained induced width, i: i-bound for WMBE
- WMBE upper bound of optimal MEU, AO: time in seconds by AND/OR graph search
- AOBB+MBE: time in seconds by AND/OR branch and bound search with Mini-bucket heuristic
- AOBB+WMBE: time in seconds by AND/OR branch and bound search with Weighted mini-bucket heuristic

# Conclusion

- AND/OR search with decomposition-based heuristic for solving influence diagrams.
- AOBB-WMBE improved search compared with AO or AOBB-MBE
- Future work: develop efficient search strategies and tighter variational bounds