

Scaling Up Abstraction Sampling

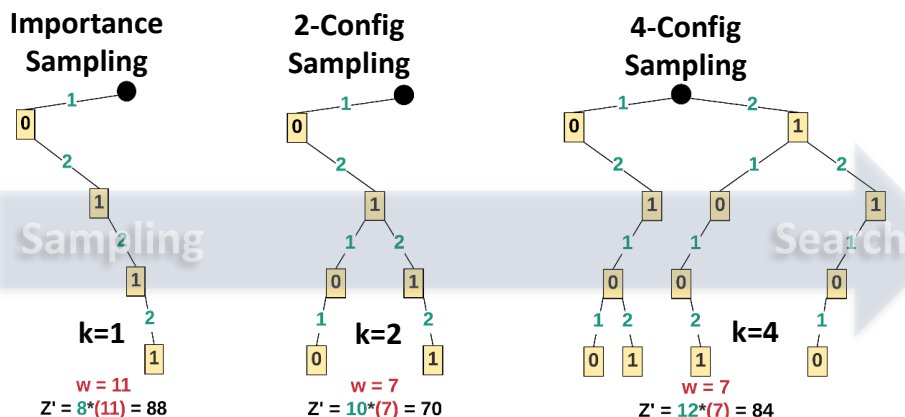
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Summary

We present AOAS: an unbiased and scalable AND/OR Abstraction Sampling scheme for computing the partition function (Z)

$$Z = \sum_X \prod_{\alpha} f_{\alpha}(X_{\alpha})$$

Interpolating Between Sampling and Search

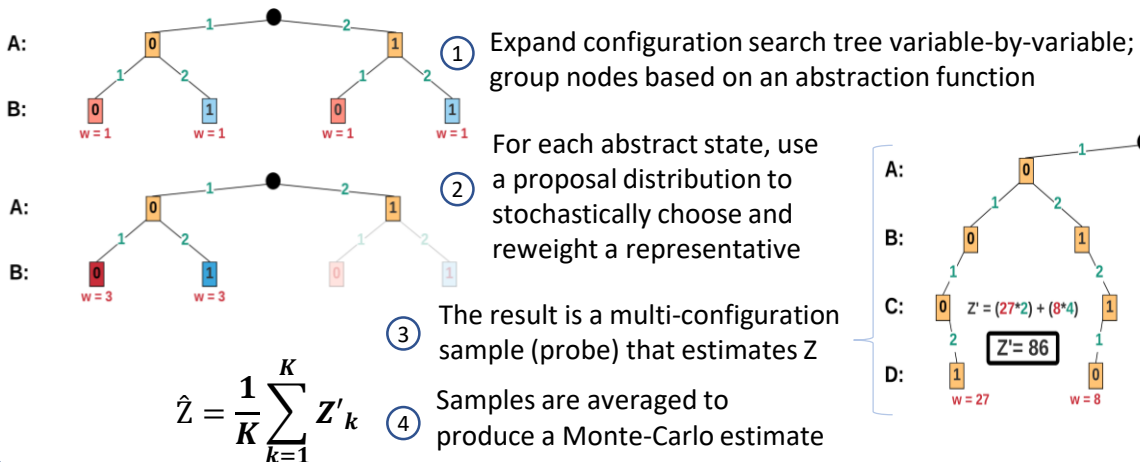


$$\hat{Z} = \frac{k}{N} \sum_{i=1}^{N/k} Z'(i)$$

Variance Reduction:
 $k^2 \text{Var}(\hat{Z})$

[Rizzo, 2007]

Abstraction Sampling



$$\hat{Z} = \frac{1}{K} \sum_{k=1}^K Z'_k$$

Algorithm 1: AOAS.

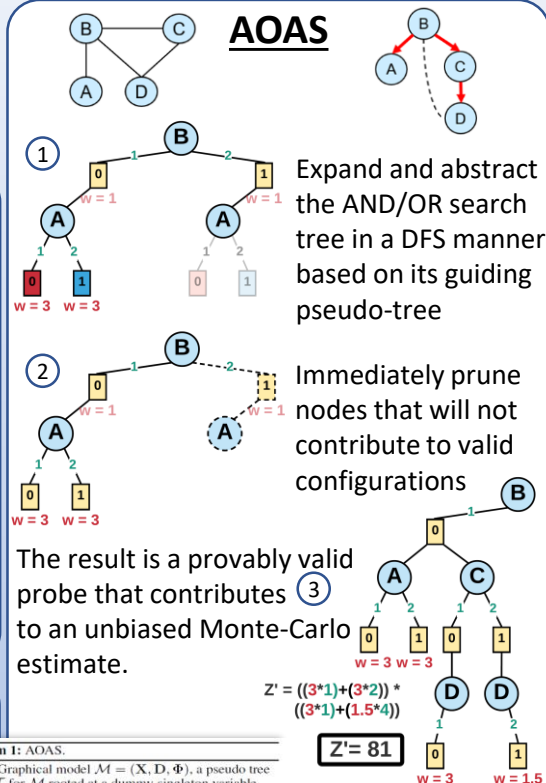
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Input: Graphical model  $\mathcal{M} = (X, D, \Phi)$ , a pseudo tree  $T$  for  $\mathcal{M}$  rooted at a dummy singleton variable  $D$ , an abstraction function  $a$ , heuristic function  $h$ . For any node  $n$ ,  $g(n)$  is its path cost,  $w(n)$  is its importance weight, and  $\hat{V}(n)$  is its estimated value (initialized to  $h(n)$ ).
Output:  $\hat{Z}_{\mathcal{M}}$ , an estimate of the partition function of  $\mathcal{M}$ 

Function AOAS( $T, h, a$ )
begin
   $PROBE \leftarrow n_D, g(n_D), w(n_D), r(n_D), \hat{V}(n_D) + 1$ 
   $STACK \leftarrow push(empty\ stack, D)$ 
  while  $STACK$  is not empty do
     $X \leftarrow top(STACK)$ 
    if  $X$  has unvisited children in  $T$  then
       $Y \leftarrow$  the next unvisited child of  $X$ 
       $PROBE \leftarrow PROBE$  expanded from  $n_X$  to  $Y$ 
       $F_Y \leftarrow$  newly added AND nodes of  $Y \in PROBE$ 
      foreach  $n_Y \in F_Y$  do
         $w(n_Y) \leftarrow w(n_X)$ 
         $g(n_Y) \leftarrow g(n_X) + c(n_Y)$ 
         $r(n_Y) \leftarrow r(n_X)$ 
         $\hat{V}(n_Y) \leftarrow \hat{V}(n_X) \cdot \prod_{(S \neq Y) \in ch_Y(X)} \hat{V}(S_{n_X})$ 
      end
    end
     $\hat{Z}_{\mathcal{M}} \leftarrow \hat{Z}_{\mathcal{M}} + \hat{V}(D)$ 
  end
  return  $\hat{Z}_{\mathcal{M}}$ 
end
foreach  $A_i \in \mathcal{A}_i$  do
  foreach  $n \in A_i$  do
     $p(n) \leftarrow \frac{w(n) \cdot g(n) \cdot h(n) \cdot r(n)}{\sum_{m \in A_i} w(m) \cdot g(m) \cdot h(m) \cdot r(m)}$ 
  end
   $n_{Y_i} \propto_p A_i$  // randomly select
   $w(n_{Y_i}) \leftarrow w(n_{Y_i}) / p(n_{Y_i})$ 
   $\hat{V}(n_{Y_i}) \leftarrow 1$ 
   $PROBE \leftarrow PROBE \cup A_i \cup \{n_{Y_i}\}$ 
end
 $pop(STACK), W \leftarrow top(STACK)$ 
 $PROBE \leftarrow PROBE$  s.t. all  $n_W$  without descendants are pruned
foreach  $n_W$  in  $PROBE$  do
   $\hat{V}(n_W) \leftarrow \hat{V}(n_W) \cdot \sum_{n_X \leftarrow child(n_W)} \hat{V}(n_X) \cdot c(n_X) \cdot \frac{w(n_X)}{w(n_W)}$ 
end
end
if  $X = D$  then  $\hat{Z}_{\mathcal{M}} = \hat{V}(D)$ ;
end
end
return  $\hat{Z}_{\mathcal{M}}$ 
end

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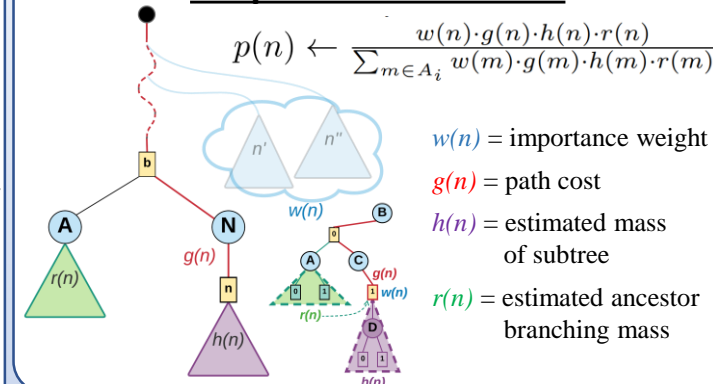
AOAS



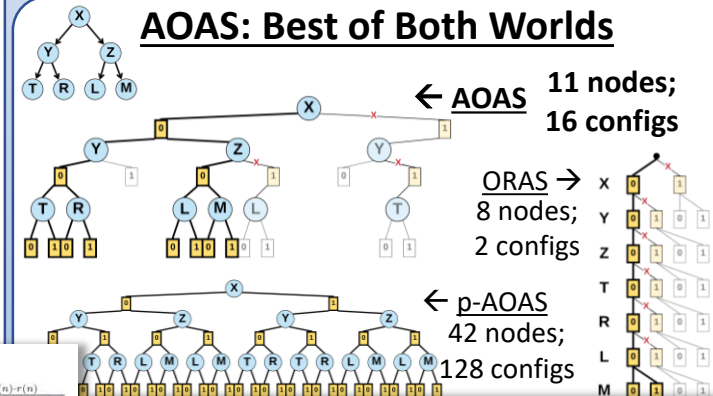
$$Z' = ((3*1)+(3*2)) * ((3*1)+(1.5*4))$$

$Z' = 81$

Proposal Distribution



AOAS: Best of Both Worlds



Bmk	Sz	Graph Scheme	Context Scheme	Abs	DIS Unable to Solve (3600 sec)						
					n*	log(err)	error distr.	#probes	#nodes/probe		
Linkage-Type4 [LARGE, n:10822, d:1.8, w:24, h:581/5745]	AOAS	RelCB		0	7	-32.487	0	0	0	5.52E+05	829
				4	26	-14.174	1	1	10	1.92E+05	16472
		RandCB		8	41	-11.090	3	10	26	1.20E+04	463468
				16	26	-18.582	0	0	0	2.50E+05	10532
	pAOAS (k=5)	RelCB		0	8	-27.742	0	0	0	1.14E+07	759
				2	13	-27.201	0	0	1	6.96E+04	280176
		RandCB		4	11	-25.427	0	0	0	8.90E+02	34177733
				16	12	-24.204	0	0	1	4.24E+03	8820722
	ORAS-DFS	RelCB		256	14	-30.930	0	0	0	2.58E+03	24532116
				0	7	-29.820	0	0	0	1.42E+06	2520
		RandCB		4	15	-20.802	0	0	2	5.14E+04	26863
				8	16	-18.971	0	0	5	2.44E+03	355404