BREN:ICS INFORMATION AND COMPUTER SCIENCES

UNIVERSITY *of* CALIFORNIA IRVINE

Decomposition Bounds for Influence Diagrams

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Final Defense

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Outline

Contributions

Backgrounds

- Basic Concepts
- Variational Upper Bounds in GMs
- Earlier Bounding Methods for IDs

Direct Decomposition Bounds for IDs: Algorithms

- Using Valuation Algebra for IDs
- Using Exponentiated Utility Functions

Direct Decomposition Bounds for IDs: Evaluation

- Synthetic Domains
- SysAdmin MDP/POMDP Domains

Contributions

- Decomposition Bounds for Influence Diagrams

Bounding Methods \ Decomposition Methods	Propagate Message over Join-Graph	Propagate Message over Mini-bucket Tree
Use Valuation Algebra	JGD-ID (GDD)	WMBE-ID (WMB/GDD)
Use Exponentiated Utility Functions	JGD-EXP (GDD)	WMBMM-EXP (WMBMM)

- AND/OR Search for Marginal MAP
  - Empirical Study on MMAP Search Algorithms

Background

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- Basic Concepts
  - Influence Diagrams
  - Graphical Models
  - Probabilistic Reasoning Problem
- Variational Upper Bounds in GMs
  - Mini-bucket based methods
  - Dual Decomposition based methods
- Earlier Bounding Methods for IDs
  - Reduction to MMAP

Influence Diagrams (IDs) [Howard, Matheson, 1984]

Basic Concepts and Notations

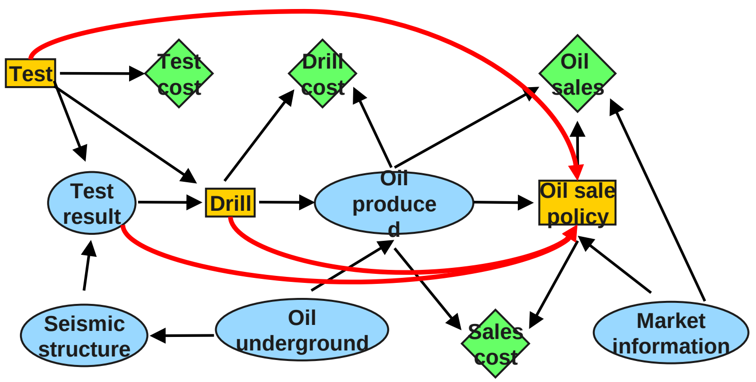
Influence Diagram $\mathcal{M} := \langle \mathbf{X}, \mathbf{D}, \mathbf{P}, \mathbf{U}, \mathcal{O} \rangle$

Chance variables $\mathbf{X} = \{X_1, X_2, \ldots, X_n\}$

Decision variables $\mathbf{D} = \{D_1, D_2, \ldots, D_m\}$

Probability functions $\mathbf{P} = \{P_1, P_2, \ldots, P_n\}$

Utility functions $\mathbf{U} = \{U_1, U_2, \ldots, U_r\}$



Precedence relations $\mathcal{O} = \{\mathrm{pa}(D_1) \prec D_1 \prec \cdots \prec \mathrm{pa}(D_m) \prec D_m\}$

Policy functions $\mathbf{\Delta} = \{\Delta_1, \ldots \Delta_m\}$ $\Delta_i(D_i | \mathrm{hist}(D_i))$

Maximum expected Utility $\max_{\mathbf{\Delta}} \mathbb{E}_{P(\mathbf{X},\mathbf{D})}[\sum_{U_i \in \mathbf{U}} U_i]$ $\mathbf{\Delta}^* = \mathrm{argmax}_{\mathbf{\Delta}} \mathbb{E}[\sum_{U_i \in \mathbf{U}} U_i]$

$$P(\mathbf{X}, \mathbf{D}) = \prod_{P_i \in \mathbf{P}} P_i \times \prod_{\Delta_i \in \mathbf{\Delta}} \Delta_i$$

Graphical Models

Graphical model $\mathcal{M} := \langle \mathbf{X}, \mathbf{D}, \mathbf{F} \rangle$

- Variables $\mathbf{X} = \{X_1, X_2, \ldots, X_n\}$
- Domains $\mathbf{D} = \{D_1, D_2, \ldots, D_n\}$
- Functions $\mathbf{F} = \{F_1, F_2, \ldots, F_r\}$

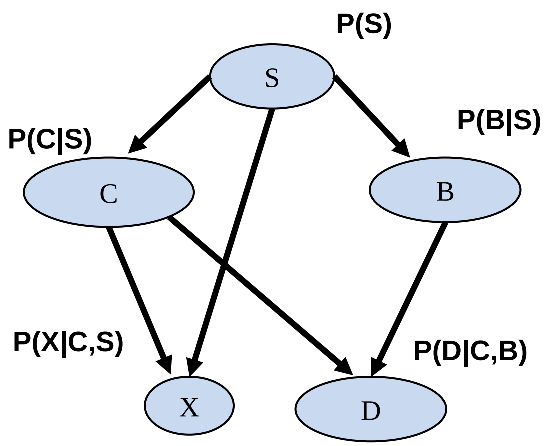
Combination operator: $\otimes, \times, +, \bowtie$

Elimination operator: $\sum, \max, \min$

$$F(\mathbf{X}) = \prod_{F_i \in \mathbf{F}} F_i(\mathbf{X}_{F_i})$$

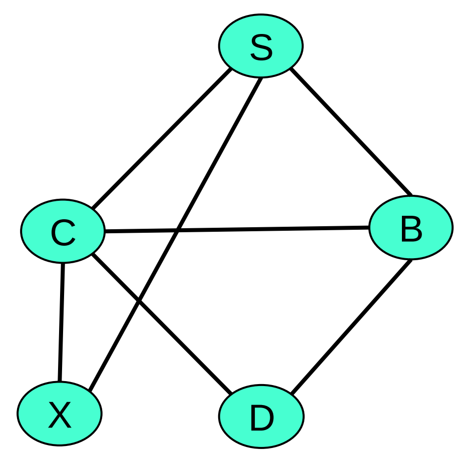
$$Z = \sum_{\mathbf{X}} \prod_{F_i \in \mathbf{F}} F_i(\mathbf{X}_{F_i})$$

Bayesian Networks [Pearl 1998]



P(S, C, B, X, D) = P(S) P(C|S) P(B|S) P(X|C,S) P(D|C,B)

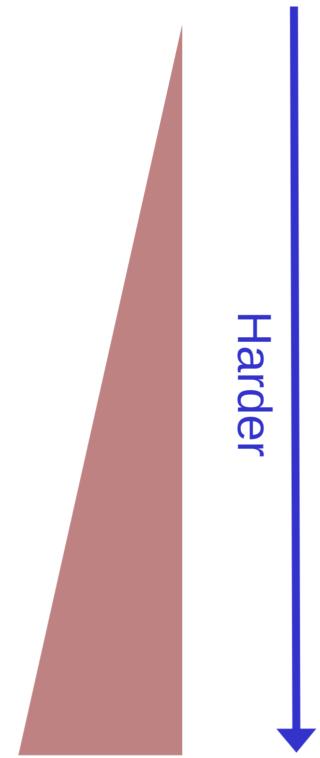
Primal Graph

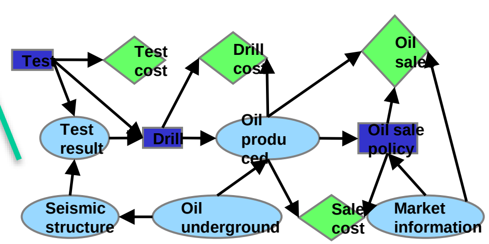


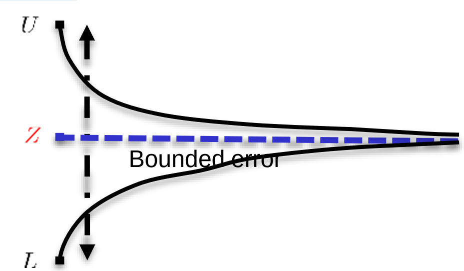
Probabilistic Reasoning Problems

Exact Algorithm: Bucket Elimination, Complexity $k^{\text{tree-width}}$

Max-Inference (most likely config.)	$f(\mathbf{x}^*) = \max_{\mathbf{x}} \prod_{\alpha} f_\alpha(\mathbf{x}_\alpha)$
Sum-Inference (data likelihood)	$Z = \sum_{\mathbf{x}} \prod_{\alpha} f_\alpha(\mathbf{x}_\alpha)$
Mixed-Inference (optimal prediction)	$f(\mathbf{x}_M^*) = \max_{\mathbf{x}_M} \sum_{\mathbf{x}_S} \prod_{\alpha} f_\alpha(\mathbf{x}_\alpha)$
Mixed-Inference (maximum expected utility)	$\mathrm{MEU} = \max_{\mathbf{\Delta}} \mathbb{E}_{P(\mathbf{X},\mathbf{D})}[\sum_{U_i \in \mathbf{U}} U_i]$







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Variational Upper Bounds in GM

Max-Inference $$f(\mathbf{x}^*) = \max_{\mathbf{x}} \prod_{\alpha} f_\alpha(\mathbf{x}_\alpha)$$

[Komodakis, Paragios, Tziritas, **Dual Decomposition**, 2007
[Sontag, Meltzer, Globerson, Jakkola, Weiss, LP relaxation for MAP, 2008]
[Ihler, Flerova, Dechter, Otten, **Join-Graph** LP, **MBE-Moment Matching** 2012]

Sum-Inference $$Z = \sum_{\mathbf{x}} \prod_{\alpha} f_\alpha(\mathbf{x}_\alpha)$$

[Wainwright, Jakkola, Wilsky, Tree-Reweighted BP, 2003]
[Liu and Ihler, **Weighted Mini-bucket**, 2011]

Mixed-Inference $$f(\mathbf{x}_M^*) = \max_{\mathbf{x}_M} \sum_{\mathbf{x}_S} \prod_{\alpha} f_\alpha(\mathbf{x}_\alpha)$$

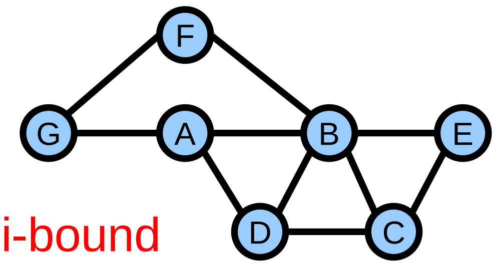
[Liu and Ihler, Mixed-product BP, 2013]
[Marinescu, Ihler, Dechter, **WMB for MMAP, WMB-Moment Matching**, 2014]
[Ping, Liu and Ihler, **Generalized Dual Decomposition** for MMAP, 2015]

Bucket and Mini-Bucket Elimination

[Dechter 1999; Dechter & Rish, 2003, Liu & Ihler 2011]

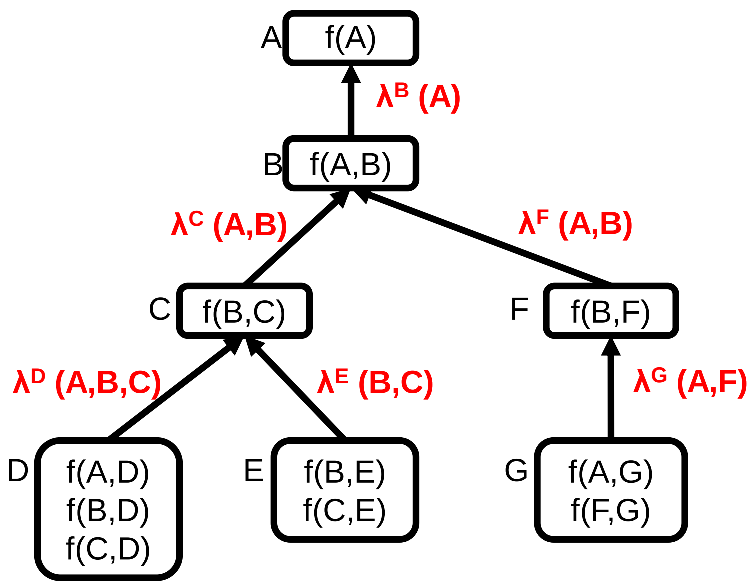
$$\sum_X F(X)$$

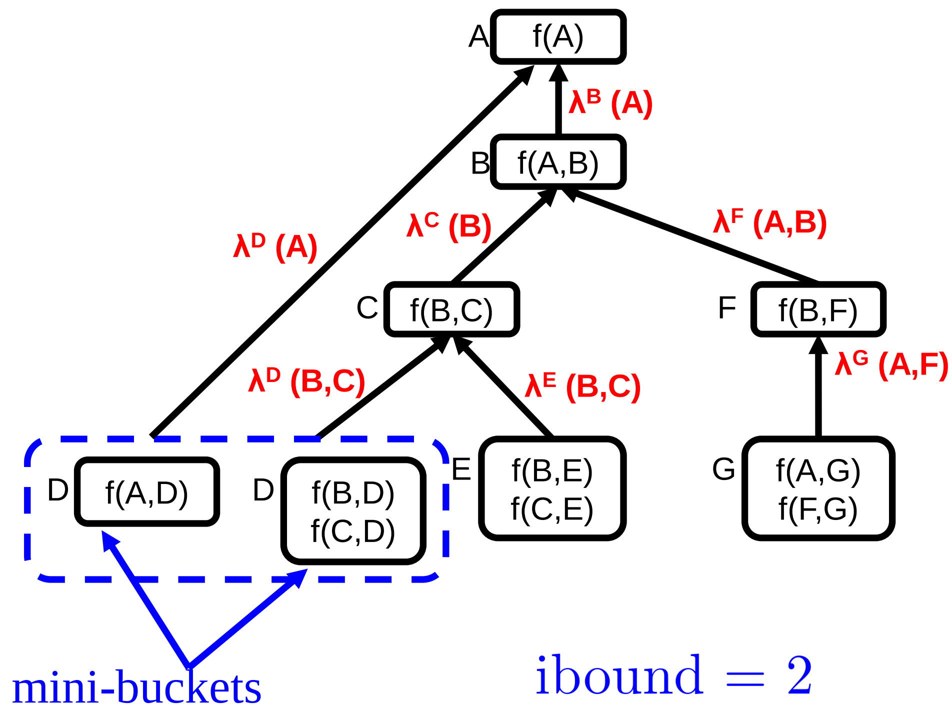
$$F(\mathbf{X}) = f(A)f(A,B)f(A,D)f(A,G)f(B,C)f(B,D)$$
$$f(B,E)f(B,F)f(C,D)f(C,E)f(E,G)$$



Time and space exponential in the induced-width/tree-width w*

Exponential in i-bound

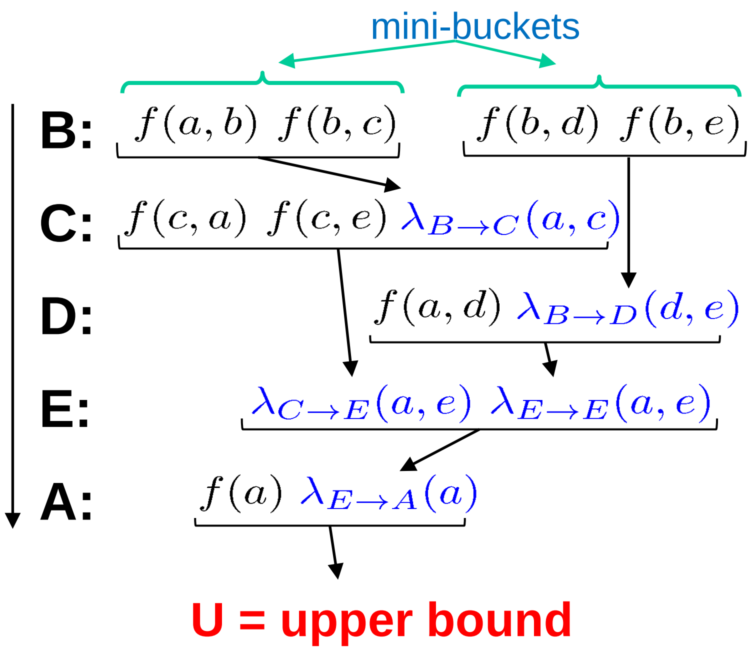




$\text{ibound} = 2$

Weighted Mini-bucket Elimination and Powered-Summation

[Liu & Ihler 2011]



Mini-bucket relaxation via Holder's inequality:

$$\sum_x f_1(x)\cdot f_2(x) \quad < \quad \Big[\sum_x f_1(x)^{\frac{1}{w_1}}\Big]^{w_1} \cdot \Big[\sum_x f_2(x)^{\frac{1}{w_2}}\Big]^{w_2}$$

$$w_1 + w_2 = 1$$

Define Elimination Operator using Powered-Summation

$$\sum_{x_1}^{w_1} f(x_1) - \Big[\sum_{x_1} f(x_1)^{\frac{1}{w_1}}\Big]^{w_1}$$

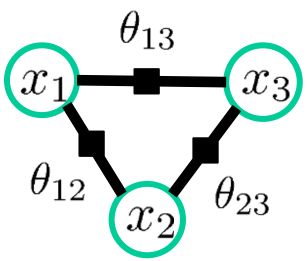
Weighted Mini-bucket

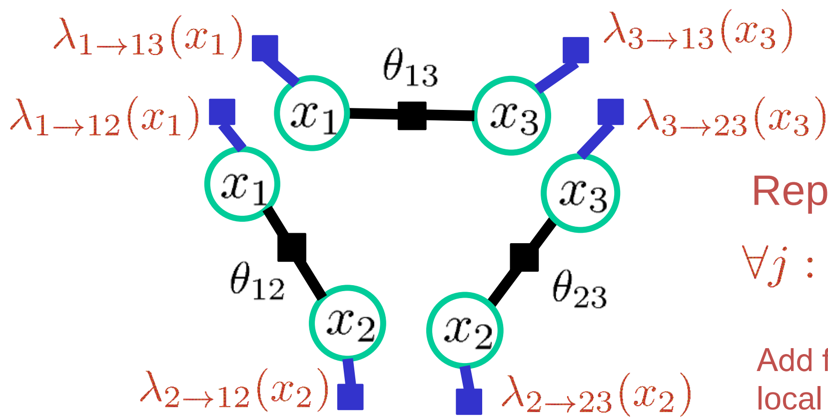
- Generalize Mini-bucket with "weights"
- Tighter bounds for sum-inference
- Powered-sum unifies max and sum operation

$$\lim_{w\to 0+} \sum_x^w f(x) = \max_x f(x)$$

Dual Decomposition for Max-Inference

[Komodakis et. al. 2007]





Reparameterization:

$$\forall j : \sum_{\alpha \ni j} \lambda_{j\to\alpha}(x_j) = 0$$

Add factors that "adjust" each local term, but cancel out in total

$$\log f(\mathbf{x}^*) = \max_{\mathbf{x}} \sum_{\alpha} \theta_\alpha(\mathbf{x}_\alpha) \le \min_{\{\lambda_{i\to\alpha}\}} \sum_{\alpha} \max_{\mathbf{x}_\alpha} \Big[\theta_\alpha(\mathbf{x}_\alpha) + \sum_{i\in\alpha} \lambda_{i\to\alpha}(x_i)\Big]$$

Bound solution using decomposed optimization

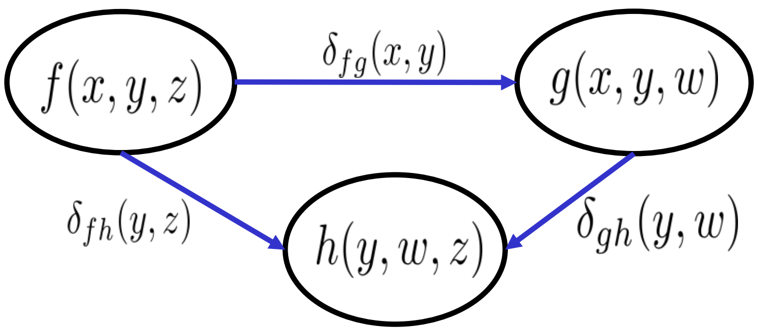
Solve independently: optimistic bound

Tighten the bound by re-parameterization

Enforces equality constraints using Lagrange multipliers

Generalized Dual Decomposition for MMAP [Ping, Liu, Ihler 2015]

Generalizes "Dual decomposition" to the summation and maximization by the powered-summation $\sum_{x}^{w} f(\mathbf{x}) := [\sum_{x} f(\mathbf{x})^{\frac{1}{w}}]^{w}$



$$\max_{x,y} \sum_{w,z} f(x,y,z) + g(x,y,w) + h(y,w,z)$$

$$\leq \min_{\boldsymbol{\delta},\mathbf{w}} \Big[\sum_{x_f}^{w_{x_f}} \sum_{y_f}^{w_{y_f}} \sum_{z_f}^{w_{z_f}} [f(x_f,y_f,z_f) - \delta_{fg} - \delta_{fh}] +$$

$$\sum_{x_g}^{w_{x_g}} \sum_{y_g}^{w_{y_g}} \sum_{w_g}^{w_{w_g}} [g(x_g,y_g,w_g) + \delta_{fg} - \delta_{gh}] +$$

$$\sum_{y_h}^{w_{y_h}} \sum_{w_h}^{w_{w_h}} \sum_{z_h}^{w_{z_h}} [h(y_h,w_h,z_h) + \delta_{fh} + \delta_{gh}] \Big]$$

Tighten the upper bound by optimizing the **weights** and **reparameterization**

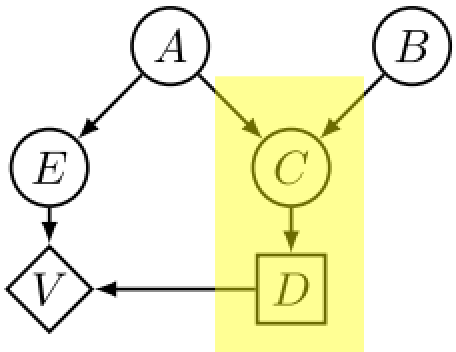
$$w_{x_f} + w_{x_g} = 0 \qquad w_{y_f} + w_{y_g} + w_{y_h} = 0$$

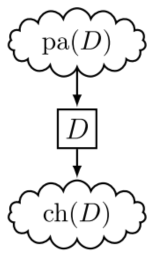
$$w_{w_g} + w_{w_h} = 1 \qquad w_{z_f} + w_{z_h} = 1$$

all $w > 0$

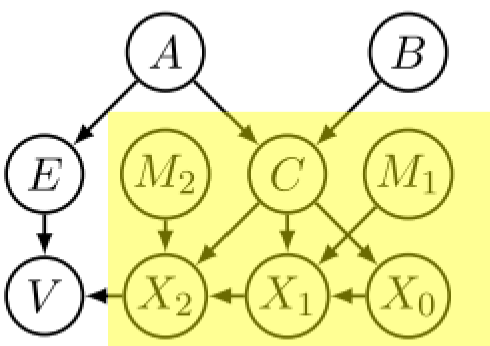
Translate IDs to BNs and Apply MMAP Inference

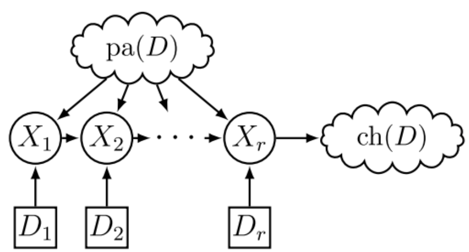
Polynomial Reductions [Maua, Equivalence between MMAP and MEU, 2016]





$$\text{MEU} = \sum_{C} \max_{D} \sum_{A,B,E} [P(A)P(B)P(C|A,B) \; P(E|A)V(E,D)]$$





$$\text{MMAP} = \max_{M_1,M_2} \sum_{A,B,C,X_0,X_1,X_2,E} [P(A)P(B)P(C|A,B) \; P(E|A)P(M_1)P(M_2)P(X_0) \; P(X_1|X_0,M_1,C) \; P(X_2|X_1,M_2,C)V(E,X_2)]$$

Practically, reduction schemes make inference task intractable!

Auxiliary variables and functions $O(|\text{Decision Variables}| \cdot k^{\text{pa}(|\text{Decision Variables}|)})$

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Direct Decomposition Bounds for IDs: Algorithms

- Using Valuation Algebra for IDs
  - Extending Powered Summation in Valuation Algebra
  - Decomposition Bounds based on Valuation Algebra
  - Join-Graph Decomposition Bounds for IDs
  - Weighted Mini-bucket Elimination Bounds for IDs
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  - Bounding MEU by MMAP Task
  - Combining GDD Bounds for MMAP
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Direct Decomposition Bounds for Influence Diagrams:

Algorithms

Contributions

Mixed-Inference
(maximum expected utility)

$$\text{MEU} = \max_{\mathbf{\Delta}} \mathbb{E}_{P(\mathbf{X},\mathbf{D})}[\textstyle\sum_{U_i \in \mathbf{U}} U_i]$$

How to extend variational upper bounds to IDs?

Bounding Methods \ Decomposition Methods	Propagate Message over Join-Graph	Propagate Message over Mini-bucket Tree
Use Valuation Algebra	JGD-ID (GDD)	WMBE-ID (WMB/GDD)
Use Exponentiated Utility Functions	JGD-EXP (GDD)	WMBMM-EXP (WMBMM)

Decomposition Methods in IDs

Bounding Methods \ Decomposition Methods	Propagate Message over Join-Graph	Propagate Message over Mini-bucket Tree
Use Valuation Algebra	JGD-ID (GDD)	WMBE-ID (WMB/GDD)
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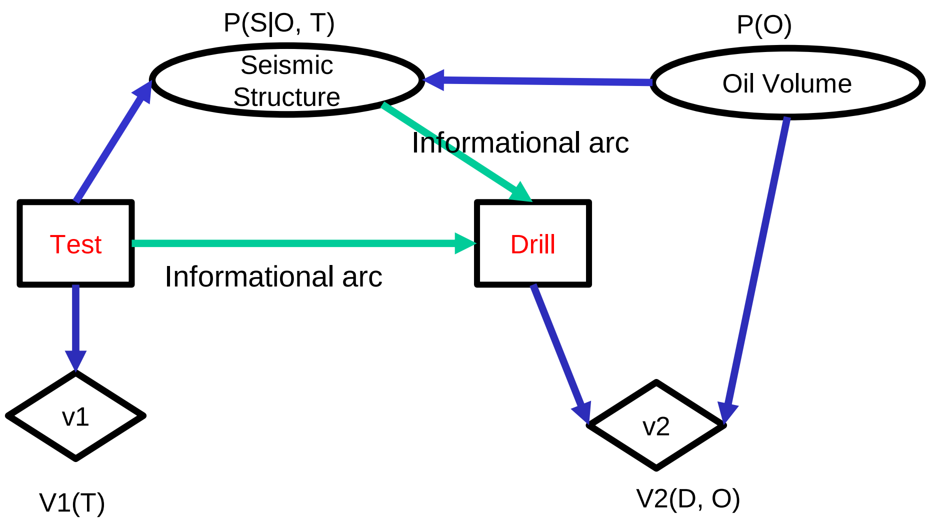
Valuation Algebra for IDs

Simplified Oil Wildcatter problem



$$\max_{\mathbf{T}} \sum_{\mathbf{S}} \max_{\mathbf{D}} \sum_{\mathbf{O}} P(S|O,T)P(O)[V_1(T) + V_2(D,O)]$$

How to handle additive utility functions?

Valuation Algebra for IDs

Jensen's Potential: $\Psi(\mathbf{X}_\Psi) = (P(\mathbf{X}_\Psi), V(\mathbf{X}_\Psi))$ [Jensen 1994]

Combination: $\Psi_1(\mathbf{X}_1) \otimes \Psi_2(\mathbf{X}_2) := (P_1(\mathbf{X}_1)P_2(\mathbf{X}_2), P_1(\mathbf{X}_1)V_2(\mathbf{X}_2) + P_2(\mathbf{X}_2)V_1(\mathbf{X}_1))$

Marginalization: $\sum_{\mathbf{Y}} \Psi(\mathbf{X}) := (\sum_{\mathbf{Y}} P(\mathbf{X}), \sum_{\mathbf{Y}} V(\mathbf{X}))$

$\max_{\mathbf{Y}} \Psi(\mathbf{X}) := (\max_{\mathbf{Y}} P(\mathbf{X}), \max_{\mathbf{Y}} V(\mathbf{X}))$

Comparison: $\Psi_1(\mathbf{X}_1) \leq \Psi_2(\mathbf{X}_2) \iff P_1(\mathbf{X}_1) \leq P_2(\mathbf{X}_2) \;\&\; V_1(\mathbf{X}_1) \leq V_2(\mathbf{X}_2).$

Maximum Expected Utility (MEU) Task

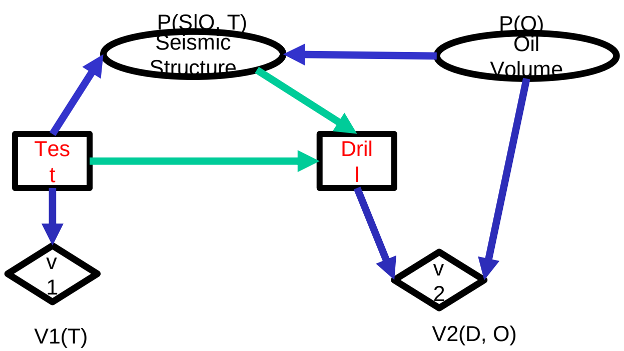
$$\sum_{\mathrm{pa}(D_1)} \max_{D_1} \cdots \sum_{\mathrm{pa}(D_M)} \max_{D_M} \sum_{\mathbf{X} \setminus \mathrm{pa}(D_M)} \bigotimes_{\alpha \in \mathcal{I}_{\mathbf{\Psi}}} \Psi_\alpha(\mathbf{X}_\alpha),$$

$$\mathbf{\Psi} = \{(P_i, 0) | P_i \in \mathbf{P}\} \cup \{(1, U_i) | U_i \in \mathbf{U}\}.$$

The valuation algebra with Jensen's potential allows extending existing graphical model algorithms by extending "factors" to "potentials"

Bucket/Mini-Bucket Elimination for IDs with VA

BE: Time and space exponential
in the **constrained** induced-width/tree-width
MBE: Exponential in i-bound



i-bound = 2

Mini-bucket | Mini-bucket

O: $(p(O),0)\quad(p(S|O,T),0)$ | $(1, v_2(D,O))$

$\lambda_O(S,T) = \sum_O P(O)P(S|O,T)$ | $\theta_O(D) = \sum_O v_2(D,O)$

D: $(1, \theta_O(D))$

$\theta_D = \max_D \theta_O(D)$

S: $(\lambda_O(S,T), 0)$

$\lambda_S(T) = \sum_S \lambda_O(S,T)$

T: $(1, (v_1(T))\quad(\lambda_S(T),0)$

$\theta_T = \max_T \lambda_S(T) v_1(T)$

$(1,\theta_T) \otimes (1, \theta_D) \qquad MEU \le \theta_T + \theta_D$

$$\text{Expected Util}(\max_T \sum_S \max_D \sum_O (p(O),0) \otimes (p(S|O,T),0) \otimes (1, v_1(T)) \otimes (1, v_2(D,O))$$

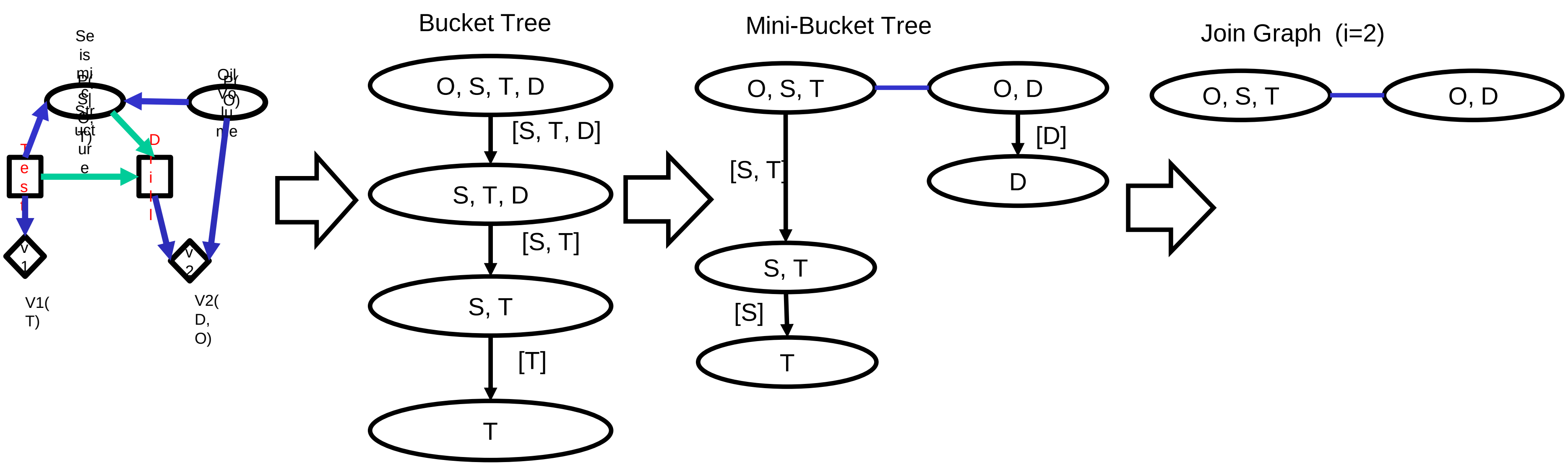
$$\le \text{Expected Util}(\max_T (1, v_1(T)) \otimes \sum_S \max_D \sum_{O_1} (p(O_1),0) \otimes (p(S|O_1,T),0) \otimes \sum_{O_2} (1, v_2(D, O_2))$$

Join-graph Decomposition for IDs with VA

Join graph decomposition

- Graph of Clusters and Separators
- Running intersection property

Join graph structuring from Mini-bucket Tree Example



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Motivations for Developing JGD-ID

Reduction to MMAP task with “auxiliary variables” and “relations” inflates the problem, making hard problems even harder

Don’t translate to Bayesian networks.

Reuse core components in variational decomposition bounds, such as decomposition methods and techniques of developing parameterized bounds

Extend valuation algebra to develop decomposition bounds for IDs

Extending Powered Sum to Valuation Algebra

$$Given\ \Psi(\mathbf{X}) := \big(P(\mathbf{X}), V(\mathbf{X})\big)$$

Component-wise elimination

$$\sum_{X}^{w} (P(\mathbf{X}), V(\mathbf{X})) := ([\sum_{X} |P(\mathbf{X})|^{1/w}]^{w}, [\sum_{X} |V(\mathbf{X})|^{1/w}]^{w})$$

This makes negative utility to positive!

Modified powered-sum for IDs, when computing **Expected Utility**

Shift utility by a constant and subtract it back

Truncate negative utility values

$$\sum_{\mathbf{X}}^{(\mathbf{w},A)} (P(\mathbf{X}), V(\mathbf{X})) := \Big(\sum_{\mathbf{X}}^{\mathbf{w}} P(\mathbf{X}), \sum_{\mathbf{X}}^{\mathbf{w}} h_{(P(\mathbf{X}),V(\mathbf{X}),A)}(\mathbf{X})\Big) \otimes (1, -A)$$

$$h_{(P(\mathbf{X}),V(\mathbf{X}),A)}(\mathbf{X}) = \begin{cases} P(\mathbf{X})(\frac{V(\mathbf{X})}{P(\mathbf{X})} + A) & if\ \frac{V(\mathbf{X}=\mathbf{x})}{P(\mathbf{X}=\mathbf{x})} + A > 0 \\ 0, & otherwise. \end{cases}$$

Decomposition Bounds for IDs with VA

Given an ID $\mathcal{M}' := \langle \mathbf{X}, \mathbf{\Psi}, \mathcal{O} \rangle$, MEU can be bounded by

$$(1, \mathrm{MEU}) := \sum_{\mathcal{O}}^{\mathbf{w}} \otimes_{\alpha \in \mathcal{I}_{\mathbf{\Psi}}} \Psi_{\alpha}(\mathbf{X}_{\alpha}) \leq \otimes_{\alpha \in \mathcal{I}_{\mathbf{\Psi}}} \sum_{\mathcal{O}}^{(\mathbf{w}^{\alpha},\, \mathbf{A})} \Psi_{\alpha}(\mathbf{X}_{\alpha})$$

Marginalize the global potential

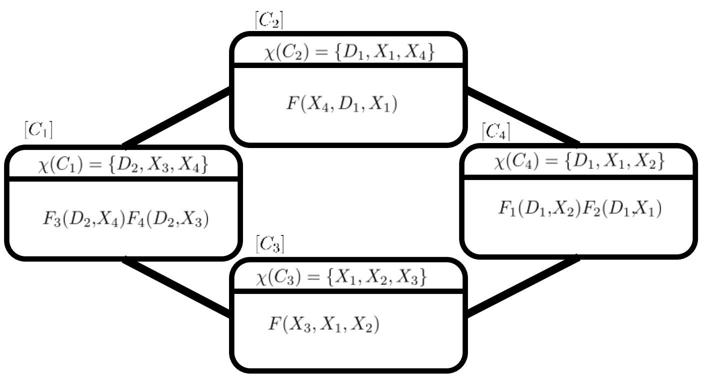
Combine fully marginalized local potentials

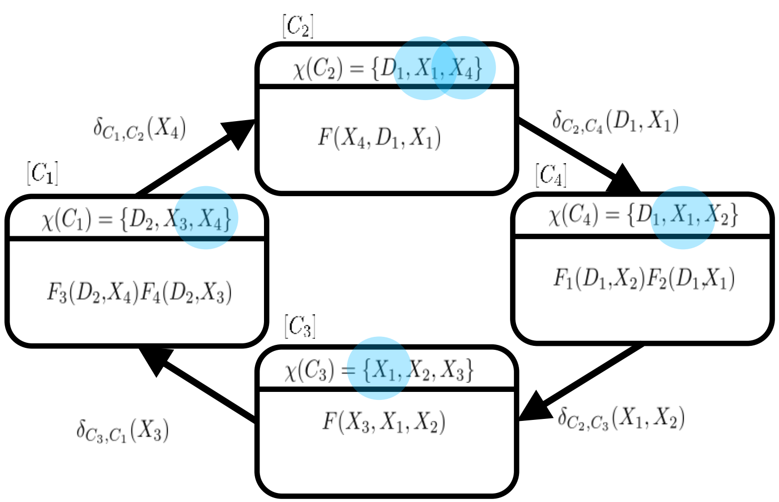
with $w_i = \sum_{\alpha \in \mathcal{I}_{\Psi}} w_i^{\alpha}$ for $\mathbf{w}$ and $\mathbf{w}^{\alpha}$ $\forall \alpha \in \mathcal{I}_{\mathbf{\Psi}}$

$$\mathbf{A} := \{ A_{\alpha} | \forall \alpha \in \mathcal{I}_{\mathbf{\Psi}} \}$$

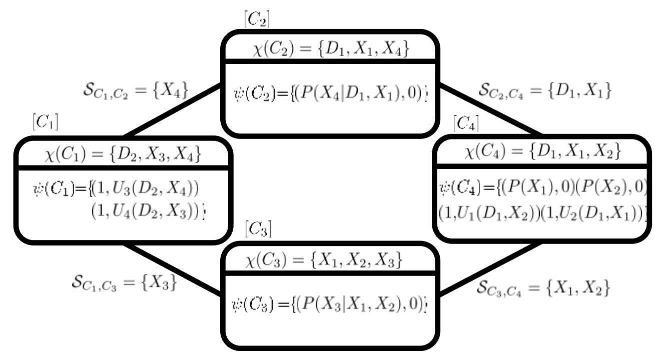
Parameterizing Decomposition Bounds for IDs over Join-Graph

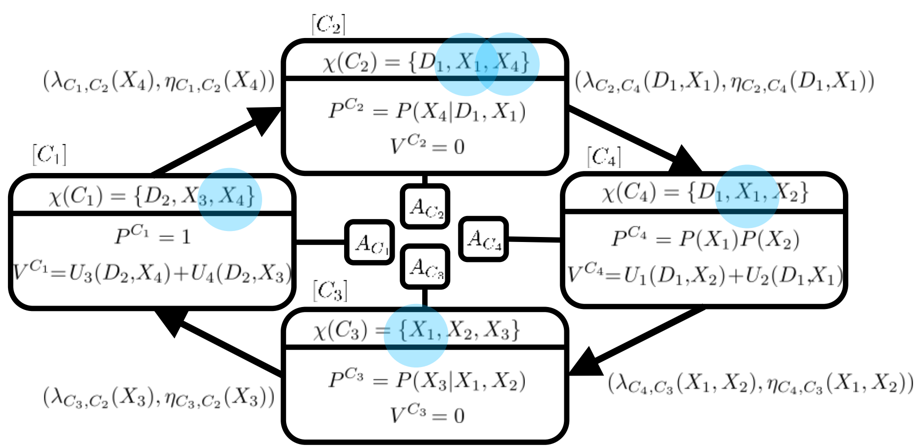
Generalized Dual Decomposition for MMAP





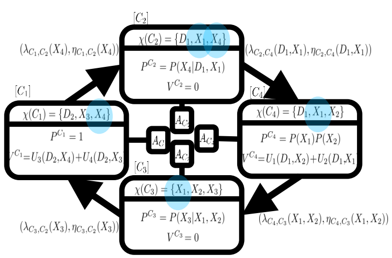
Generalized Dual Decomposition for MEU in IDs





Parameterizing Decomposition Bounds for IDs over Join-Graph

$$\min_{\mathbf{w},\boldsymbol{\lambda},\boldsymbol{\eta},\mathbf{A}} U_{\text{value}}$$



$$U_{value} = \sum_{C_i \in \mathcal{C}} \Big(\sum_{\mathcal{O}}^{\mathbf{w}^{C_i}} h_{(\tilde{P}_{C_i}(\mathbf{X}_{C_i}), \tilde{V}_{C_i}(\mathbf{X}_{C_i}), A_{C_i})}(\mathbf{X}_{C_i}) \Big) \cdot \Big(\prod_{C_j \neq C_i} \sum_{\mathcal{O}}^{\mathbf{w}^{C_j}} \tilde{P}_{C_j}(\mathbf{X}_{C_j}) \Big) - A^{C_i}$$

$$\tilde{P}_C(\mathbf{X}_{C_i}) = P_C(\mathbf{X}_{C_i}) \prod_{(C_i, C_j) \in \mathcal{S}} \lambda_{C_i,C_j}(\mathbf{X}_{C_i,C_j})$$

$$\tilde{V}_{C_i}(\mathbf{X}_{C_i}) = P_{C_i}(\mathbf{X}_{C_i}) \Big(\frac{V_{C_i}(\mathbf{X}_{C_i})}{P_{C_i}(\mathbf{X}_{C_i})} + \sum_{(C_i,C_j) \in \mathcal{S}} \frac{\eta_{C_i,C_j}(\mathbf{X}_{C_i,C_j})}{\lambda_{C_i,C_j}(\mathbf{X}_{C_i,C_j})} \Big)$$

Optimization parameters $\mathbf{w}^{C_i}$ for all $C_i \in \mathcal{C}$

$$\Psi_{C_i,C_j}(\mathbf{X}_{C_i,C_j}) \triangleq (\lambda_{C_i,C_j}(\mathbf{X}_{C_i,C_j}), \eta_{C_i,C_j}(\mathbf{X}_{C_i,C_j})).$$

A^{C_i} for all $C_i \in \mathcal{C}$

Constraints

$$\lambda_{C_i,C_j}(\mathbf{X}_{C_i,C_j}) \in \mathcal{R}^{|\mathbf{X}_{C_i,C_j}|} \quad \eta_{C_i,C_j}(\mathbf{X}_{C_i,C_j}) \in \mathcal{R}^{|\mathbf{X}_{C_i,C_j}|}$$

$$\Psi_{C_i,C_j}(\mathbf{X}_{C_i,C_j}) \bigotimes \Psi_{C_j,C_1}(\mathbf{X}_{C_i,C_j}) = (\mathbf{1}(\mathbf{X}_{C_i,C_j}), \mathbf{0}(\mathbf{X}_{C_i,C_j})) \quad \forall (C_i, C_j) \in \mathcal{S}.$$

$$w_{X_k} > 0 \text{ for all } w_{X_k} \in \mathbf{w}^{C_i} \quad w_{X_k} = \sum_{C_i \in \mathcal{C}} w_{X_k}^{C_i}$$

$$A^{C_i} \in \mathcal{R}$$

Algorithm JGD-ID

Algorithm 3.2 Join-Graph GDD Bounds for IDs (JGD-ID)

Require: ID $\mathcal{M} = \langle \mathbf{X}, \mathbf{D}, \mathbf{\Psi}, \mathcal{O} \rangle$, weights w_{X_i} associated with each variable $X_i \in \mathbf{X}$, i-bound, iteration limit M for the outer-loop of BCD.

Ensure: an upper bound of the MEU, U_{value}

Initialization Steps of BCD

Generate a join-graph decomposition $\mathcal{G}_{\text{JG}} = \langle G(\mathcal{C}, \mathcal{S}), \chi, \psi \rangle$ with the i-bound

Execute a single pass cost-shifting by the messages generated by MBE algorithm.

Initialize the weights for each chance variable $X_i \in \mathbf{X}$ to uniform weights: $w^C_{X_i} = \frac{1}{|\{C|X_i \in \chi(C)\}|}$, and for each decision variable $D_i \in \mathbf{D}$, $w^C_{D_i} \approx 0$.

Outer-loop of BCD

$iter$=0, $U_{\text{value}} = \inf$

while $iter < M$ or U_{value} is not converged **do**

    Inner-loop of BCD

    for each variable $X_i \in \mathbf{X}$ **do**

        $U_{\text{value}} \leftarrow \min(U_{\text{value}}, \text{UPDATE-WEIGHTS}(\mathcal{G}_{\text{JG}}, X_i))$ ▷ see Algorithm 3.3

    for each edge $(C_i, C_j) \in \mathcal{S}$ **do**

        $U_{\text{value}} \leftarrow \min(U_{\text{value}}, \text{UPDATE-COSTS}(\mathcal{G}_{\text{JG}}, (C_i, C_j)))$ ▷ see Algorithm 3.4

    for each cluster $C_i \in \mathcal{C}$ **do**

        $U_{\text{value}} \leftarrow \min(U_{\text{value}}, \text{UPDATE-UTIL-CONST}(\mathcal{G}_{\text{JG}}, C_i))$ ▷ see Algorithm 3.5

    $iter = iter + 1$

return U_{value}

Time and Space Complexity is exponential in i-bound

Weighted Mini-bucket Elimination Bounds for IDs

Bounding Methods \ Decomposition Methods	Propagate Message over Join-Graph	Propagate Message over Mini-bucket Tree
Use Valuation Algebra	JGD-ID (GDD)	WMBE-ID (WMB/GDD)
Use Exponentiated Utility Functions	JGD-EXP (GDD)	WMBMM-EXP (WMBMM)

Motivations

JGD-ID doesn't perform well on high i-bound:

- optimization parameter space is too big
- non-convex optimization problem

Variable Elimination + Optimization over Mini-bucket tree
Optimizing over a chain of Mini-buckets

JGD-ID is not efficient for generating heuristics for search

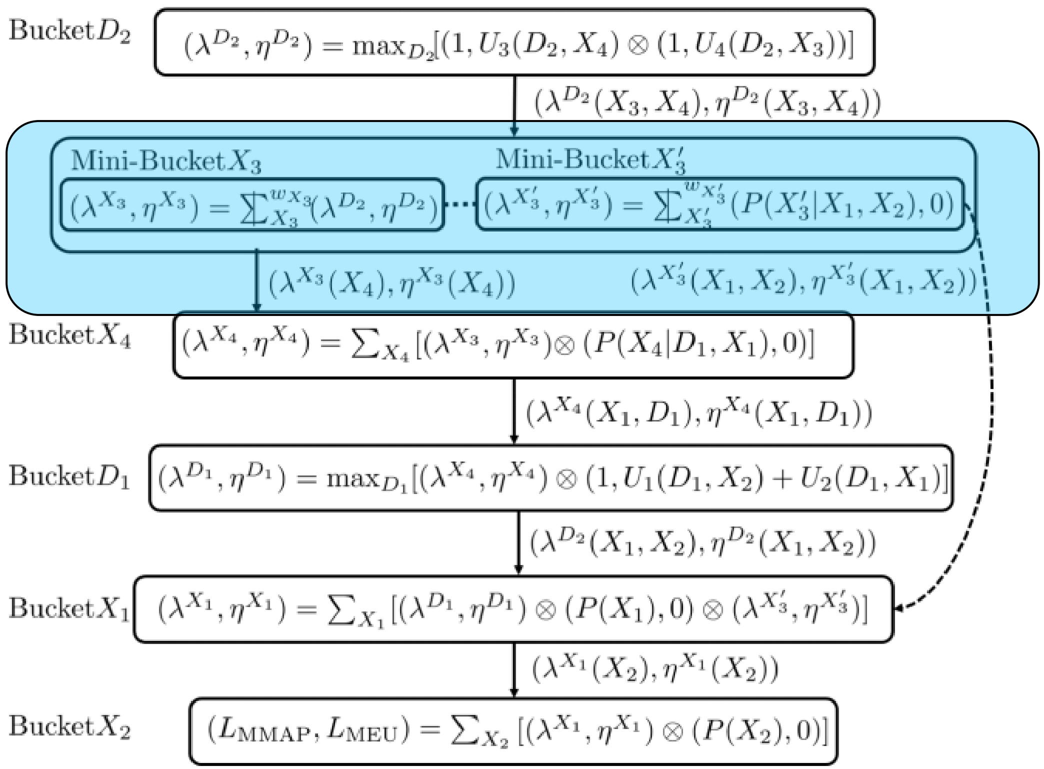
upper-bounds are not pre-compiled relative to search space

Weighted mini-bucket messages are pre-compiled heuristic functions

Weighted Mini-Bucket Elimination for IDs

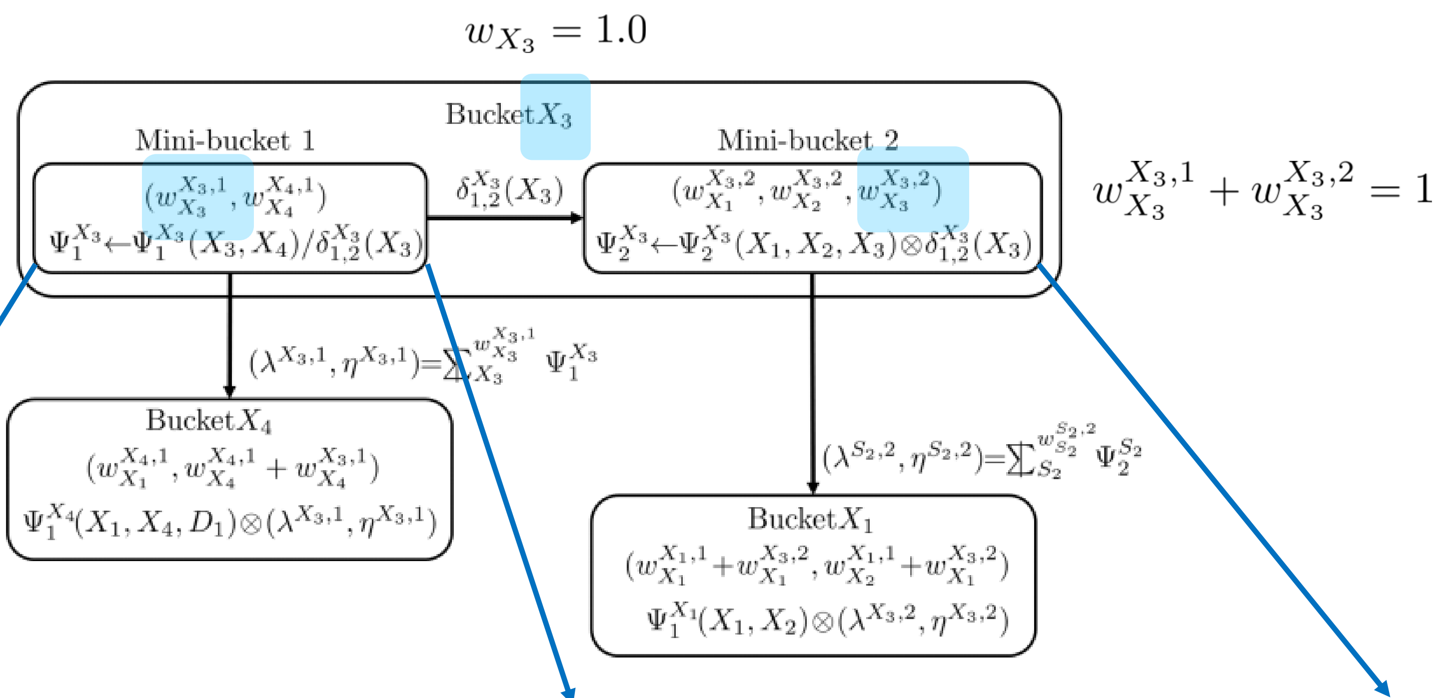
Interleave variable elimination and Optimization

Optimization



Variable Elimination

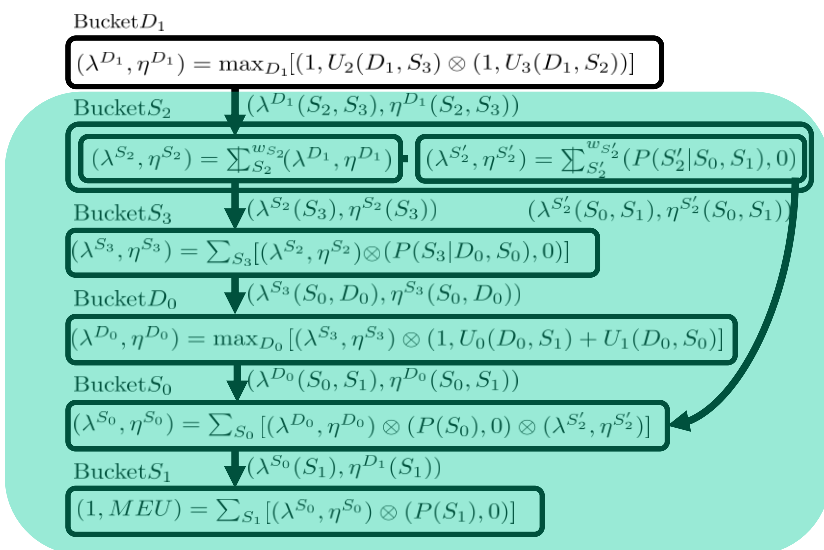
Parameterizing Decomposition Bounds for IDs over Mini-bucket Tree



$$\sum_{X_3}^{w_{X_3}} \frac{\Psi_1^{X_3}(X_3,X_4)}{\delta_{1,2}^{X_3}(X_3)} \otimes \Psi_2^{X_3}(X_1,X_2,X_3) \otimes \delta_{1,2}^{X_3}(X_3) \le \left[\sum_{X_3}^{w_{X_3}^{X_3,1}} \frac{\Psi_1^{X_3}(X_3,X_4)}{\delta_{1,2}^{X_3}(X_3)}\right] \otimes \left[\sum_{X_3}^{w_{X_3}^{X_3,2}} \Psi_2^{X_3}(X_1,X_2,X_3) \otimes \delta_{1,2}^{X_3}(X_3)\right]$$

The RHS cannot be an objective function for optimization!

Objective Function for WMBE-ID



Eliminate all the layers below

Use Decomposition Bounds for IDs

Re-use pre-computed Bounds Per Cluster

$$(1, \mathrm{MEU}) \leq \Psi_{\mathrm{MEU}_{\mathrm{WMB}}} := \sum^{1}_{S_1} \Psi_1 \sum^{1}_{S_0} \otimes \Psi_0 \sum^{0}_{D_0} \otimes \Psi_{D_0} \sum^{1}_{S_3} \otimes \Psi_3 \otimes \left[\sum^{w^{S_2,1}_{S_2}}_{S_2} \frac{\Psi^{S_2}_1(S_2, S_3)}{\delta^{S_2}_{1,2}(S_2)} \right] \otimes \left[\sum^{w^{S_2,2}_{S_2}}_{S_2} \Psi^{S_2}_2(S_0, S_1, S_2) \otimes \delta^{S_2}_{1,2}(S_2) \right]$$

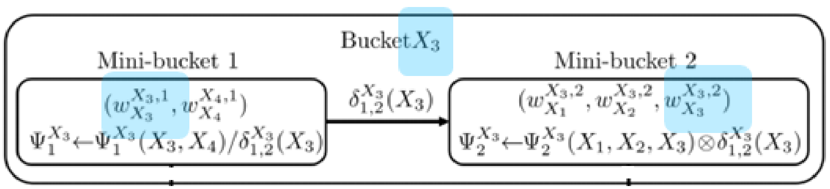
$$\leq \Psi_{\mathrm{MEU}_{\mathrm{JGDID}}} := \left[\sum^{\mathbf{w}^{S_1}}_{S_1} \Psi_1(S_1) \right] \otimes \left[\sum^{\mathbf{w}^{S_0}}_{S_1, S_0} \Psi_0(S_0, S_1) \right] \otimes \left[\sum^{\mathbf{w}^{D_0}}_{S_1, S_0, D_0} \Psi_{D_0}(S_3, D_0, S_0) \right] \otimes \left[\sum^{\mathbf{w}^{S_3}}_{S_1, S_0, D_0, S_3} \Psi_3(S_3, D_0, S_0) \right]$$

$$\otimes \left[\sum^{\mathbf{w}^{S_2,1}}_{S_1, S_0, D_0, S_3} \sum^{w^{S_2,1}_{S_2}}_{S_2} \frac{\Psi^{S_2}_1(S_2, S_3)}{\delta^{S_2}_{1,2}(S_2)} \right] \otimes \left[\sum^{\mathbf{w}^{S_2,2}}_{S_1, S_0, D_0, S_3} \sum^{w^{S_2,2}_{S_2}}_{S_2} \Psi^{S_2}_2(S_0, S_1, S_2) \otimes \delta^{S_2}_{1,2}(S_2) \right]$$

Parameterizing Decomposition Bounds for IDs over Mini-bucket Tree

$$\min_{\mathbf{w},\boldsymbol{\lambda},\boldsymbol{\eta}} U_{\text{value}}$$

$$U_{\text{value}} = \Gamma \cdot \left[\sum_{i=1}^{n-1} \left(\sum_{\alpha \in Q_{X_i}} \frac{\sum^{\mathbf{w}^{X_i;\alpha}_{1:n-1}}_{\mathbf{X}_{1:n-1}} V_\alpha^{X_i}}{\sum^{\mathbf{w}^{X_i;\alpha}_{1:n-1}}_{\mathbf{X}_{1:n-1}} P_\alpha^{X_i}} \right) + \sum_{\alpha\in Q_{X_n}} \frac{\sum^{\mathbf{w}^{X_n,\alpha}_{1:n}}_{\mathbf{X}_{1:n}} P_\alpha^{X_n} \frac{\lambda^{X_n}_{\alpha-1,\alpha}}{\lambda^{X_n}_{\alpha,\alpha+1}} \left[\frac{V_\alpha^{X_n}}{P_\alpha^{X_n}} - \frac{\eta^{X_n}_{\alpha,\alpha+1}}{\lambda^{X_n}_{\alpha,\alpha+1}} + \frac{\eta^{X_n}_{\alpha-1,\alpha}}{\lambda^{X_n}_{\alpha-1,\alpha}}\right]}{\sum^{\mathbf{w}^{X_n,\alpha}_{1:n}}_{\mathbf{X}_{1:n}} P_\alpha^{X_n} \frac{\lambda^{X_n}_{\alpha-1,\alpha}}{\lambda^{X_n}_{\alpha,\alpha+1}}} \right]$$



Optimization parameters $\delta_{1,2}^{S_2}(S_2) := (\lambda_{1,2}^{S_2}(S_2), \eta_{1,2}^{S_2}(S_2))$

$w_{S_2}^{S_2,1} \quad w_{S_2}^{S_2,2}$

Constraints $\lambda_{1,2}^{S_2}(S_2) \geq 0$ Probability is non-negative

$w_{S_2}^{S_2,1} + w_{S_2}^{S_2,2} = 1 \quad 0 \leq w_{S_2}^{S_2,1}, w_{S_2}^{S_2,2} \leq 1$

$\frac{\eta_1^{S_2}}{\lambda_1^{S_2}} - \frac{\eta_{1,2}^{S_2}}{\lambda_{1,2}^{S_2}} \geq 0 \qquad \frac{\eta_2^{S_2}}{\lambda_2^{S_2}} + \frac{\eta_{1,2}^{S_2}}{\lambda_{1,2}^{S_2}} \geq 0$ Utility is non-negative

Algorithm WMBE-ID

Algorithm 3.6 Weighted Mini-Bucket Elimination Bounds for IDs (WMBE-ID)

Require: ID $\mathcal{M} = \langle \mathbf{X}, \mathbf{D}, \mathbf{\Psi}, \mathcal{O} \rangle$, total constrained elimination order $\mathcal{O}_{\text{elim}} := \{X_N, X_{N-1}, \dots, X_1\}$, i-bound, iteration limit M,

Ensure: an upper bound of the MEU, U_{value}

Initialization Steps

Generate a mini-bucket tree decomposition $\mathcal{T}_{\text{MBT}} = \langle T(\mathcal{C}, \mathcal{S}), \chi, \psi \rangle$ with i-bound

Allocate functions to mini-buckets $\{Q_{X_i} | X_i \in \mathbf{X}\}$, where $Q_{X_i} = \{1, \dots, q_{X_i}\}$

Initialize weights of chance variables $X_i \in \mathbf{X}$ by uniform weights $w_{X_i}^{X_l, \alpha} = \frac{1}{|\{C | X_i \in \chi(C)\}|}$,

Initialize weights of decision variables $D_i \in \mathbf{D}$ by a constant close to zero, $w_{D_i}^{X_l, \alpha} \approx 0$

Interleave Variable Elimination and Reparameterization

for $i \leftarrow N$ to 1 **do**

  for each mini-bucket $\alpha \in Q_{X_i}$ **do**

    Combine potentials at the α-th mini-bucket, $\Psi_\alpha(\mathbf{X}_\alpha)$

    for each $X_k \in \text{sc}(\Psi_\alpha)$ **do**

      $w_{X_k}^{X_i, \alpha} \leftarrow w_{X_k}^{X_i, \alpha} + \sum_{l > i} w_{X_k}^{X_l, \alpha_l}$ ▷ sum up the weights of incoming messages $w_{X_k}^{X_l, \alpha_l}$ to the pre-allocated $w_{X_k}^{X_i, \alpha}$

  Outer-loop of BCD

  $iter = 0$, compute U_{value} by Eq. (3.55)

  while $iter < M$ or U_{value} is not converged **do**

    Inner-loop of BCD

    for each edge $(\alpha, \alpha + 1) \in \{(\alpha, \alpha + 1) | \alpha, \alpha + 1 \in Q_{X_i}\}$ **do**

      $U_{\text{value}} \leftarrow min(U_{\text{value}}, \text{UPDATE-COSTS}(\mathcal{T}_{\text{MBT}}, (X_i, \alpha, \alpha + 1)))$ ▷ Algorithm 3.7

    $U_{\text{value}} \leftarrow min(U_{\text{value}}, \text{UPDATE-WEIGHTS}(\mathcal{T}_{\text{MBT}}, X_i))$

  for each mini-bucket $\alpha \in Q_{X_i}$ **do**

    $\Psi^{X_i} \leftarrow \sum_{X_i}^{w_{X_i}^{X_i, \alpha}} \Psi_\alpha(\mathbf{X}_\alpha)$ ▷ Compute message at the α-th mini-bucket

    Send message Ψ^{X_i} downward to the destination mini-bucket

return U_{value}

Outline

Direct Decomposition Bounds for IDs: Algorithms

- Using Valuation Algebra for IDs
  - Extending Powered Summation in Valuation Algebra
  - Decomposition Bounds based on Valuation Algebra
  - Join-Graph Decomposition Bounds for IDs
  - Weighted Mini-bucket Elimination Bounds for IDs

- Using Exponentiated Utility Functions
  - Bounding MEU by MMAP Task
  - Combining GDD Bounds for MMAP
  - Combining Weighted Mini-bucket with Moment Matching Bounds for MMAP

Exponentiated Utility Bounds

Bounding Methods \ Decomposition Methods	Propagate Message over Join-Graph	Propagate Message over Mini-bucket Tree
Use Valuation Algebra	JGD-ID (GDD)	WMBE-ID (WMB/GDD)
Use Exponentiated Utility Functions	JGD-EXP (GDD)	WMBMM-EXP (WMBMM)

Motivation

The formulation is “too” complicated

Non-convex formulation of JGD-ID and WMBE-ID has scalability issue

Find an alternative formulation that gives convex formulation

Exponentiating Utility Functions

Stochastic optimal control

$$\xi(\mathbf{x}) = \frac{1}{\rho}\mathbb{E}\Big(\mathcal{J}(\mathbf{x})\Big) = \frac{1}{\rho}\log E_{\mathbb{P}}\Big[\exp\left(\rho\mathcal{J}(\mathbf{x})\right)\Big]$$

Equality holds in limit as $\rho \to 0$

LHS: Expectation of value function

RHH: Log partition function form of exponentiated value function

Path integral control [Kappen 2012]

Duality in control [Theodorous, Divijotham, Todorov, 2013]

Bounding Scheme using Exponentiated Utility Functions

Jensen's Inequality applied to exponential function

$$e^{\mathbb{E}[X]} \leq \mathbb{E}[e^X]$$

$$\begin{aligned}
\text{MEU} := \max_{\boldsymbol{\Delta}} \mathbb{E}\Big[\sum_{U_i \in \mathbf{U}} U_i(\mathbf{X}_{U_i})\Big] &\leq \max_{\boldsymbol{\Delta}} \log \mathbb{E}\Big[e^{\sum_{U_i \in \mathbf{U}} U_i(\mathbf{X}_{U_i})}\Big] \\
&= \log \max_{\boldsymbol{\Delta}} \mathbb{E}\Big[e^{\sum_{U_i \in \mathbf{U}} U_i(\mathbf{X}_{U_i})}\Big] \\
&= \log \max_{\boldsymbol{\Delta}} \mathbb{E}\Big[\prod e^{U_i(\mathbf{X}_{U_i})}\Big]
\end{aligned}$$

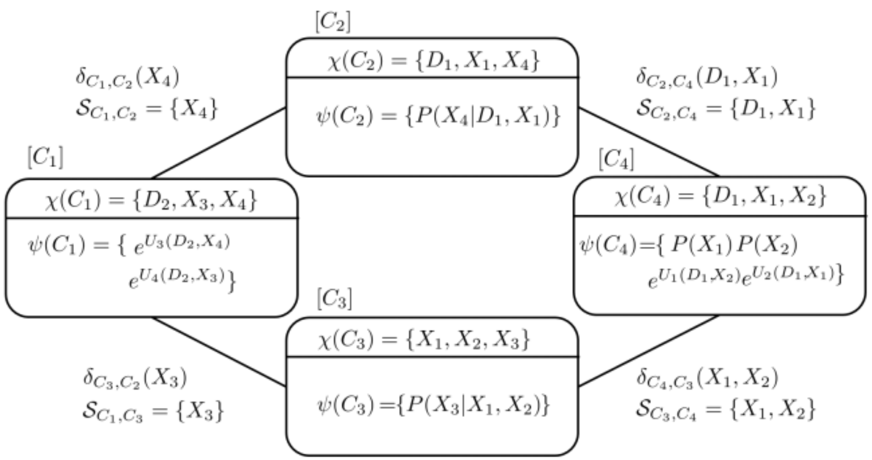
Bound MEU by log-partition function of MMAP (mixed inference) task

JGD-EXP and WMBMM-EXP

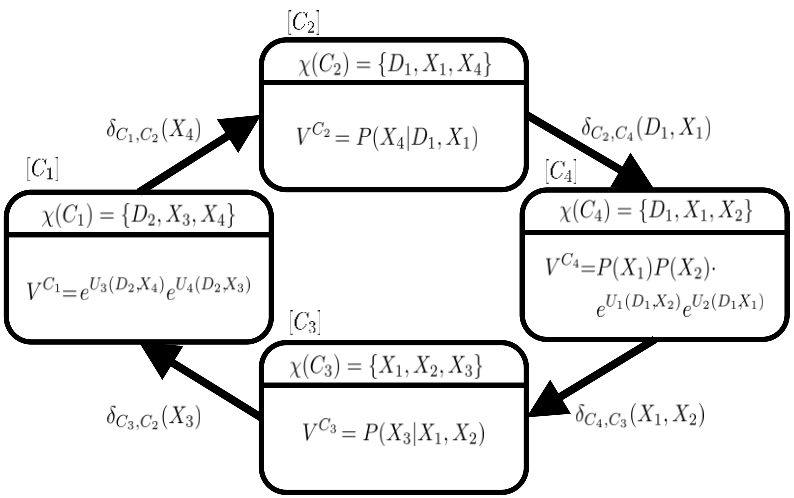
Bounding Methods \ Decomposition Methods	Propagate Message over Join-Graph	Propagate Message over Mini-bucket Tree
Use Valuation Algebra	JGD-ID (GDD)	WMBE-ID (WMB/GDD)
Use Exponentiated Utility Functions	JGD-EXP (GDD)	WMBMM-EXP (WMBMM)

JGD-EXP

Bound MEU by MMAP Task

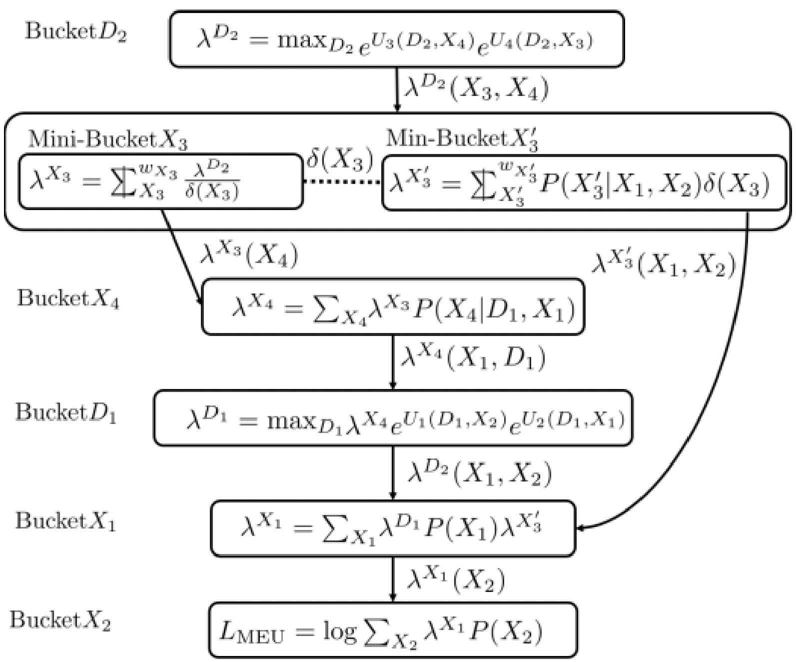


Use GDD Bounds for MMAP

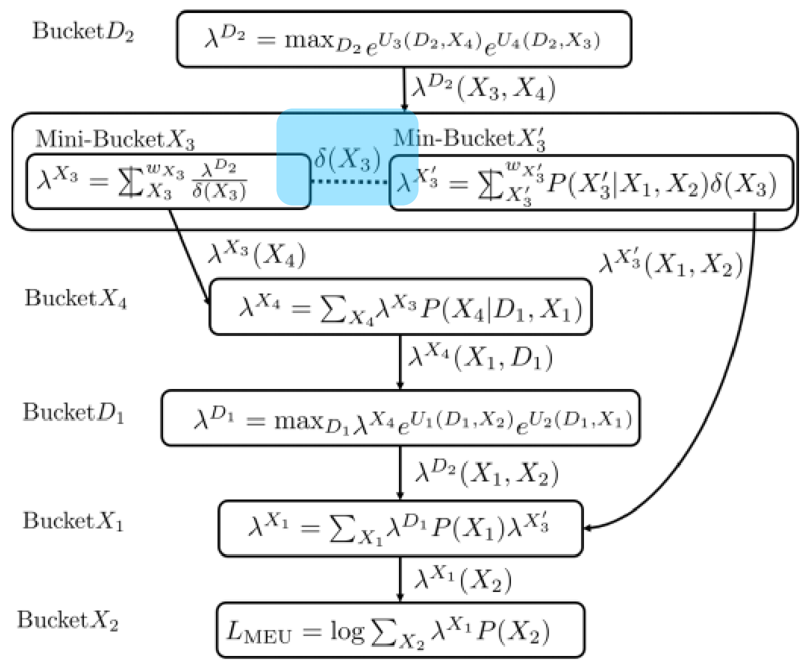


WMBMM-EXP

Bound MEU by MMAP Task



Use WMBMM Bounds for MMAP



Direct Decomposition Bounds for Influence Diagrams:

Empirical Evaluations

Outline

Direct Decomposition Bounds for IDs: Evaluation

- Experiment on Synthetic Domains
  - List of Algorithms and Benchmark Domains
  - Convergence Behavior
  - Summary of Bounding Performance

- Case Study on SysAdmin Domains
  - SysAdmin MDP/POMDP Domains
  - Evaluation Results

Evaluated Algorithms

List of Algorithms

Algorithm	Decomposition Methods	Bounding Method	Iteration	Translation
JGD-ID	Join-graph	GDD over Potential	Yes	No
WMBE-ID	Mini-bucket Tree	WMB,GDD over Potential	Yes	No
JGD-EXP	Join-graph	GDD over $\mathcal{M}^e$	Yes	No
WMBMM-EXP	Mini-bucket Tree	WMBMM over $\mathcal{M}^e$	No	No
GDD-MI	Join-graph	GDD	Yes	Yes
WMBMM-MMAP	Mini-bucket Tree	WMBMM	No	Yes
MBE-ID	Mini-bucket Tree	MBE	No	No

Hyper Parameters for Controlling Iterations

Iteration parameter	JGD-ID	WMBE-ID	JGD-EXP	GDD-MI
BCD outer-loop limit M	1000	5	1000	1000
BCD outer-loop convergence ϵ	$1e^{-3}$	$1e^{-3}$	$1e^{-3}$	$1e^{-3}$
Inner-loop optimizer	gradient descent	SLSQP	gradient descent	gradient descent
Inner-loop iteration limits	20	100	10	20

Benchmark Domains

Finite Horizon MDP

	n	c	d	f	p	u	k	s	w
Min	25	20	3	30	20	10	2	4	5
Average	105.7	99.6	6.1	134.1	99.6	34.5	3.1	7.1	25.5
Max	170	160	10	240	160	80	5	9	43

Finite Horizon POMDP

	n	c	d	f	p	u	k	s	w
Min	15	12	3	18	12	6	2	3	10
Average	55.9	52.4	3.5	73.5	52.4	21.1	2.4	5.5	28
Max	96	92	5	140	92	48	3	9	46

Random Influence Diagrams

	n	c	d	f	p	u	k	s	w
Min	22	20	2	22	20	2	2	3	6
Average	56	47	9	56	47	9	2	3	17
Max	91	70	21	91	70	21	2	3	34

IDs converted from BN

	n	c	d	f	p	u	k	s	w
Min	54	48	3	54	48	3	2	6	12
Average	84	77	7	84	77	7	2	8	21
Max	115	109	12	115	109	12	2	10	42

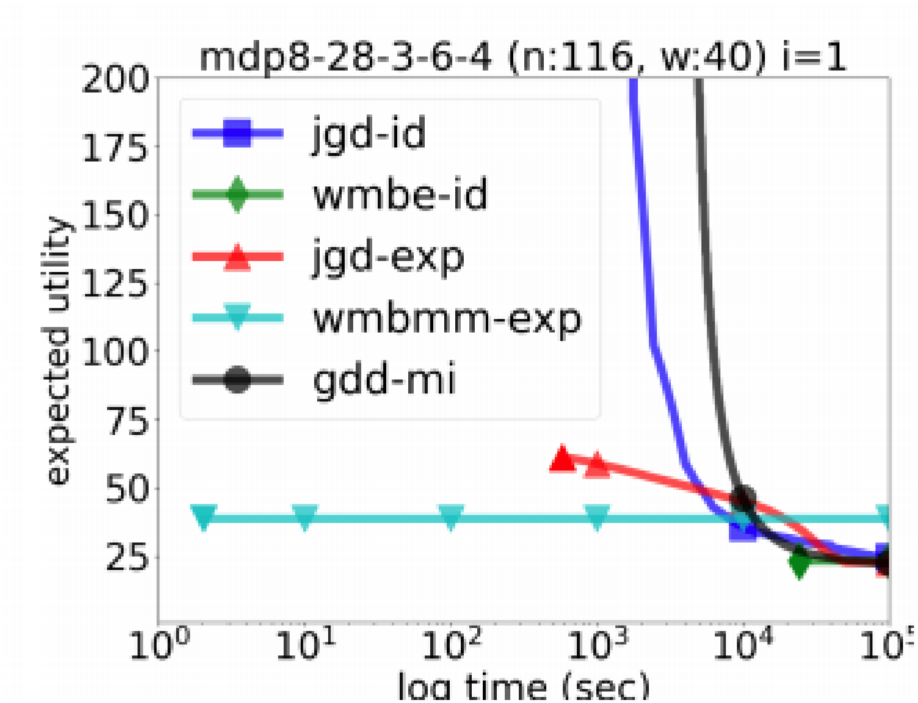
Average Quality of Upper bounds

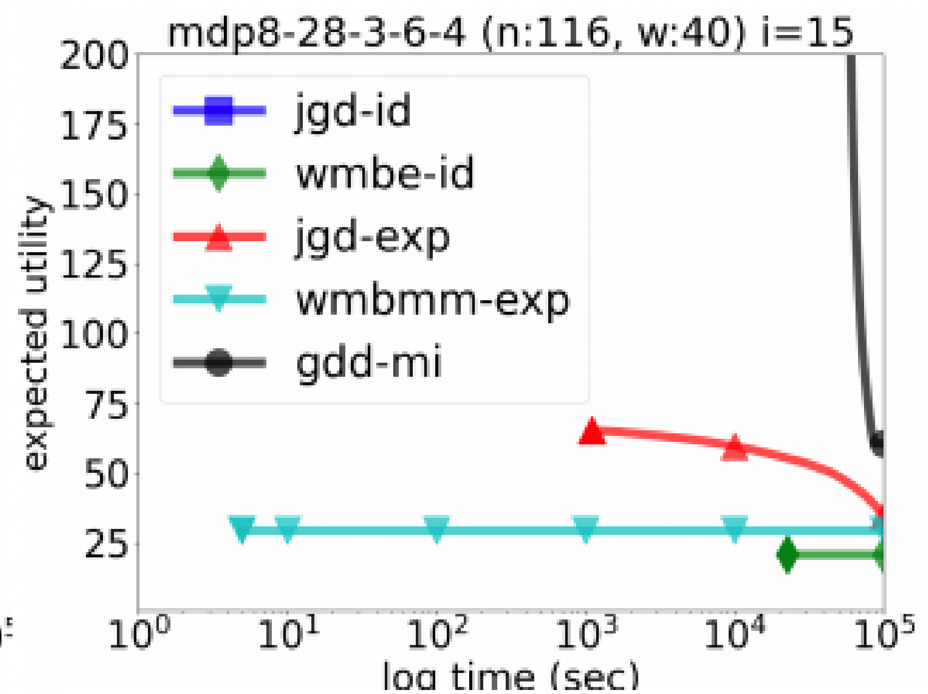
Domain	i-bd	time (sec)	JGD -ID	WMBE -ID	JGD -EXP	WMBMM -EXP	GDD -MI	WMBMM -MMAP	MBE -ID
All	1	10E+1	0.017	0.0	0.086	**0.708**	0.011	0.005	0.002
n:75.6 f:87.1		10E+2	0.048	0.006	0.504	**0.708**	0.055	0.005	0.002
k:2.4 s:5.9		10E+3	0.112	0.059	0.67	**0.708**	0.105	0.005	0.002
w:25.9		10E+4	0.584	0.096	**0.73**	0.708	0.195	0.005	0.002
		10E+6	0.663	0.181	**0.786**	0.708	0.272	0.005	0.002
	5	10E+1	0.045	0.0	0.019	**0.753**	0.01	0.022	0.066
		10E+2	0.078	0.065	0.457	**0.753**	0.036	0.022	0.066
		10E+3	0.175	0.152	0.686	**0.753**	0.095	0.022	0.066
		10E+4	0.449	0.262	**0.759**	0.753	0.176	0.022	0.066
		10E+6	0.539	0.305	**0.808**	0.753	0.288	0.022	0.066
	10	10E+1	0.062	0.0	0.034	**0.818**	0.008	0.087	0.2
		10E+2	0.151	0.099	0.449	**0.818**	0.027	0.087	0.2
		10E+3	0.252	0.284	0.67	**0.818**	0.071	0.087	0.2
		10E+4	0.46	0.441	0.749	**0.818**	0.108	0.087	0.2
		10E+6	0.468	0.486	0.816	**0.818**	0.226	0.087	0.2
	15	10E+1	0.062	0.025	0.034	**0.798**	0.007	0.184	0.33
		10E+2	0.147	0.099	0.227	**0.861**	0.025	0.184	0.33
		10E+3	0.266	0.283	0.574	**0.861**	0.055	0.184	0.33
		10E+4	0.433	0.489	0.682	**0.861**	0.083	0.184	0.33
		10E+6	0.525	0.631	0.77	**0.861**	0.106	0.184	0.33
	20	10E+1	0.062	0.0	0.019	**0.557**	0.005	0.271	0.478
		10E+2	0.184	0.124	0.137	**0.829**	0.02	0.271	0.478
		10E+3	0.262	0.249	0.335	**0.879**	0.056	0.275	0.478
		10E+4	0.363	0.445	0.568	**0.904**	0.072	0.275	0.478
		10E+6	0.479	0.705	0.614	**0.904**	0.075	0.275	0.478

Instance	i-bd	time (sec)	JGD -ID	WMBE -ID	JGD -EXP	WMBMM -EXP	GDD -MI	WMBMM -MMAP	MBE -ID
fh-mdp	1	10E+1	0.041	0.0	0.048	**0.583**	0.004	0.0	0.001
n:105.7 f:134.1		10E+2	0.084	0.0	0.122	**0.583**	0.048	0.0	0.001
k:3.1 s:7.1		10E+3	0.155	0.152	0.473	**0.583**	0.137	0.0	0.001
w:25.9		10E+4	0.463	0.28	**0.631**	0.583	0.465	0.0	0.001
		10E+6	0.779	0.619	**0.841**	0.583	0.77	0.0	0.001
	5	10E+1	0.086	0.0	0.0	**0.645**	0.0	0.0	0.1
		10E+2	0.1	0.1	0.113	**0.645**	0.016	0.0	0.1
		10E+3	0.101	0.185	0.485	**0.645**	0.113	0.0	0.1
		10E+4	0.154	0.562	**0.659**	0.645	0.321	0.0	0.1
		10E+6	0.507	0.734	**0.841**	0.645	0.723	0.0	0.1
	10	10E+1	0.086	0.0	0.0	**0.738**	0.0	0.0	0.2
		10E+2	0.1	0.1	0.073	**0.738**	0.006	0.0	0.2
		10E+3	0.196	0.2	0.453	**0.738**	0.074	0.0	0.2
		10E+4	0.196	0.609	0.613	**0.738**	0.16	0.0	0.2
		10E+6	0.208	0.79	**0.842**	0.738	0.606	0.0	0.2
	15	10E+1	0.086	0.1	0.0	**0.734**	0.0	0.001	0.4
		10E+2	0.1	0.1	0.07	**0.81**	0.003	0.001	0.4
		10E+3	0.196	0.2	0.26	**0.81**	0.041	0.001	0.4
		10E+4	0.295	0.394	0.458	**0.81**	0.088	0.001	0.4
		10E+6	0.457	**0.959**	0.691	0.81	0.182	0.001	0.4
	20	10E+1	0.086	0.0	0.0	0.387	0.0	0.002	**0.4**
		10E+2	0.1	0.1	0.07	**0.773**	0.004	0.002	0.4
		10E+3	0.196	0.2	0.195	**0.873**	0.061	0.002	0.4
		10E+4	0.295	0.3	0.313	**0.873**	0.079	0.002	0.4
		10E+6	0.394	0.8	0.361	**0.873**	0.079	0.002	0.4

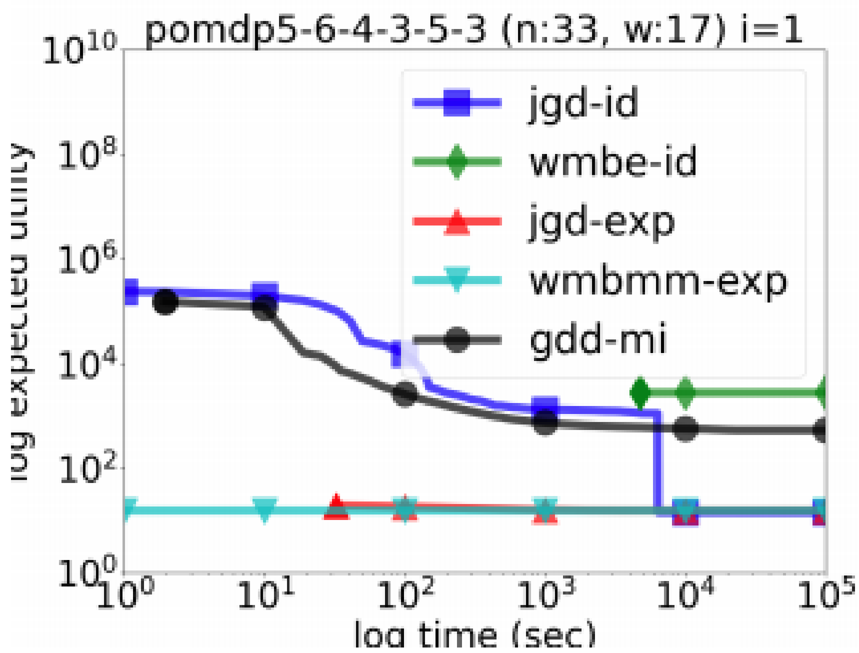
Instance	i-bd	time (sec)	JGD -ID	WMBE -ID	JGD -EXP	WMBMM -EXP	GDD -MI	WMBMM -MMAP	MBE -ID
rand	1	10E+1	0.016	0.0	0.143	**0.821**	0.025	0.01	0.004
n:56.2 f:56.2		10E+2	0.048	0.025	0.82	**0.821**	0.064	0.01	0.004
k:2.0 s:3.0		10E+3	0.105	0.051	**0.823**	0.821	0.093	0.01	0.004
w:20.4		10E+4	0.774	0.051	**0.823**	0.821	0.105	0.01	0.004
		10E+6	0.775	0.051	**0.823**	0.821	0.106	0.01	0.004
	5	10E+1	0.08	0.0	0.0	**0.826**	0.012	0.057	0.117
		10E+2	0.121	0.096	0.808	**0.826**	0.057	0.057	0.117
		10E+3	0.319	0.273	0.817	**0.826**	0.114	0.057	0.117
		10E+4	0.711	0.273	0.819	**0.826**	0.136	0.057	0.117
		10E+6	0.713	0.273	0.819	**0.826**	0.148	0.057	0.117
	10	10E+1	0.079	0.0	0.062	**0.828**	0.01	0.201	0.325
		10E+2	0.281	0.096	0.803	**0.828**	0.045	0.201	0.325
		10E+3	0.413	0.443	0.81	**0.828**	0.115	0.201	0.325
		10E+4	0.733	0.582	0.812	**0.828**	0.148	0.201	0.325
		10E+6	0.747	0.582	0.812	**0.828**	0.152	0.201	0.325
	15	10E+1	0.079	0.0	0.062	**0.829**	0.01	0.336	0.461
		10E+2	0.252	0.096	0.437	**0.829**	0.045	0.336	0.461
		10E+3	0.487	0.462	0.807	**0.829**	0.098	0.336	0.461
		10E+4	0.637	0.692	0.811	**0.829**	0.137	0.336	0.461
		10E+6	0.764	0.692	0.811	**0.829**	0.123	0.336	0.461
	20	10E+1	0.079	0.0	0.0	**0.729**	0.005	0.51	0.704
		10E+2	0.347	0.196	0.284	**0.829**	0.03	0.512	0.704
		10E+3	0.455	0.494	0.633	**0.829**	0.084	0.512	0.704
		10E+4	0.548	0.781	0.808	**0.829**	0.124	0.512	0.704
		10E+6	0.705	**0.896**	0.811	0.829	0.124	0.512	0.704

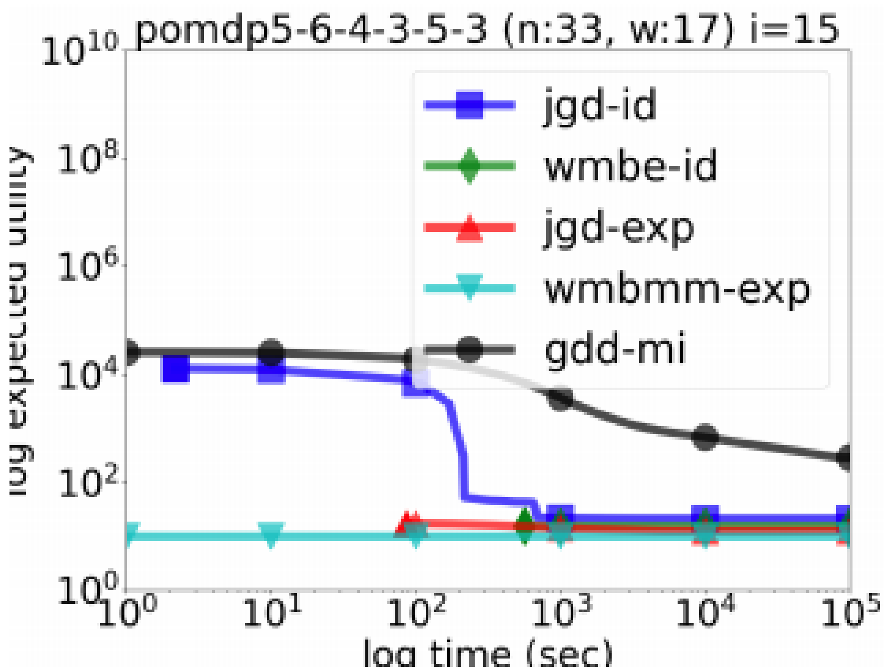
Comparing Upper Bounds



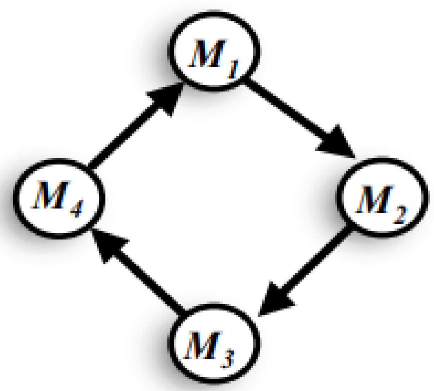


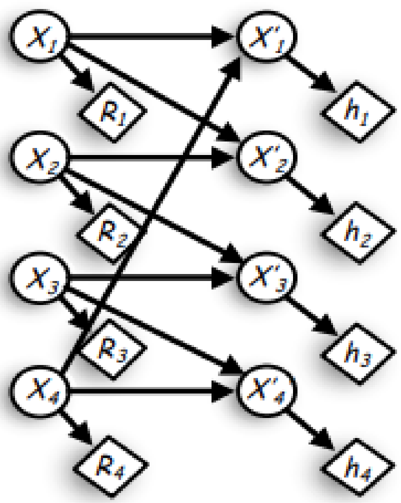
Comparing Upper Bounds





SysAdmin MDP/POMDP Domains [Guestrin, et. al 2003]





$P(X'_i = t \mid X_i, X_{i-1}, A)$:

	Action is reboot:	
	machine i	other machine
$X_{i-1} = f \wedge X_i = f$	1	0.0238
$X_{i-1} = f \wedge X_i = t$	1	0.475
$X_{i-1} = t \wedge X_i = f$	1	0.0475
$X_{i-1} = t \wedge X_i = t$	1	0.95

Evaluation

UB: WMBMM-EXP (i=20)

LB: Online planner to obtain lower bounds

$$\text{gap} = 1 - \frac{\text{LB}}{\text{UB}}$$

SysAdmin MDP

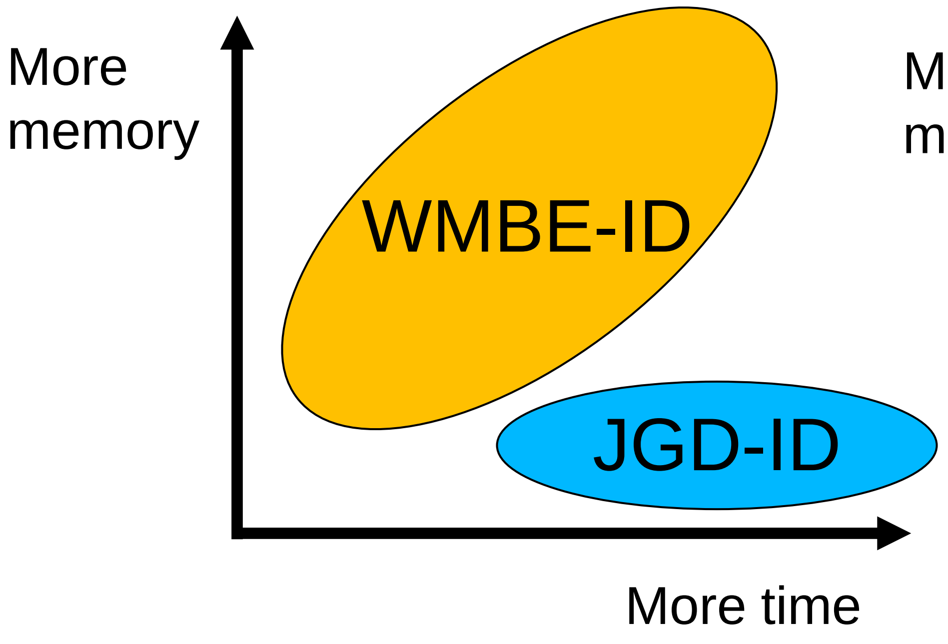
Instance	c	d	p	u	k	s	w	utime (sec)	ub wmbmm	ltime (sec)	lb sogbofa	gap ($\frac{ub-lb}{ub}$)
.												
mdp9-s50-t3	227	150	227	300	3	8	107	217	161.292	240	142.931	11%
mdp9-s50-t4	286	200	286	400	3	8	108	241	218.210	320	184.250	16%
mdp9-s50-t5	345	250	345	500	3	8	109	298	274.907	400	222.925	19%
mdp9-s50-t6	404	300	404	600	3	8	109	353	329.248	480	261.319	21%
mdp9-s50-t7	463	350	463	700	3	8	109	49406	383.290	560	296.269	23%
mdp9-s50-t8	522	400	522	800	3	8	109	530	438.786	640	328.550	25%
mdp9-s50-t9	581	450	581	900	3	8	108	370	496.466	720	355.263	28%
mdp9-s50-t10	640	500	640	1000	3	8	109	129	547.757	800	385.263	30%
mdp10-s50-t3	218	150	218	300	3	11	112	149	162.368	240	142.731	12%
mdp10-s50-t4	274	200	274	400	3	11	112	184	217.515	320	183.650	16%
mdp10-s50-t5	330	250	330	500	3	11	113	257	273.332	400	221.450	19%
mdp10-s50-t6	386	300	386	600	3	11	113	19741	327.268	480	257.394	21%
mdp10-s50-t7	442	350	442	700	3	11	113	28013	383.312	560	291.988	24%
mdp10-s50-t8	498	400	498	800	3	11	113	41748	439.826	640	316.600	28%
mdp10-s50-t9	554	450	554	900	3	11	113	34739	494.662	720	345.844	30%
mdp10-s50-t10	610	500	610	1000	3	11	113	58270	549.867	800	364.569	34%

SysAdmin POMDP

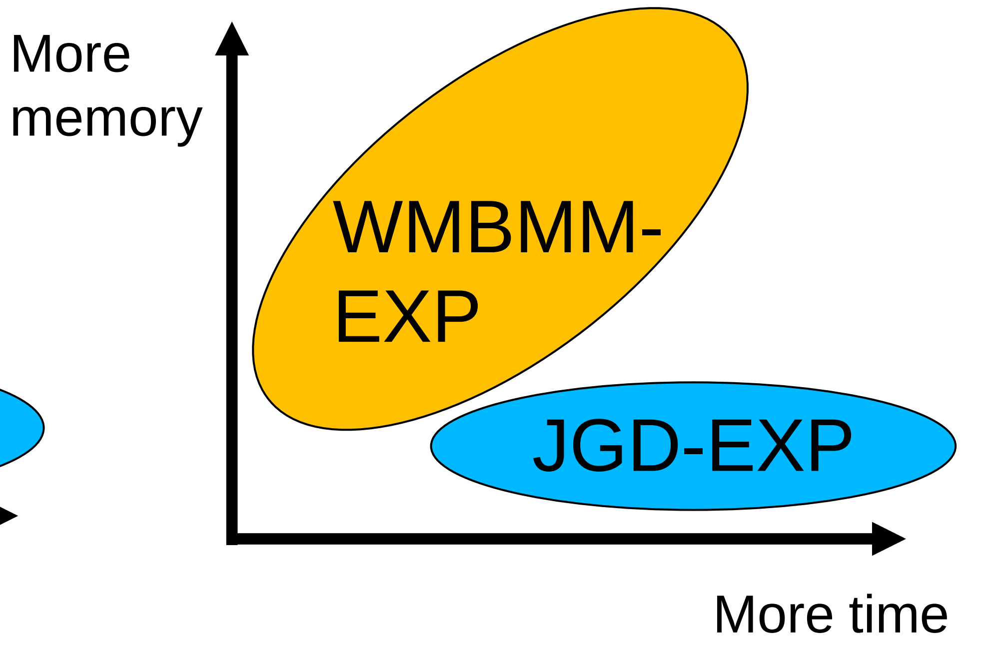
Instance	c	d	p	u	k	s	w	utime (sec)	ub wmbmm	ltime (sec)	lb snap	gap ($\frac{ub-lb}{ub}$)
pomdp9-s50-t3	371	150	371	300	3	10	300	7660	177.597	240	141.575	20%
pomdp9-s50-t4	478	200	478	400	3	10	400	20747	244.415	320	182.775	25%
pomdp9-s50-t5	585	250	585	500	3	10	501	27369	315.351	400	223.000	29%
pomdp9-s50-t6	692	300	692	600	3	10	600	69085	384.860	480	257.350	33%
pomdp9-s50-t7	799	350	799	700	3	10	701	107660	454.910	560	290.425	36%
pomdp9-s50-t8	906	400	906	800	3	10	800	224727	528.582	640	323.575	39%
pomdp9-s50-t9	1013	450	1013	900	3	10	900	207883	599.923	720	348.325	42%
pomdp9-s50-t10	1120	500	1120	1000	3	10	1000	231511	668.883	800	372.750	44%
pomdp10-s50-t3	371	150	371	300	3	10	300	6249	180.257	240	141.950	21%
pomdp10-s50-t4	478	200	478	400	3	10	400	14333	253.043	320	184.925	27%
pomdp10-s50-t5	585	250	585	500	3	10	501	36078	331.108	400	221.775	33%
pomdp10-s50-t6	692	300	692	600	3	10	600	66848	409.456	480	258.525	37%
pomdp10-s50-t7	799	350	799	700	3	10	701	121312	480.291	560	290.300	40%
pomdp10-s50-t8	906	400	906	800	3	10	800	116597	563.324	640	321.425	43%
pomdp10-s50-t9	1013	450	1013	900	3	10	900	290003	633.134	720	346.500	45%
pomdp10-s50-t10	1120	500	1120	1000	3	10	1000	244446	707.226	800	375.900	47%

Summary of Performance

Easy problems



Hard problems



Conclusion

Decomposition Bounds for Influence Diagrams

- Extending Valuation Algebra
- Using Exponentiated Utility Functions

- AND/OR Search for Marginal MAP
  - Evaluated Various AND/OR Search Strategies

Future Work

Inference Algorithm for the MEU Task

- Advancing approx. inference by optimization

Search Algorithms for the MEU Task

- Heuristic AND/OR Search with Proposed Upper Bounds

Publications on Influence Diagrams

1. **Junkyu Lee, Radu Marinescu, and Rina Dechter.** "Submodel Decomposition Bounds for Solving Influence Diagrams" *accepted to AAAI 2021*.
2. **Radu Marinescu, Junkyu Lee, and Rina Dechter.** “A New Bounding Scheme for Influence Diagrams" *accepted to AAAI 2021*.
3. **Junkyu Lee.** "Submodel Decomposition for Solving Limited Memory Influence Diagrams (Student Abstract)" *in Proceedings of AAAI 2020.*
4. **Junkyu Lee, Radu Marinescu, and Rina Dechter.** "Heuristic AND/OR Search for Solving Influence Diagrams (Extended Abstract)" *in Proceedings of the Symposium on Combinatorial Search (SoCS 2020)*.
5. **Junkyu Lee, Radu Marinescu, Alexander Ihler, and Rina Dechter.** "A Weighted Mini-Bucket Bound for Solving Influence Diagrams" *in Proceedings of UAI 2019.*
6. **Junkyu Lee, Alexander Ihler, and Rina Dechter.** "Join Graph Decomposition Bounds for Influence Diagrams" *in Proceedings of UAI 2018.*
7. **Junkyu Lee, Alexander Ihler, and Rina Dechter.** "Generalized Dual Decomposition for Bounding Maximum Expected Utility of Influence Diagrams with Perfect Recall" *in AAAI-18 Workshop on Planning and Inference.*
8. **Junkyu Lee.** “Probabilistic Planning and Influence Diagrams" *in AAAI-18 Doctoral Consortium*

Publications on Marginal MAP

1. **Radu Marinescu, Junkyu Lee, Rina Dechter, and Alexander Ihler.** "AND/OR Search for Marginal MAP" *Journal of Artificial Intelligence Research (JAIR) volume 63, 2018.*
2. **Radu Marinescu, Junkyu Lee, Alexander Ihler, and Rina Dechter.** "Anytime Best+Depth-First Search for Bounding Marginal MAP" *in Proceedings of AAAI 2017.*
3. **Junkyu Lee, Radu Marinescu, and Rina Dechter.** "Applying Search Based Probabilistic Inference Algorithms to Probabilistic Conformant Planning: Preliminary Results" *in proceedings of International Symposium on Artificial Intelligence and Mathematics (ISAIM 2016).*
4. **Junkyu Lee, Radu Marinescu, Rina Dechter and Alexander Ihler.** "From Exact to Anytime Solutions for Marginal MAP" *in proceedings of AAAI 2016.*
5. **Junkyu Lee, Radu Marinescu, and Rina Dechter.** "Applying Marginal MAP Search to Probabilistic Conformant Planning: Initial Results." in *the 4th International Workshop on Statistical Realtional AI (STARAI 2014).*
6. **Junkyu Lee, William Lam, and Rina Dechter.** "Benchmark on DAOOPT and GUROBI with the PASCAL2 Inference Challenge Problems" *in DISCML 2013*