BREN:ICS INFORMATION AND COMPUTER SCIENCES — UNIVERSITY of CALIFORNIA IRVINE

Modern Exact and Approximate MAP algorithms for Graphical Models

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Harvard, 10/12

Outline

- Graphical models and the MAP task
- Solving exactly: Variable elimination and search
- Upper bound approximations
  - The mini-bucket scheme
  - The cost-shifting or reparameterization scheme
- New Algorithms combinations
- Experiments

Sample Applications for Graphical Models

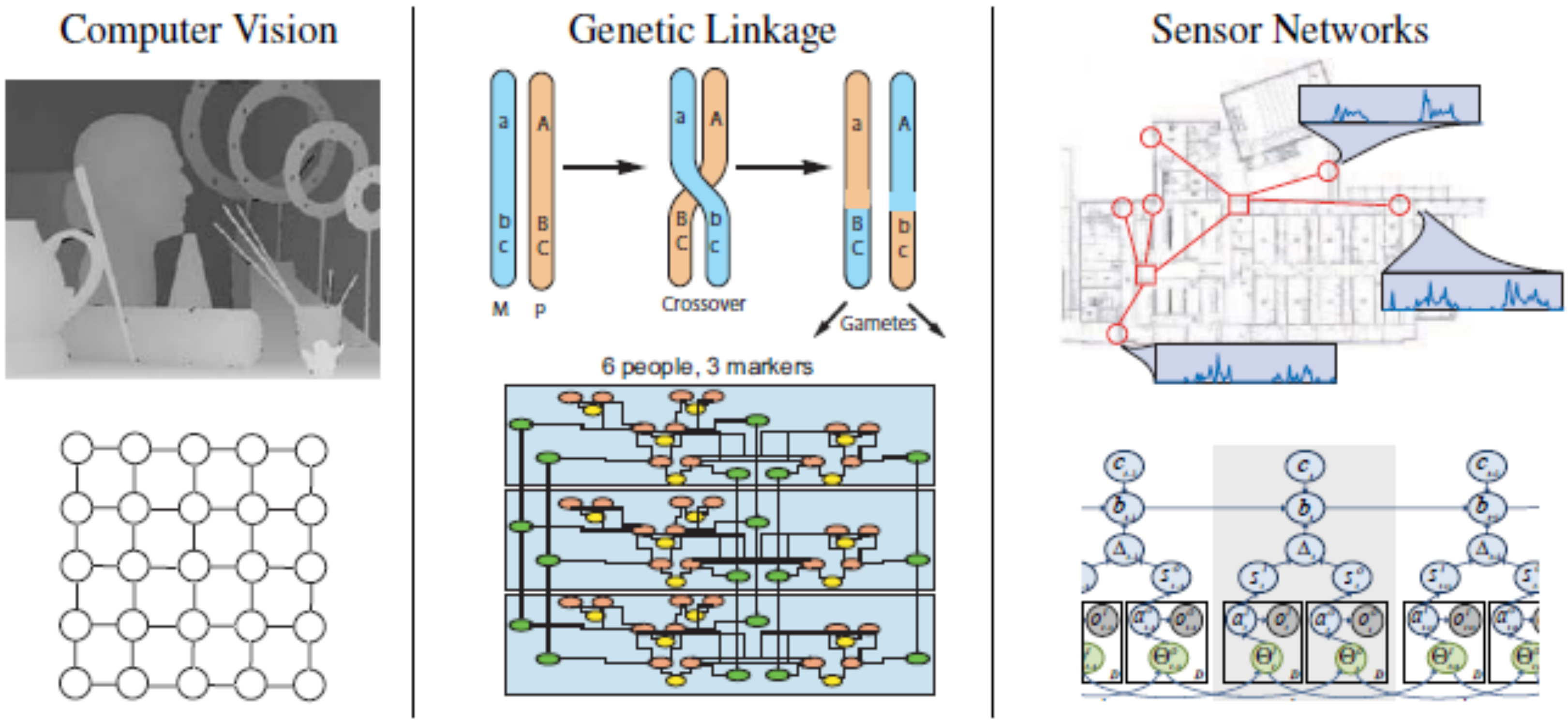
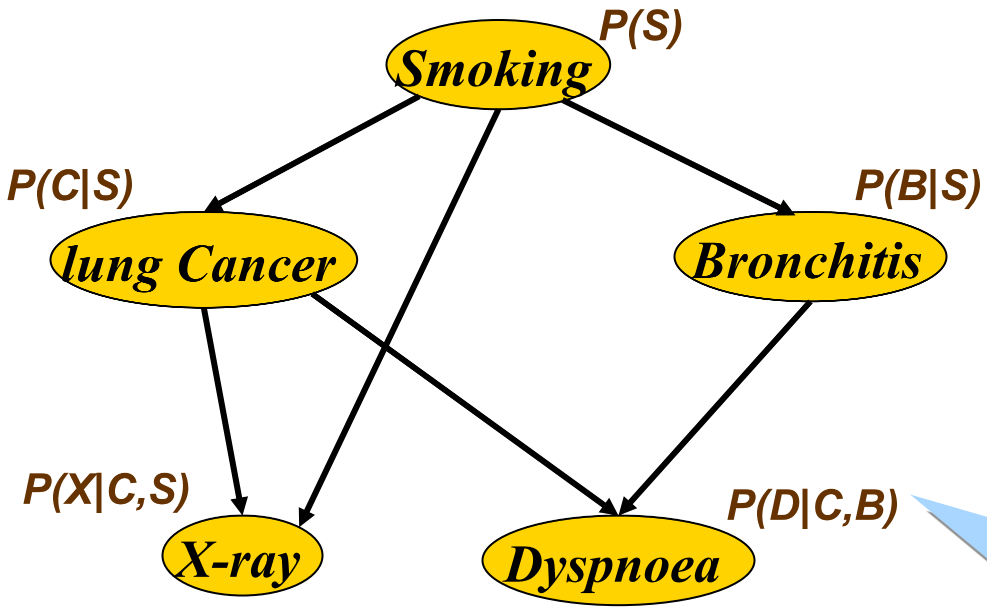


Figure 1: Application areas and graphical models used to represent their respective systems: (a) Finding correspondences between images, including depth estimation from stereo; (b) Genetic linkage analysis and pedigree data; (c) Understanding patterns of behavior in sensor measurements using spatio-temporal models.

Bayesian Networks (Pearl 1988)



BN = (G, Θ)

CPD:

C	B	P(D|C,B)	
0	0	0.1	0.9
0	1	0.7	0.3
1	0	0.8	0.2
1	1	0.9	0.1

$P(S, C, B, X, D) = P(S)\, P(C|S)\, P(B|S)\, P(X|C,S)\, P(D|C,B)$

$$P(x_1 \ldots x_n) = \prod_i p(x_i \mid pa(x_i))$$

$$P(e) = \sum_{X-E} \prod_i p(x_i \mid pa(x_i))$$

$$mpe = \max_x P(x)$$

Constraint Optimization Problems
for Graphical Models

A *finite* COP is a triple $R = \langle X, D, F \rangle$ where:

$X = \{X_1, ..., X_n\}$ - variables

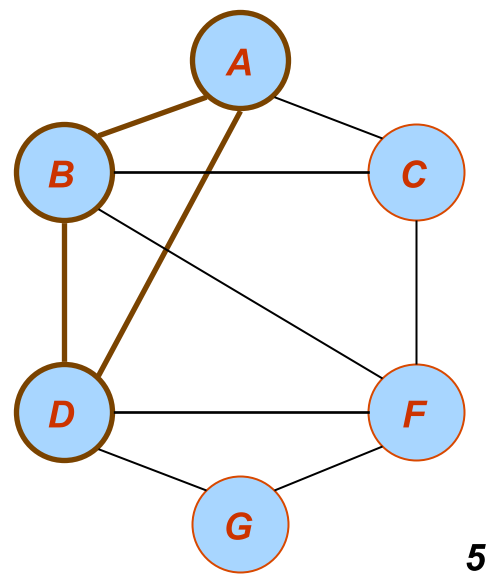
$D = \{D_1, ..., D_n\}$ - domains

$F = \{f_1, ..., f_m\}$ - cost functions

f(A,B,D) has scope {A,B,D}

A	B	D	Cost
1	2	3	3
1	3	2	2
2	1	3	∞
2	3	1	0
3	1	2	5
3	2	1	0

Primal graph =
Variables --> nodes
Functions, Constraints -→ arcs



$F(a,b,c,d,f,g) = f_1(a,b,d)+f_2(d,f,g)+f_3(b,c,f)$

Global Cost Function

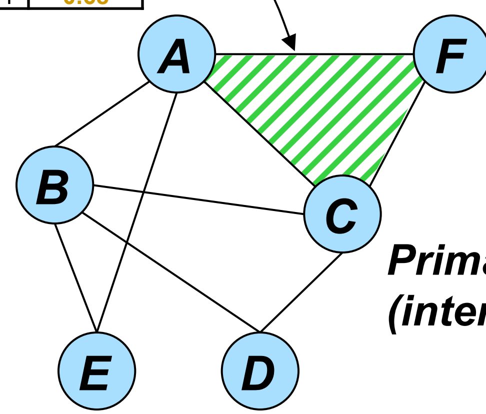
$$F(X) = \sum_{i=1}^{m} f_i(X)$$

Graphical Models

Conditional Probability or potential

- A graphical model **(X,D,F)**:
  - **X** = $\{X_1,\dots X_n\}$ variables
  - **D** = $\{D_1, \dots D_n\}$ domains
  - **F** = $\{f_1,\dots,f_r\}$ functions
    (potential, factors, CPTS, CNFs …)

A	C	F	P(F|A,C)
0	0	0	0.14
0	0	1	0.96
0	1	0	0.40
0	1	1	0.60
1	0	0	0.35
1	0	1	0.65
1	1	0	0.72
1	1	1	0.68



Primal graph (interaction graph)

- Tasks:
  - **Belief updating**: $\Sigma_{X-y} \prod_j P_i$
  - **MPE**: $\max_X \prod_j P_j$

The MRF

$\boldsymbol{x} = (x_1, \dots, x_n)$ to all the variables which maximizes the sum of the factors:

$$\mathrm{MAP}(\boldsymbol{\theta}) = \max_{\boldsymbol{x}} \sum_{i \in V} \theta_i(x_i) + \sum_{f \in F} \theta_f(\boldsymbol{x}_f). \quad (1.1)$$

PASCAL 2011 Probabilistic Inference Challenge

- http://www.cs.huji.ac.il/project/PASCAL/
- Evaluates solvers in three categories:
  - PR: Probability of evidence / partition function
  - MAR: Posterior node marginals
  - MPE: Most probable explanation (our entry)
- Three tracks each: 20 sec, 20 min, 1 hour.
- Variety of benchmark domains:
  - CSPs, Deep Belief Nets, Image Alignment and Segmentation, Object Detection, Protein Folding, …

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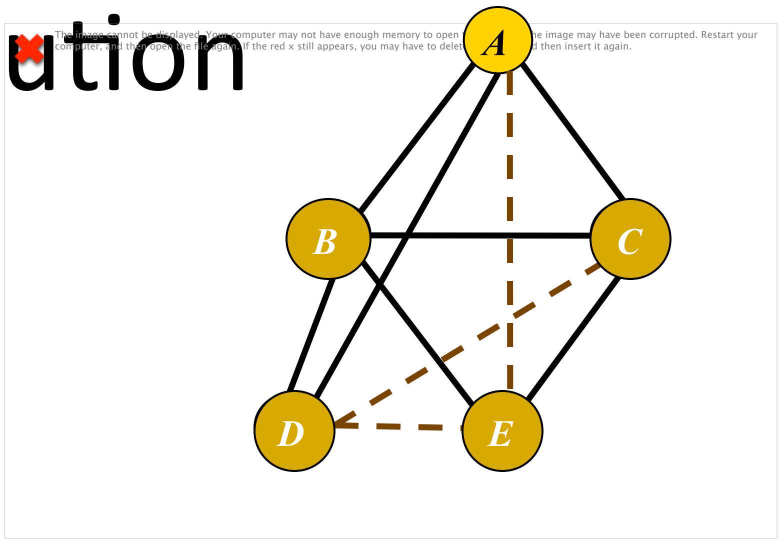
Computing the Optimal Cost Solution



$$OPT = \min_{e=0,d,c,b} f(a,b)f(a,c)f(a,d)f(b,c)f(b,d)f(b,e)f(c,e) =$$

Combination

$$\min_{e=0}\min_{d} f(a,d) \cdot \min_{c} f(a,c)f(c,e) \min_{b} f(a,b)f(b,c)f(b,d)f(b,e)$$

$$h^{B}(a,d,c,e)$$

Variable Elimination

Inference by Variable Elimination

Algorithm BE-mpe *(Dechter 1996, Bertele and Briochi, 1977)*

$$MPE = \max_{a,e,d,c,b} P(a)P(c|a)P(b|a)P(d|a,b)P(e|b,c)$$

$$\max_X \prod$$

bucket B: $P(b|a)$ $P(d|b,a)$ $P(e|b,c)$

bucket C: $P(c|a)$ $h^B(a,d,c,e)$

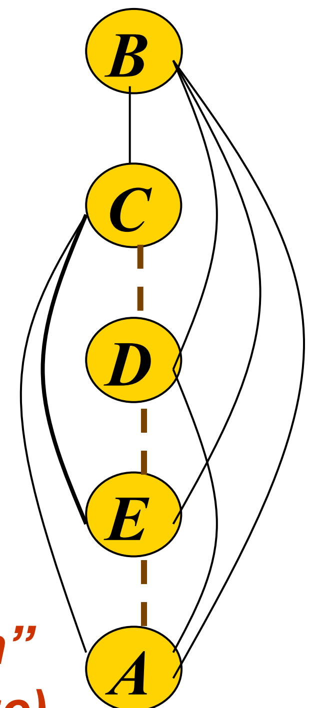
bucket D: $h^C(a,d,e)$

bucket E: $e=0$ $h^D(a,e)$

bucket A: $P(a)$ $h^E(a)$

OPT

W*=4
"induced width"
(max clique size)



Generating the MPE-tuple

5. $b' = \arg\max_b P(b \mid a') \times P(d' \mid b, a') \times P(e' \mid b, c')$

4. $c' = \arg\max_c P(c \mid a') \times h^B(a', d', c, e')$

3. $d' = \arg\max_d h^C(a', d, e')$

2. $e' = 0$

1. $a' = \arg\max_a P(a) \cdot h^E(a)$

B: $P(b|a)$ $P(d|b,a)$ $P(e|b,c)$

C: $P(c|a)$ $h^B(a,d,c,e)$

D: $h^C(a,d,e)$

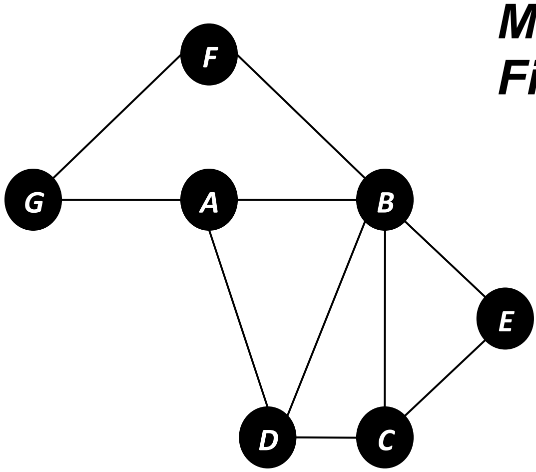
E: $e=0$ $h^D(a,e)$

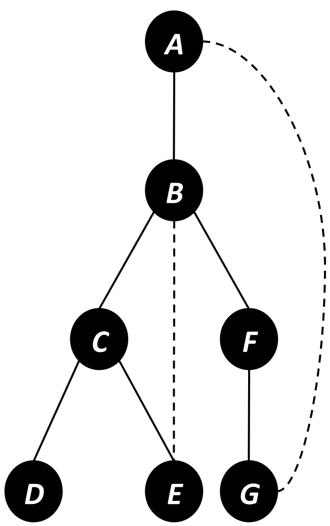
A: $P(a)$ $h^E(a)$

Return (a', b', c', d', e')

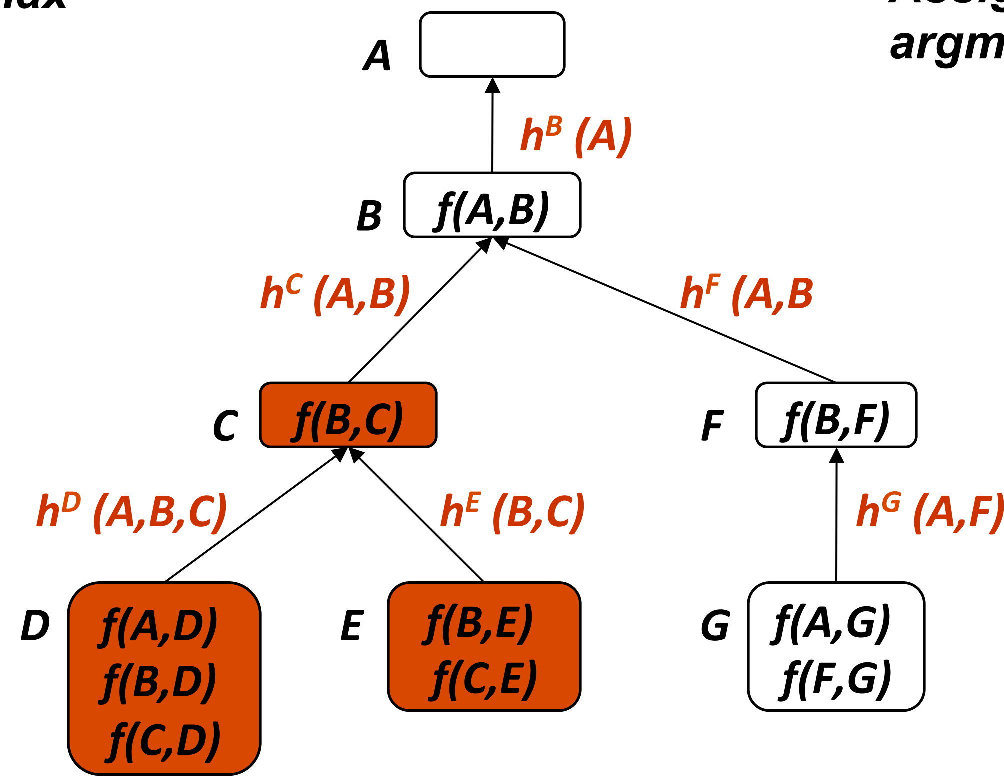
Bucket Elimination

$$\min_{a,b,c,d,e,f,g} f(a,b)+f(a,d)+f(b,c)+f(a,d)+f(b,d)+f(c,d)+f(b,e)+f(c,e)+f(b,f)+f(a,g)+f(f,g)=$$





Messages Finding max



Assignment argmax

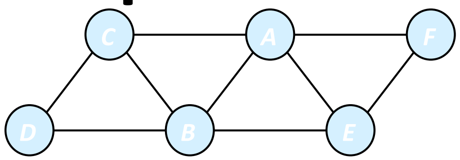
$$O(n\exp(w^*(d)))$$

Ordering: (A, B, C, D, E, F, G)

Outline

- Introduction
- Inference
- **Search (OR)**
  - Branch-and-Bound and Best-First search
- Lower-bounds and relaxations
- Exploiting problem structure in search
- Software

An Optimal Solution



A	B	f_1
0	0	2
0	1	0
1	0	1
1	1	4

A	C	f_2
0	0	3
0	1	0
1	0	0
1	1	1

A	E	f_3
0	0	0
0	1	3
1	0	2
1	1	0

A	F	f_4
0	0	2
0	1	0
1	0	0
1	1	2

B	C	f_5
0	0	0
0	1	1
1	0	2
1	1	4

B	D	f_6
0	0	4
0	1	2
1	0	1
1	1	0

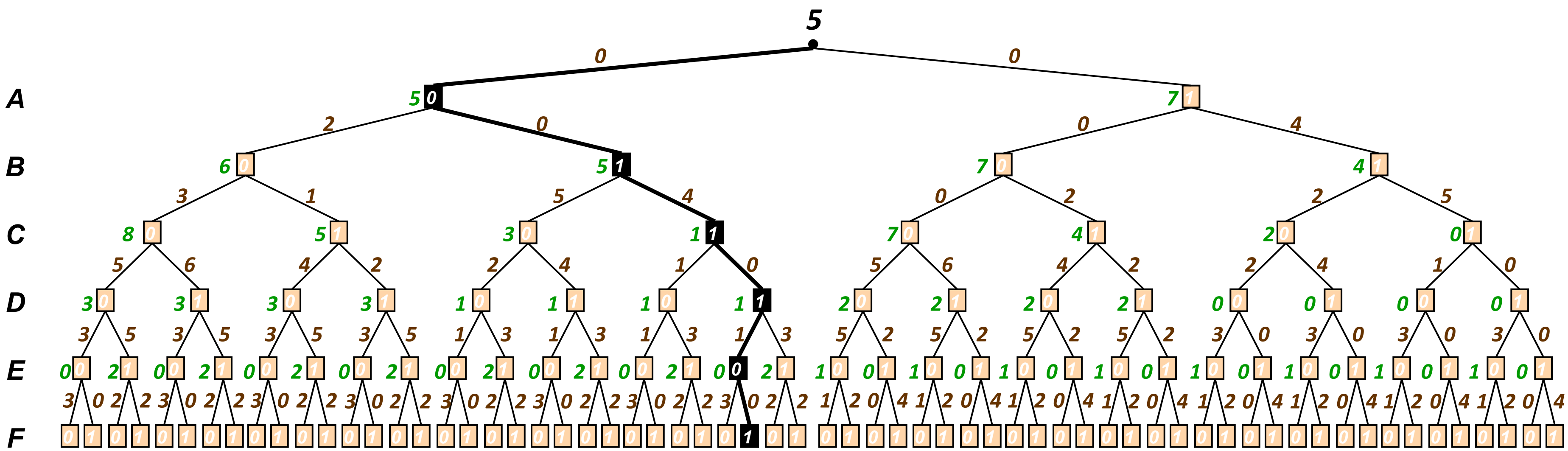
B	E	f_7
0	0	3
0	1	2
1	0	1
1	1	0

C	D	f_8
0	0	1
0	1	4
1	0	0
1	1	0

E	F	f_9
0	0	1
0	1	0
1	0	0
1	1	2

$$f(\mathbf{X}) = \sum_{i=1}^{9} f_i(\mathbf{X})$$

$$\min_{a,b,c,d,e,f} f_1(a,b) + f_2(a,c) + f_3(a,f) + f_4(b,c) + f_5(b,d) + f_6(b,e) + f_7(c,d) + f_8(e,f)$$

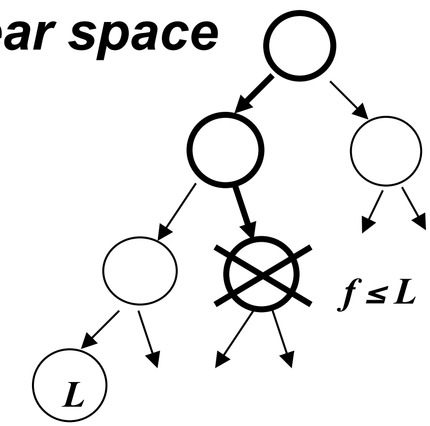


An optimal assignment is A=0, B=1, C=1, D=1, E=0, F=1 with cost 5

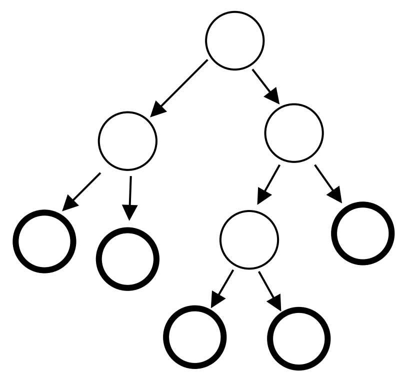
Basic Heuristic Search Schemes

Heuristic function $f(x^p)$ computes a lower bound on the best extension of x^p and can be used to guide a heuristic search algorithm. We focus on:

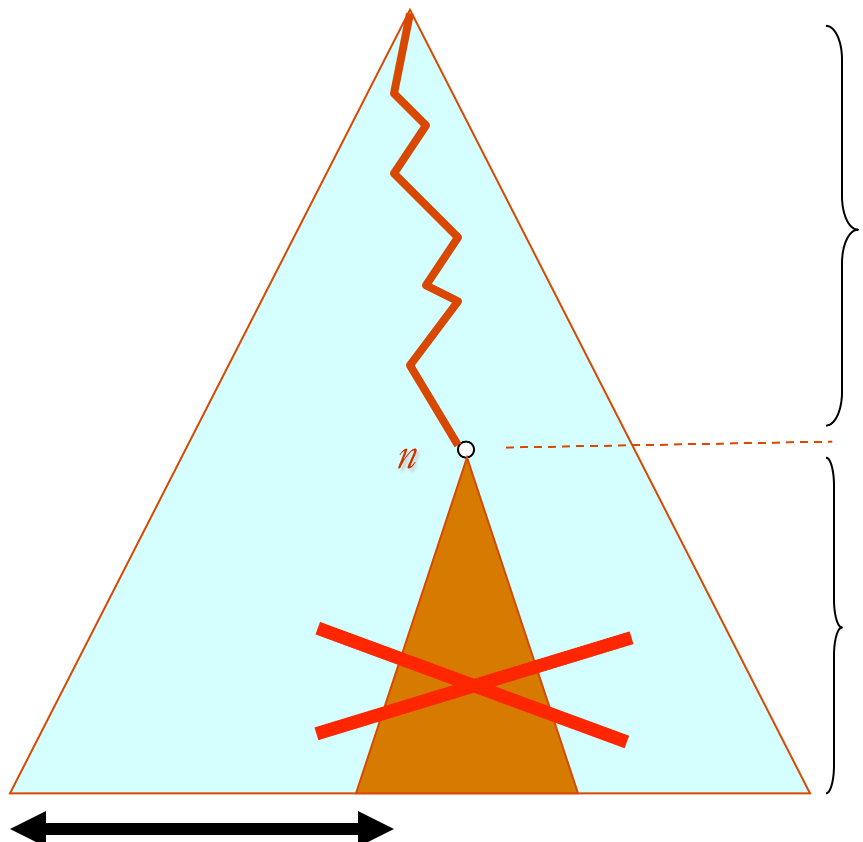
1. Branch-and-Bound
Use heuristic function $f(x^p)$ to prune the depth-first search tree
Linear space



2. Best-First Search
Always expand the node with the highest heuristic value $f(x^p)$ needs lots of memory



Classic Branch-and-Bound



g(n)

f(n) = g(n) + h(n)
f(n) = lower bound

Prune if f(n) ≥ UB

h(n) - under-estimates
Optimal cost below n

(UB) Upper Bound = best solution so far

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- Bound (lower or upper) approximations
  - The mini-bucket scheme
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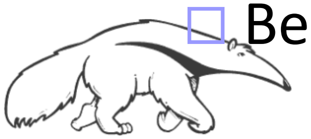
Primary Bounding Schemes

- Goal: bound $\min_x \sum_i f_i(x)$ or $\max_x \prod_i f_i(x)$
- **Node duplication control:**
  - Mini-bucket scheme: (Dechter and Rish 1997,2003, Kask and Dechter, 1999, Rollon and Dechter 2010)
- **Reparameterization schemes:**
  - Soft arc-consistency (Bistareli, 2000, Sciex 2000)
  - Linear relaxation/ Dual-decomposition: (Globerson and Jaakkola 2007, Sontag, Globerson and Jaakkola, 2010, Kovalevsky et al. 1975)
  - Belief Propagation can be viewed as re-parameterization



Mini-bucket Approximation
(Dechter and Rish, 1997, 2003)

Split a bucket into mini-buckets =>bound complexity

$$\textbf{bucket}(X) = \{ h_1, \ldots, h_r, h_{r+1}, \ldots, h_n \}$$

$$h^X = \max_X \prod_{i=1}^{n} h_i$$

$$\{ h_1, \ldots, h_r \} \qquad \{ h_{r+1}, \ldots, h_n \}$$

$$g^X = \left(\max_X \prod_{i=1}^{r} h_i \right) \cdot \left(\max_X \prod_{i=r+1}^{n} h_i \right)$$

$$h^X \le g^X$$

Exponential complexity decrease : $O(e^n) \to O(e^r) + O(e^{n-r})$

Mini-Bucket Elimination

$max_B \prod$ $max_B \prod$

Node duplication, renaming

Bucket B **P(E|B,C)** **P(B|A) P(D|A,B)**

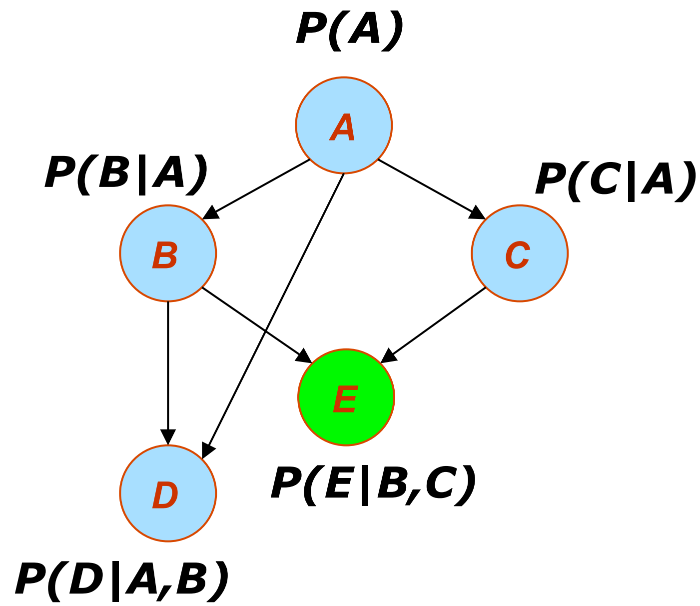
Bucket C **P(C|A) h^B (C,E)**

Bucket D **h^B (A,D)**

Bucket E **E = 0 h^C (A,E)**

Bucket A **P(A) h^E (A) h^D (A)**

W=2

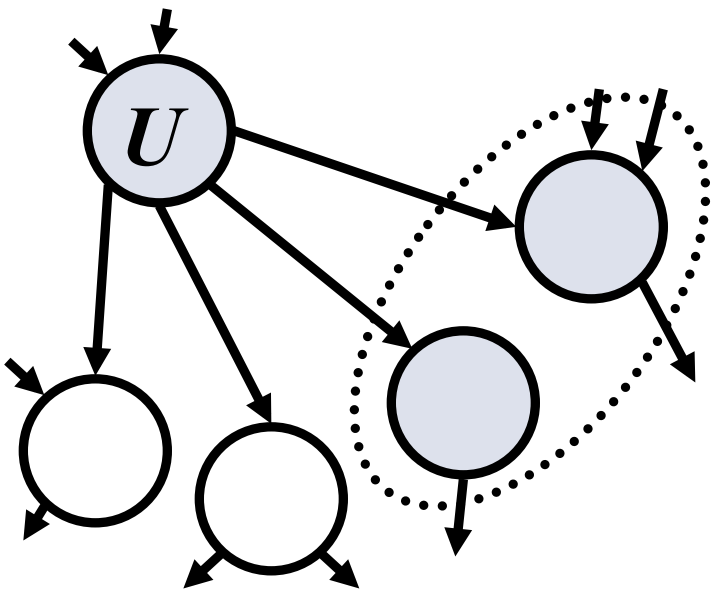


MPE* is an upper bound on MPE --U
Generating a solution yields a lower bound--L

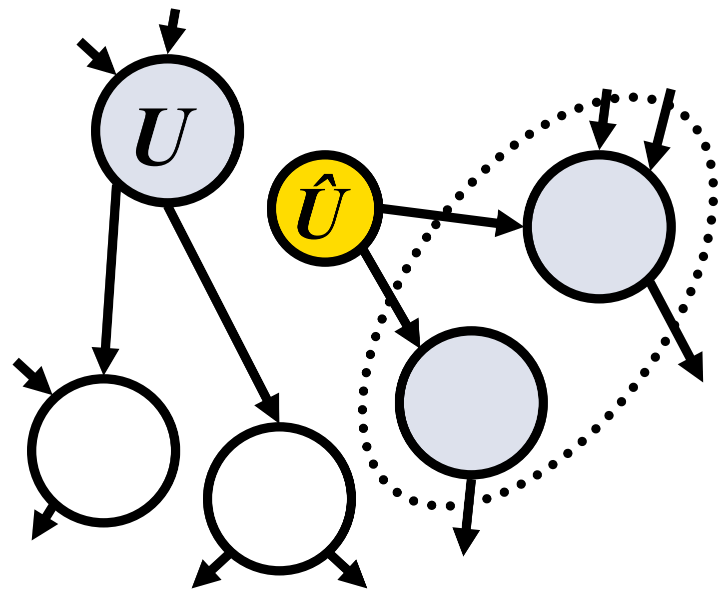
Semantics of Mini-Bucket: Splitting a Node

Variables in different buckets are renamed and duplicated
(Kask and Dechter, 2001), (Geffner et. al., 2007), (Choi, Chavira, Darwiche , 2007)

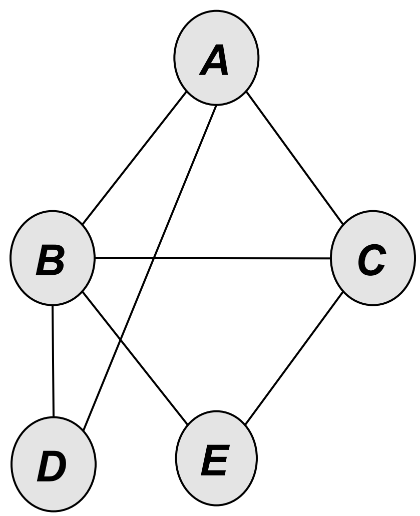
Before Splitting:
Network N



After Splitting:
Network N'



Mini-Bucket Elimination Semantic



Mini-buckets

$\min_B \Sigma$ $\min_B \Sigma$

bucket B: *F(a,b') F(b',c)*   *F(b,d) F(b,e)*

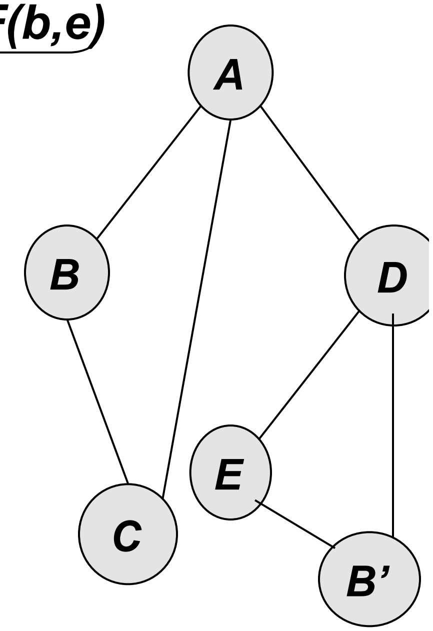
bucket C: *h^B(a,c) F(c,e) F(a,c)*

bucket D: *F(a,d) h^B(d,e)*

bucket E: *h^C(e,a) h^D(e,a)*

bucket A: *h^E(a)*

L = lower bound

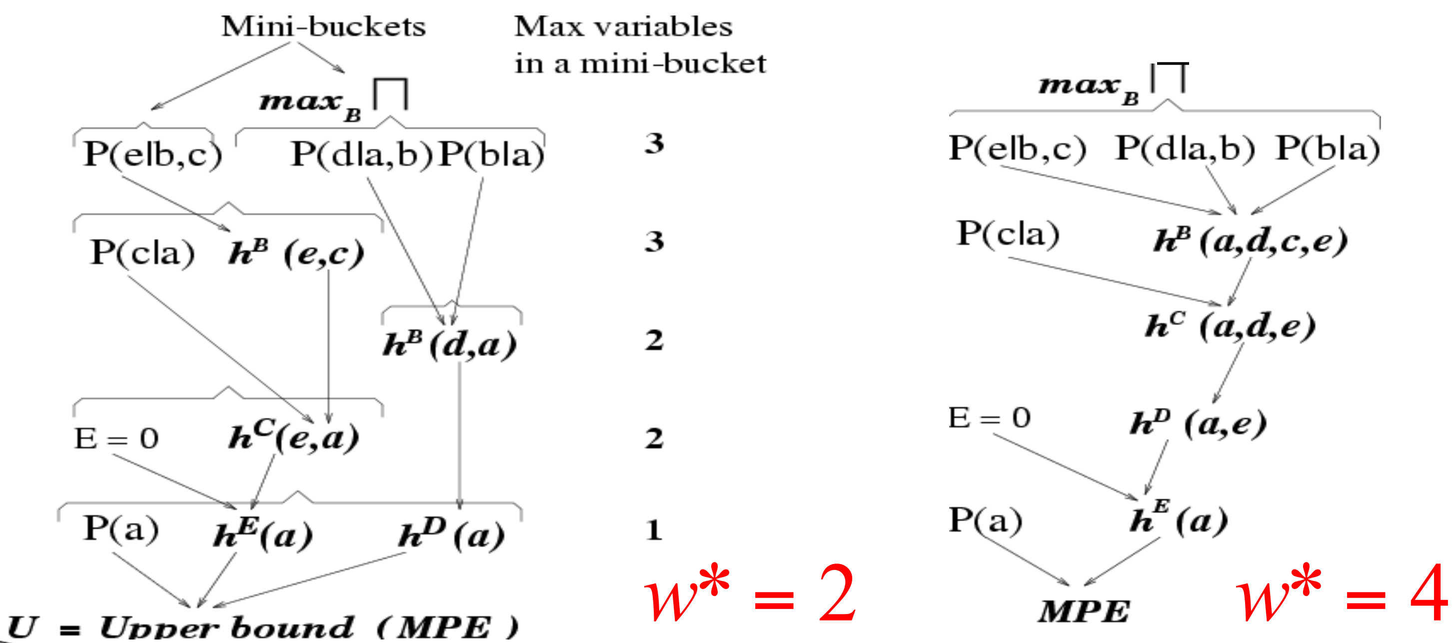


MBE-MPE(i)

Algorithm **Approx-MPE** (Dechter & Rish, 1997)

- **Input**: i – max number of variables allowed in a mini-bucket
- **Output**: [lower bound (P of a sub-optimal solution), upper bound]

Example: approx-mpe(3) versus elim-mpe

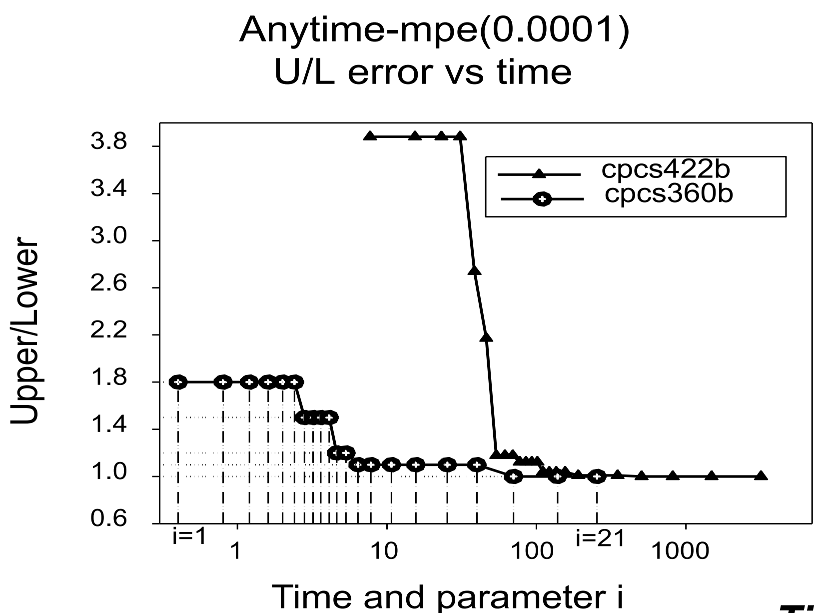


Properties of MBE(i)

- **Complexity:** *O(r exp(i))* time and *O(exp(i)) space*.
- Yields an upper-bound and a lower-bound.
- **Accuracy:** determined by upper/lower (U/L) bound.
- As ***i*** increases, both accuracy and complexity increase.
- Possible use of mini-bucket approximations:
  - As anytime algorithms
  - As heuristics in search
- Other tasks: similar mini-bucket approximations for: belief updating, MAP and MEU (Dechter and Rish, 1997)

CPCS Networks – Medical Diagnosis
(noisy-OR CPD's)

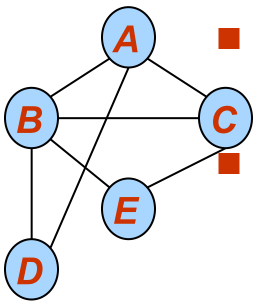
Test case: no evidence



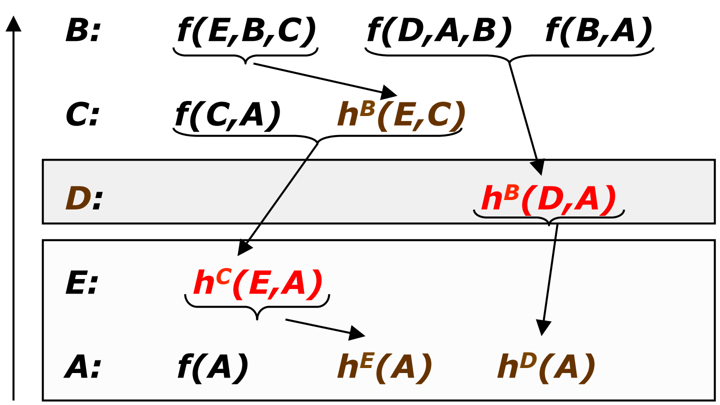
Algorithm	***Time (sec)*** ***cpcs360***	***cpcs422***
elim-mpe	***115.8***	***1697.6***
anytime-mpe(ε***),*** $\varepsilon = 10^{-4}$	***70.3***	***505.2***
anytime-mpe(ε***),*** $\varepsilon = 10^{-1}$	***70.3***	***110.5***

Static MBE Heuristics

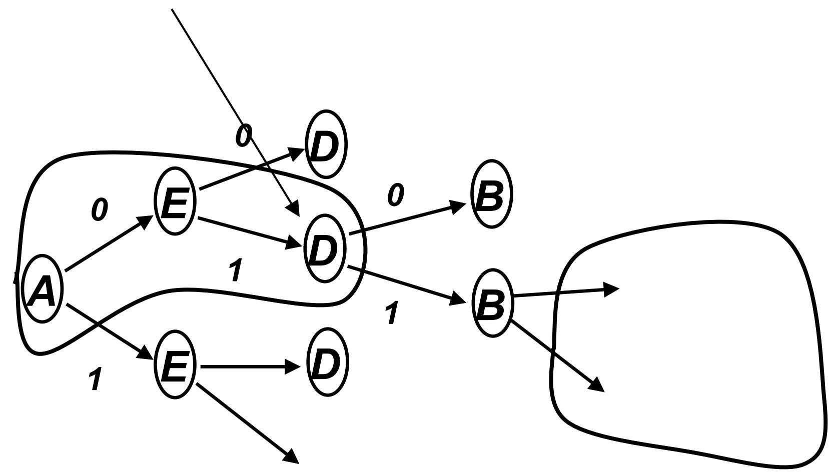
- Given a partial assignment x^p, estimate the cost of the best extension to a full solution
- The evaluation function $f(x^p)$ can be computed using function recorded by the Mini-Bucket scheme
- Heuristic is consistent and admissible.



Cost Network



$f(a,e,D))=g(a,e) + H(a,e,D)$

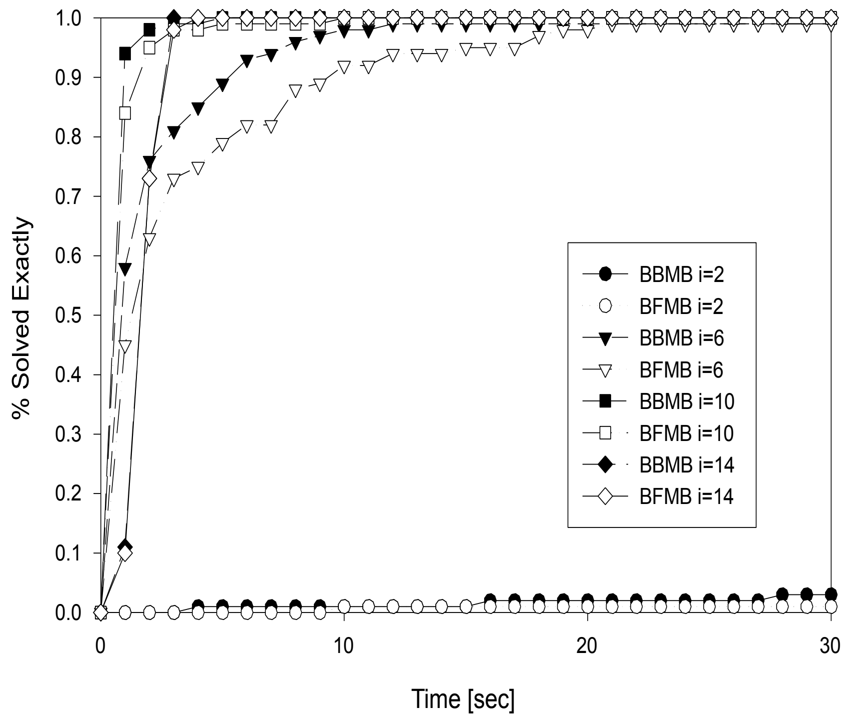


$$f(a,e,D) = \underbrace{f(a)}_{g} + \underbrace{h^B(D,a) + h^C(e,a)}_{h \text{ – is admissible}}$$

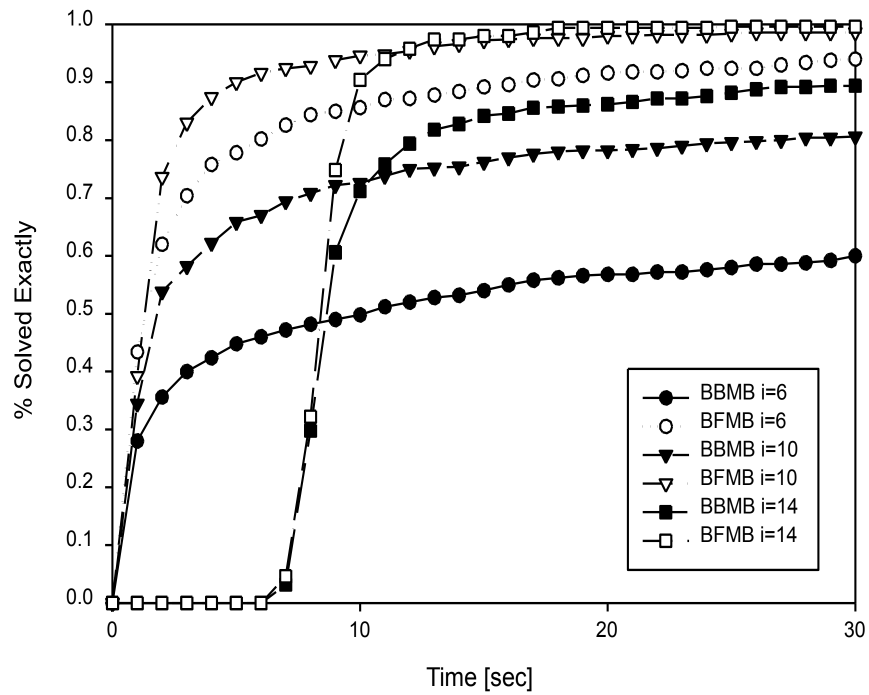
Empirical Evaluation of Mini-Bucket heuristics:

Random coding networks (Kask & Dechter, UAI'99, Aij 2000)

Random Coding, K=100, noise=0.28



Random Coding, K=100, noise=0.32



Each data point represents an average over 100 random instances

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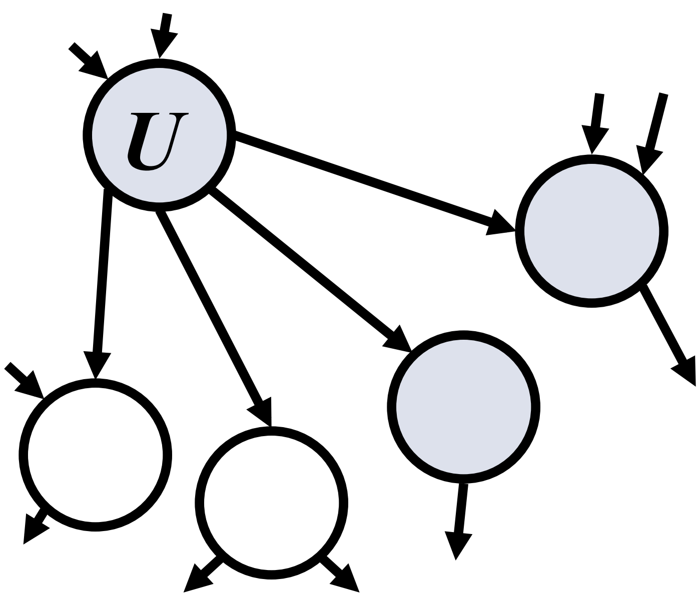
Semantics of Dual Decomposition: Each Functions is a Mini-Bucket + Reparameterization

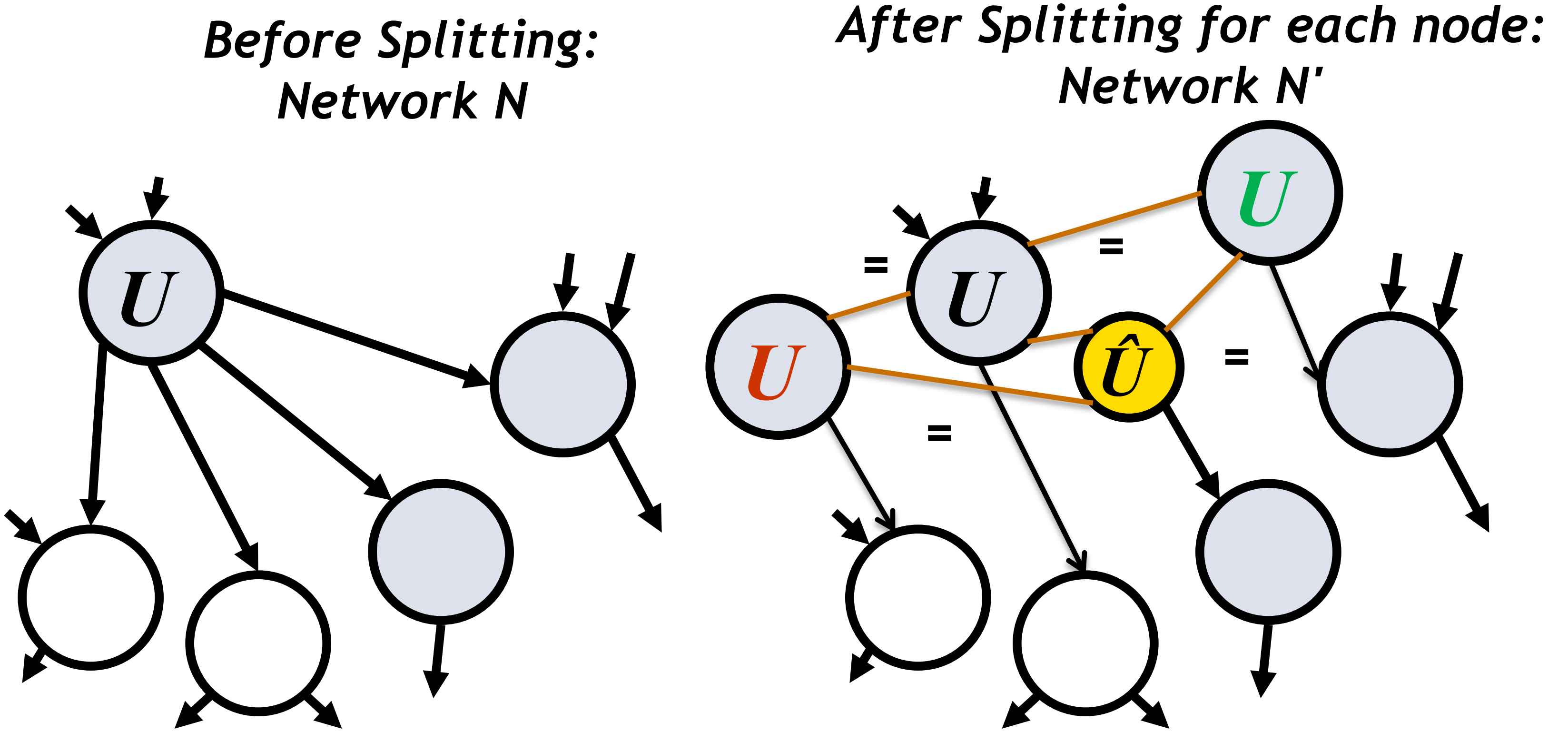
Variables in different buckets are renamed and duplicated (Globerson and Jakkola, 2008),

Before Splitting:
Network N



Re-parameterization: Duplicating a Node for Each Arc/Function

Variables in different buckets are renamed and duplicated (Globerson and Jakkola, 2008),

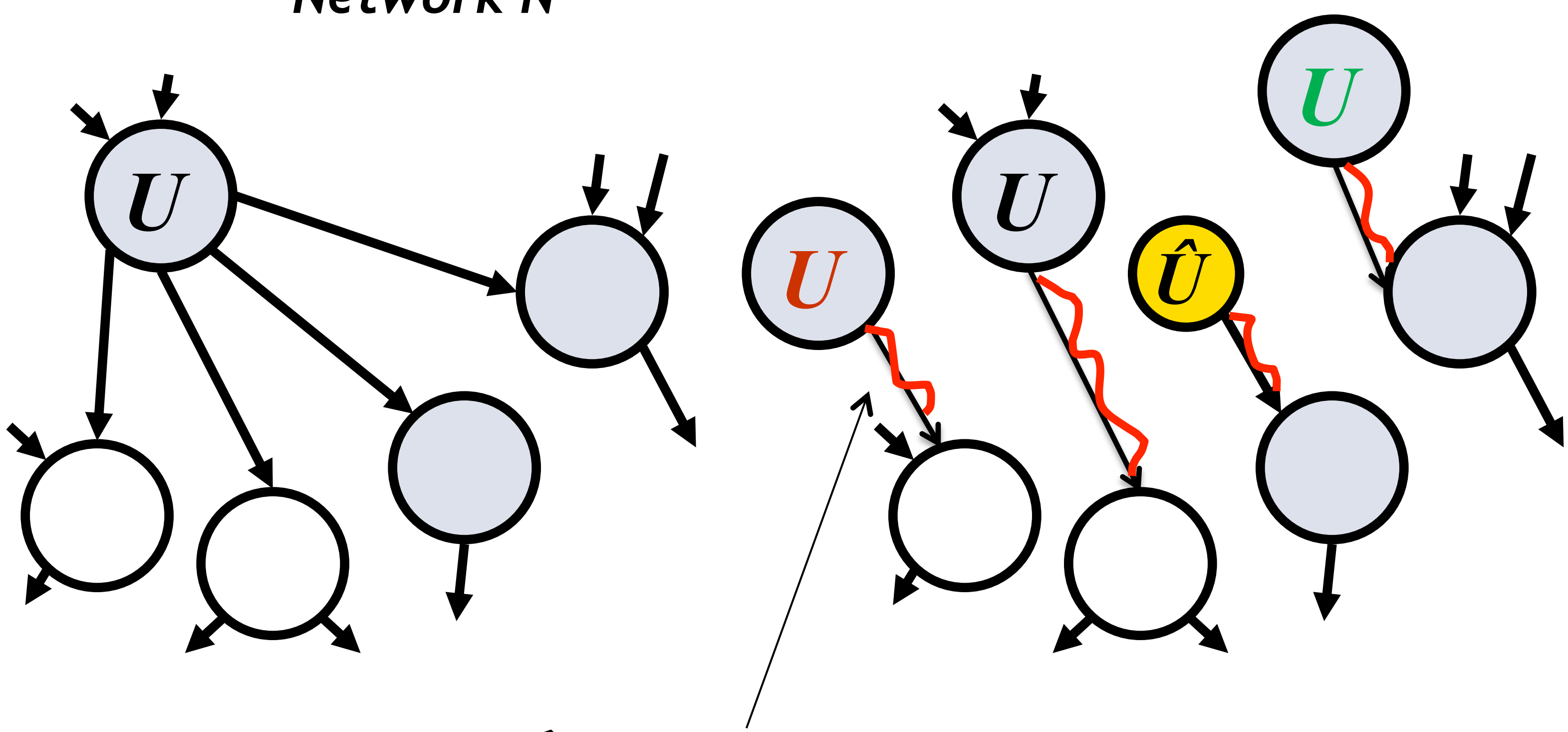


Re-parameterization: Duplicating a Node for Each Arc/Function

Variables in different buckets are renamed and duplicated
(Globerson and Jakkola, 2008)

Before Splitting:
Network N

After Splitting:
Network N'



Reparameterize by cost shifting,
optimally by linear programming

The Principle of Cost-Shifting

$$C^* = \max_{\mathbf{X}} \sum_{(ij)\in F} f_{ij}(X_i, X_j) \leq \sum_{(ij)\in F} \max_{\mathbf{X}} f_{ij}(X_i, X_j),$$

Introduce a collection of functions:

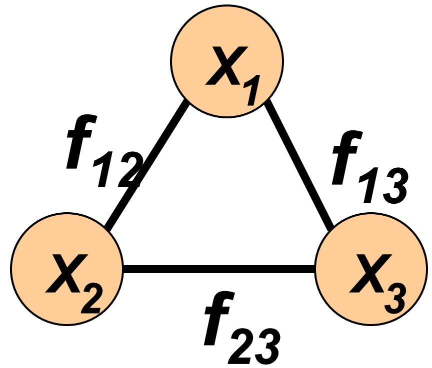
$$\lambda \in \Lambda \quad \Leftrightarrow \quad \forall i, \quad \sum_j \lambda_{ij}(X_i) = 0$$

Then, we have

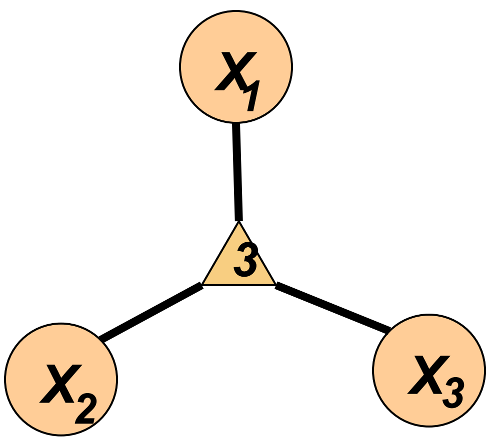
$$\begin{aligned} C^* &= \max_{\mathbf{X}} \sum_{(ij)\in F} f_{ij}(X_i, X_j) \\ &= \max_{\mathbf{X}} \sum_{(ij)\in F} f_{ij}(X_i, X_j) + \sum_i \sum_j \lambda_{ij}(X_i) \qquad (3) \\ &\leq \min_{\lambda\in\Lambda} \sum_{(ij)\in F} \max_{\mathbf{X}} \left(f_{ij}(X_i, X_j) + \lambda_{ij}(X_i) + \lambda_{ji}(X_j) \right) \end{aligned}$$

Bounding by Full Decomposition

Original problem:

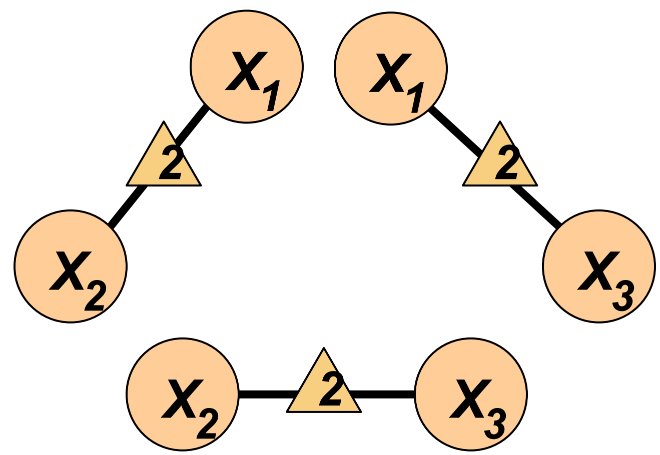


Exact solution:



$\leq$

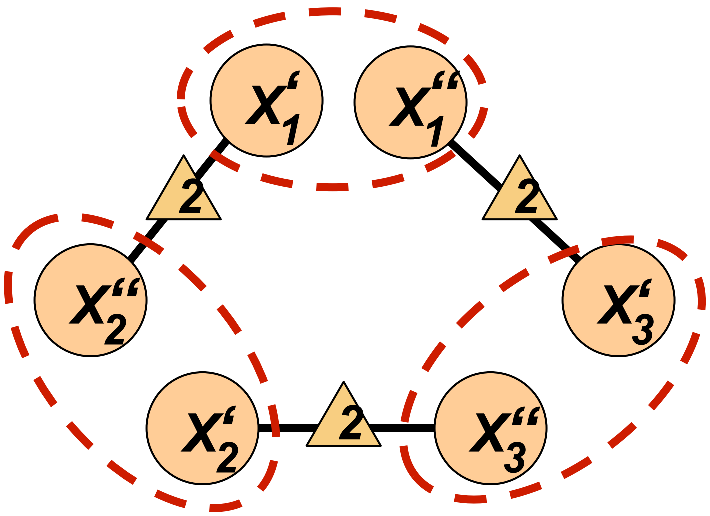
Upper bound:



$\max(f_{12}+f_{13}+f_{23})$

$\max f_{12}+\max f_{13}+\max f_{23}$

Tightening the upper bound



maximize each factor independently

$$\max f_{12} + \max f_{13} + \max f_{23}$$

subject to $X'_1 = X''_1,\ X'_2 = X''_2,\ X'_3 = X''_3$

Introduce functions
$\lambda_{ij}(X_i), \lambda_{ji}(X_j)$ *for each edge (ij)*
whose sum is 0

minimize upper bound:

$$\min_\lambda \sum_{(ij)} \max_X \left(f_{ij}(X_i, X_j) + \lambda_{ij}(X_i), \lambda_{ji}(X_j)\right)$$

re-parameterization

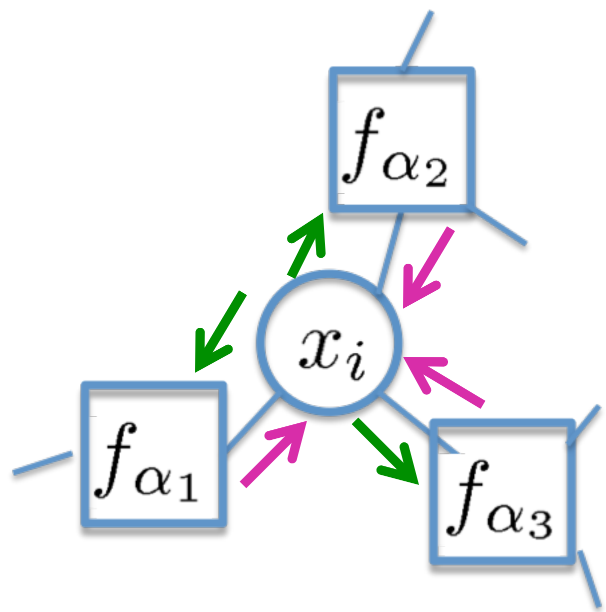
LP-tightening: use message passing, based on coordinate descent or gradient, sub-gradient approaches.

λ's   *Lagrangian multipliers*

Factor graph Linear Programming

- Update the original factors (FGLP)
  - Tighten all factors over over x_i simultaneously
  - Compute max-marginals $\forall \alpha, \ \gamma_\alpha(x_i) = \max_{x_\alpha \setminus x_i} f_\alpha$
  - & update:

$$\forall \alpha, \ f_\alpha(x_\alpha) \leftarrow f_\alpha(x_\alpha) - \gamma_\alpha(x_i) + \frac{1}{|F_i|} \sum_\beta \gamma_\beta(x_i)$$

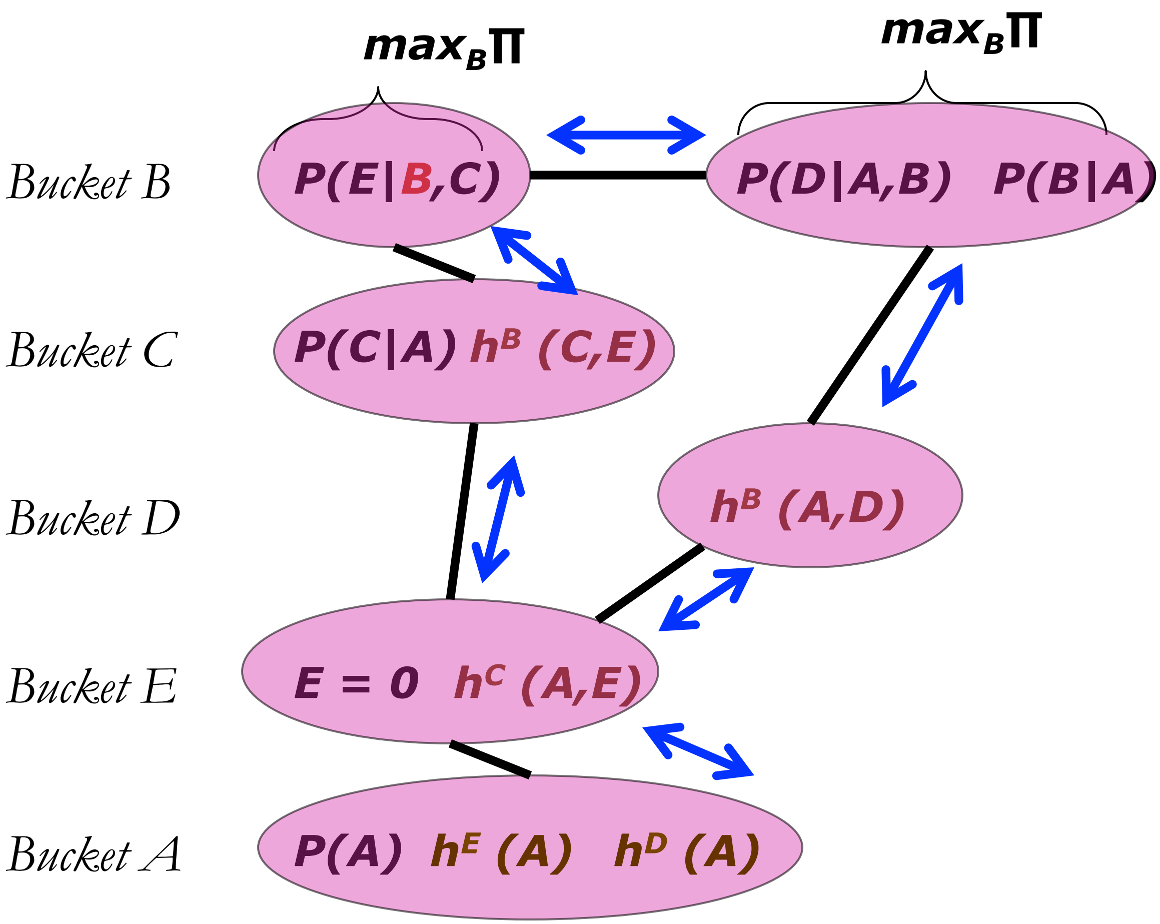


Combining Mini-buckets with cost-shifting

- Use mini-bucket as structuring a join-graph with a given i-bound: JGLP(i)

- Do mini-bucket with cost-shifting in each bucket: MBE-MM(i)

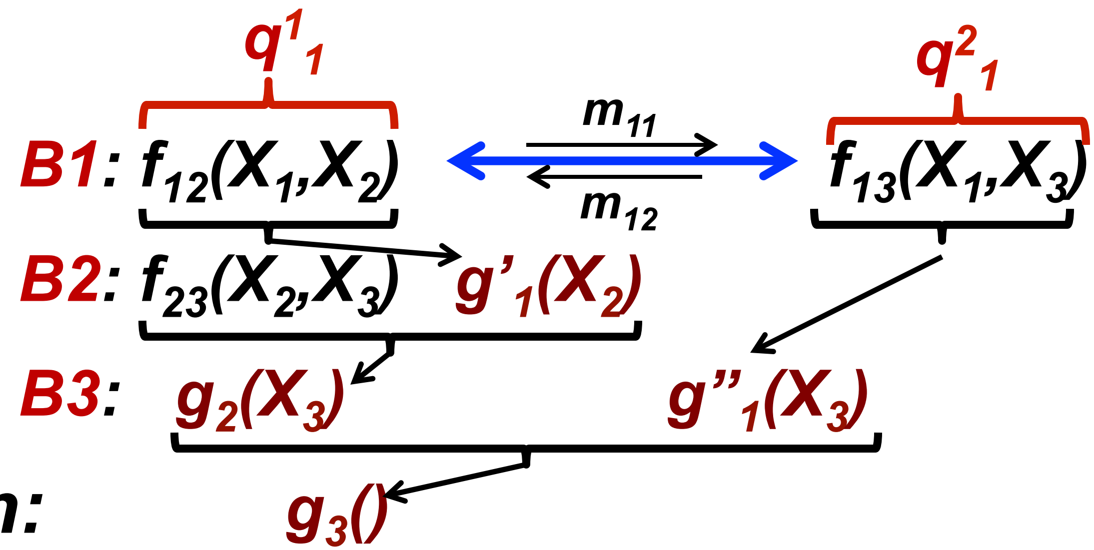
Join Graph Linear Programming (JGLP)



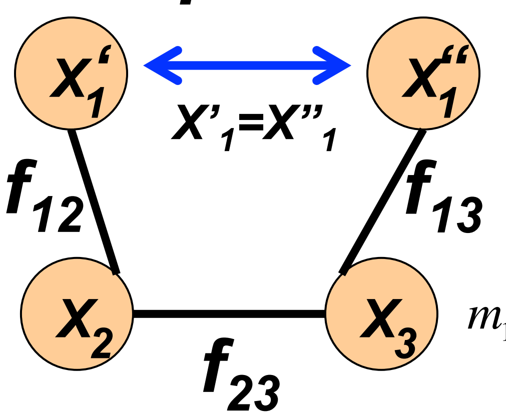
MB defines
A Join Graph

Mini-bucket elimination with moment-matching

Buckets and messages:



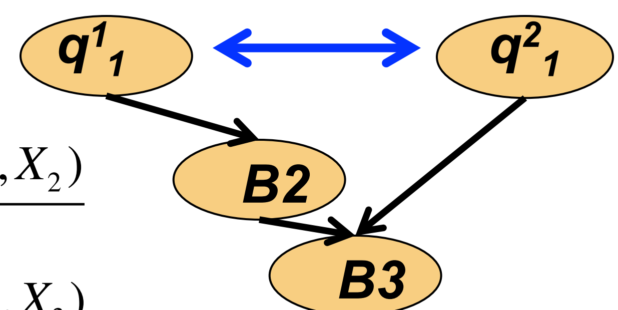
Interpretation:



$$m_{11} = \frac{\max_{X_3} f_{13}(X_1, X_3) - \max_{X_2} f_{12}(X_1, X_2)}{2}$$

$$m_{12} = \frac{\max_{X_2} f_{12}(X_1, X_2) - \max_{X_3} f_{13}(X_1, X_3)}{2}$$

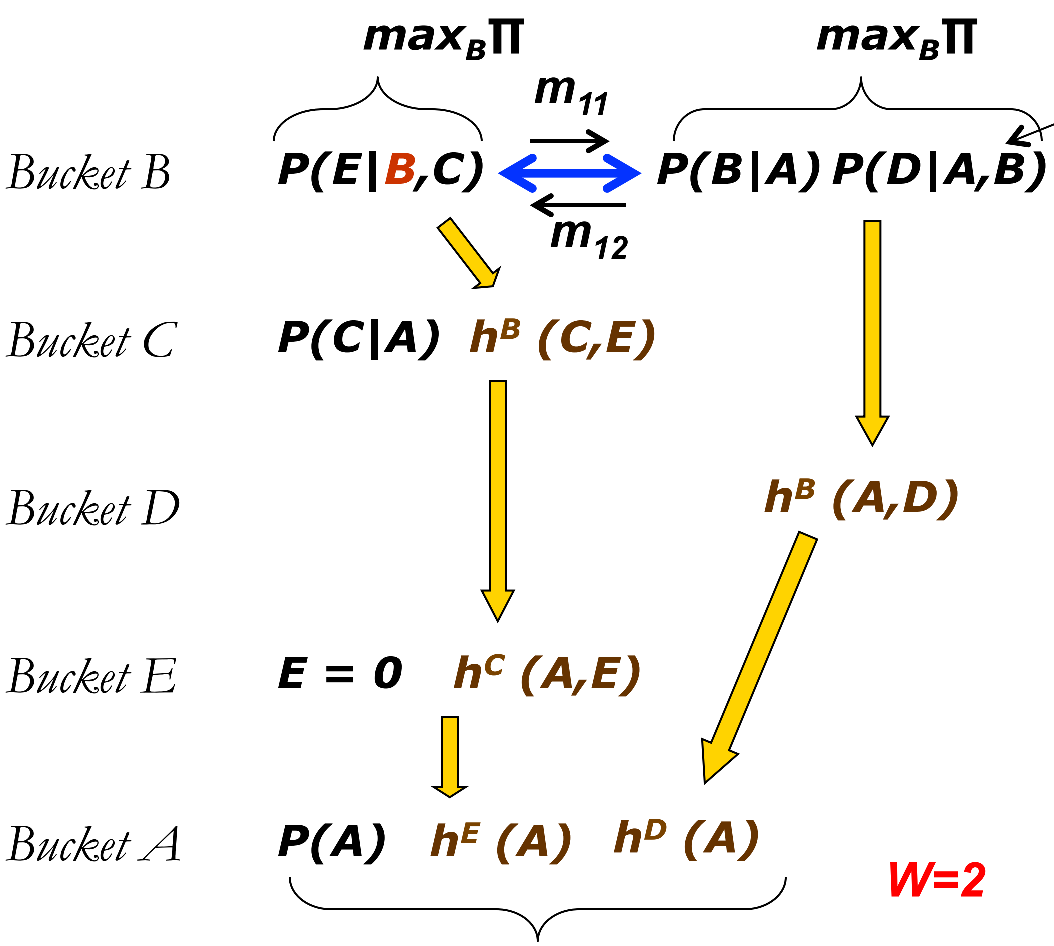
Bucket junction tree:

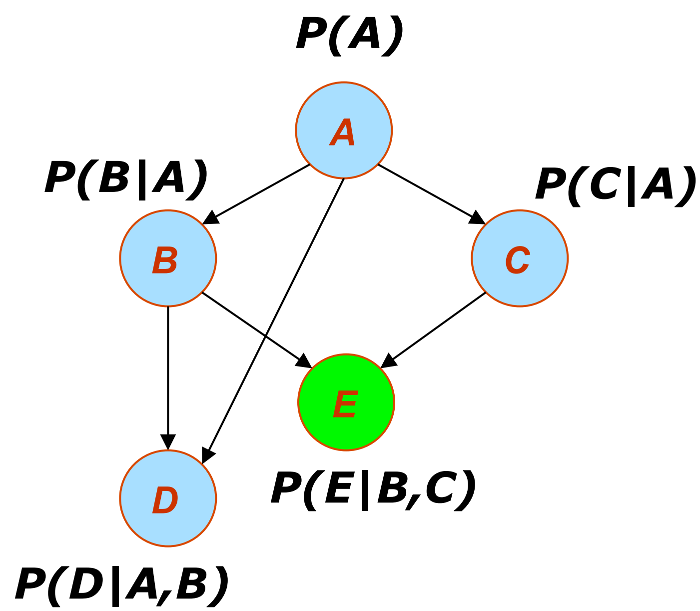


Mini-Bucket Elimination

m_{11}, m_{12}- moment-matching messages

Node duplication, renaming





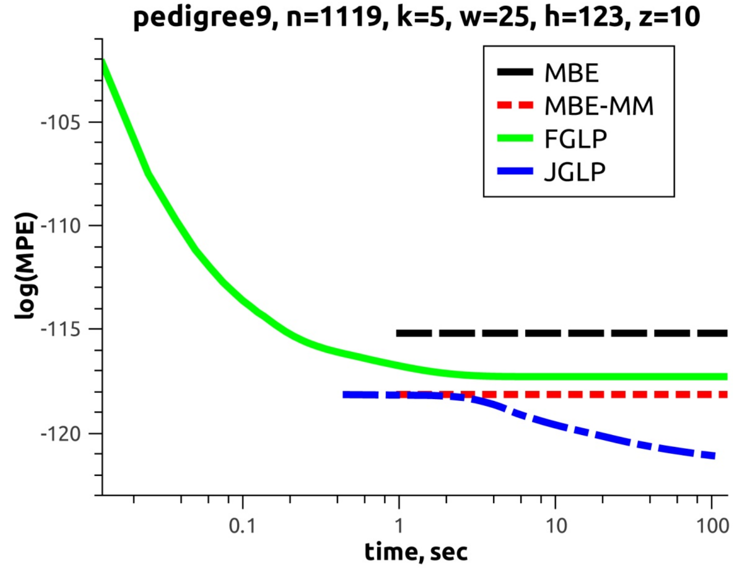
MPE* is an upper bound on MPE --U
Generating a solution yields a lower bound--L

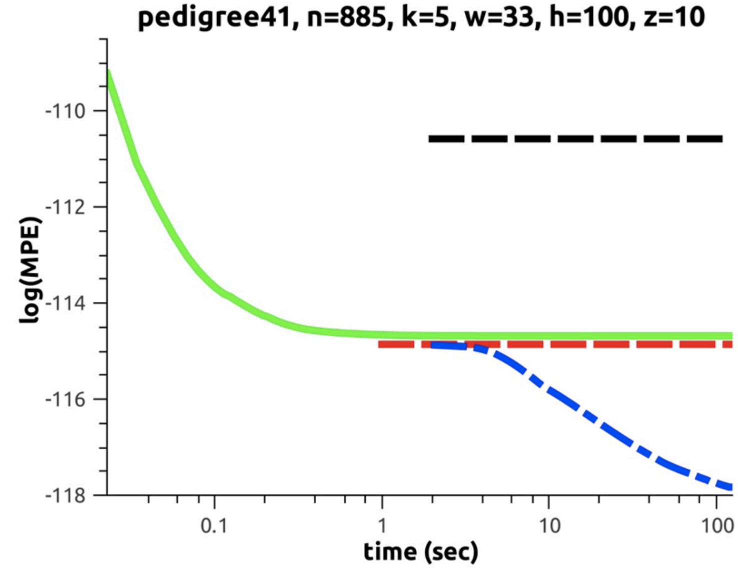
Iterative tightening as bounding schemes

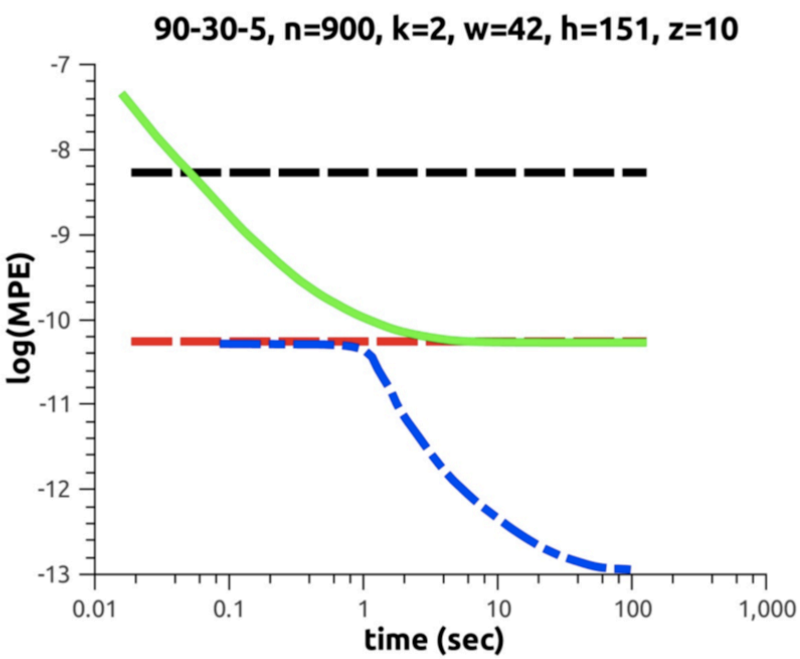
- 4 schemes: MBE, MBE-MM, FGLP, JGLP
  - Iterative schemes (FGLP, JGLP) ran for 5, 300, 3600 seconds

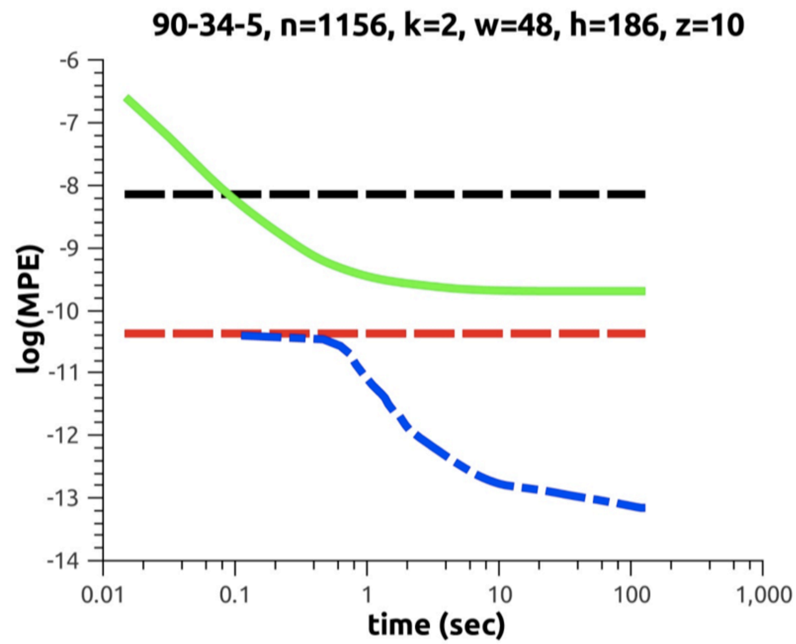
instance name	n	k	w	z	MBE	MBE-MM	FGLP time cut-offs			JGLP time cut-offs		
							5	300	3600	5	300	3600
					UB/time	UB/time	UB	UB	UB	UB	UB	UB
75-25-5	625	2	34	10 20	-15.4553/1 -17.4417/4	-18.4089/1 -20.0576/4	-16.6853	-16.6854	-16.6854	-20.0289 -20.0576	-20.8364 -20.1278	-20.8364 -20.7067
90-30-5	900	2	42	10 20	-8.2481/1 -9.7424/7	-10.2597/1 -11.6004/7	-10.2450	-10.2705	-10.2705	-11.8469 -11.6004	-12.9594 -11.6942	-13.015 -12.5259
90-34-5	1156	2	48	10 20	-8.42007/1 -9.58332/8	-10.3708/1 -12.3670/9	-9.65003	-9.69458	-9.69458	-12.3469 -12.3670	-13.2262 -12.5621	-13.2883 -13.1538
90-42-5	1764	2	60	10 20	-12.7401/1 -14.6136/13	-15.9680/1 -18.5487/14	-15.2480	-15.3653	-15.3653	-18.4100 -18.5487	-20.7714 -18.7679	-20.8136 -19.9705
largeFam4_11_51	1002	4	40	10 20	[illegible] OOM	[illegible] OOM	-201.582	-201.673	-201.673	[illegible] OOM	[illegible] OOM	[illegible] OOM
largeFam4_11_55	1114	4	38	10 20	-229.43/1 OOM	-242.489/1 OOM	-226.075	-226.328	-226.328	-242.657 OOM	-249.551 OOM	-250.453 OOM
largeFam4_12_51	1461	4	56	10 20	-218.229/2 OOM	-239.896/3 OOM	-217.564	-217.740	-217.740	-239.896 OOM	-245.900 OOM	-253.153 OOM
pedigree7	867	4	32	10 20	-105.854/1 -108.011/33	-109.569/1 -111.120/42	-110.179	-110.187	-110.187	-109.960 OOM	-110.810 OOM	-111.293 OOM
pedigree13	888	3	32	10 20	-69.0973/1 -69.8890/8	-70.0999/1 -71.1071/11	-71.8561	-71.8591	-71.8591	-70.4581 -71.1071	-71.9869 -71.1071	-72.0374 -71.3658
pedigree31	1006	5	30	10 20	-125.032/1 OOM	-126.629/1 OOM	-126.667	-126.678	-126.678	-126.644 OOM	-129.158 OOM	-129.277 OOM
pedigree41	885	5	33	10 20	-110.156/1 -112.153/29	-114.858/1 -117.638/37	-114.681	-114.681	-114.681	-115.050 OOM	-118.133 OOM	-118.419 OOM
type4_120_17	4302	5	23	10 20	-1128.22/1 -1235.94/18	-1203.08/1 -1237.95/21	-1049.34	-1049.85	-1049.86	-1203.21 -1237.95	-1221.21 OOM	-1223.69 OOM
type4_170_23	6933	5	21	10 20	-1682.9/1 -1783.18/7	-1747.18/1 -1783.76/7	-1509.96	-1511.61	-1511.65	-1747.22 -1783.76	-1769.96 -1783.76	-1772.16 -1783.76

Iterative tightening as bounding schemes





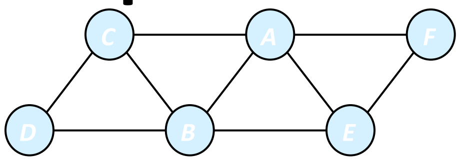




Outline

- Graphical models: reasoning principles
- Inference
- Advancing Search via AND/OR Search
- Lower Bounding schemes for inference
- Lower-bounding heuristic for AND/OR search
- Experiments

An Optimal Solution



A	B	f_1
0	0	2
0	1	0
1	0	1
1	1	4

A	C	f_2
0	0	3
0	1	0
1	0	0
1	1	1

A	E	f_3
0	0	0
0	1	3
1	0	2
1	1	0

A	F	f_4
0	0	2
0	1	0
1	0	0
1	1	2

B	C	f_5
0	0	0
0	1	1
1	0	2
1	1	4

B	D	f_6
0	0	4
0	1	2
1	0	1
1	1	0

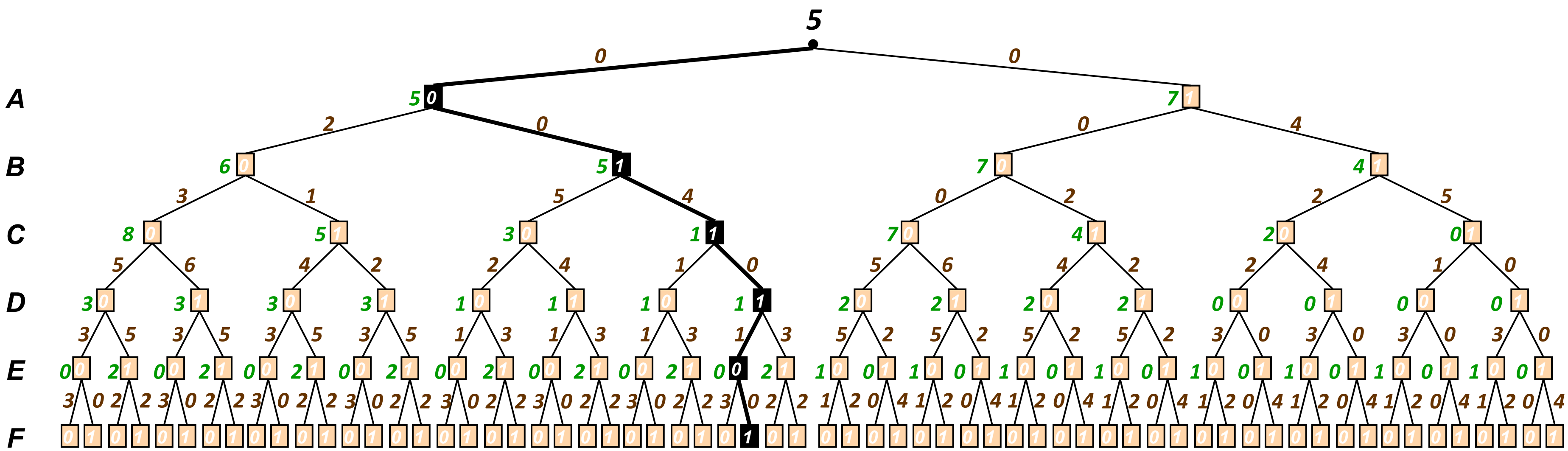
B	E	f_7
0	0	3
0	1	2
1	0	1
1	1	0

C	D	f_8
0	0	1
0	1	4
1	0	0
1	1	0

E	F	f_9
0	0	1
0	1	0
1	0	0
1	1	2

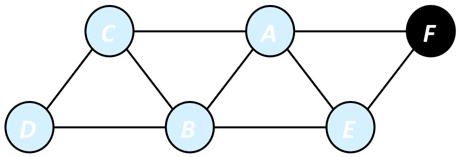
$$f(\mathrm{X}) = \sum_{i=1}^{9} f_i(\mathrm{X})$$

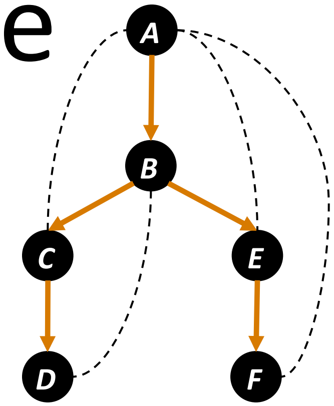
$$\min_{a,b,c,d,e,f} f_1(a,b) + f_2(a,c) + f_3(a,f) + f_4(b,c) + f_5(b,d) + f_6(b,e) + f_7(c,d) + f_8(e,f)$$



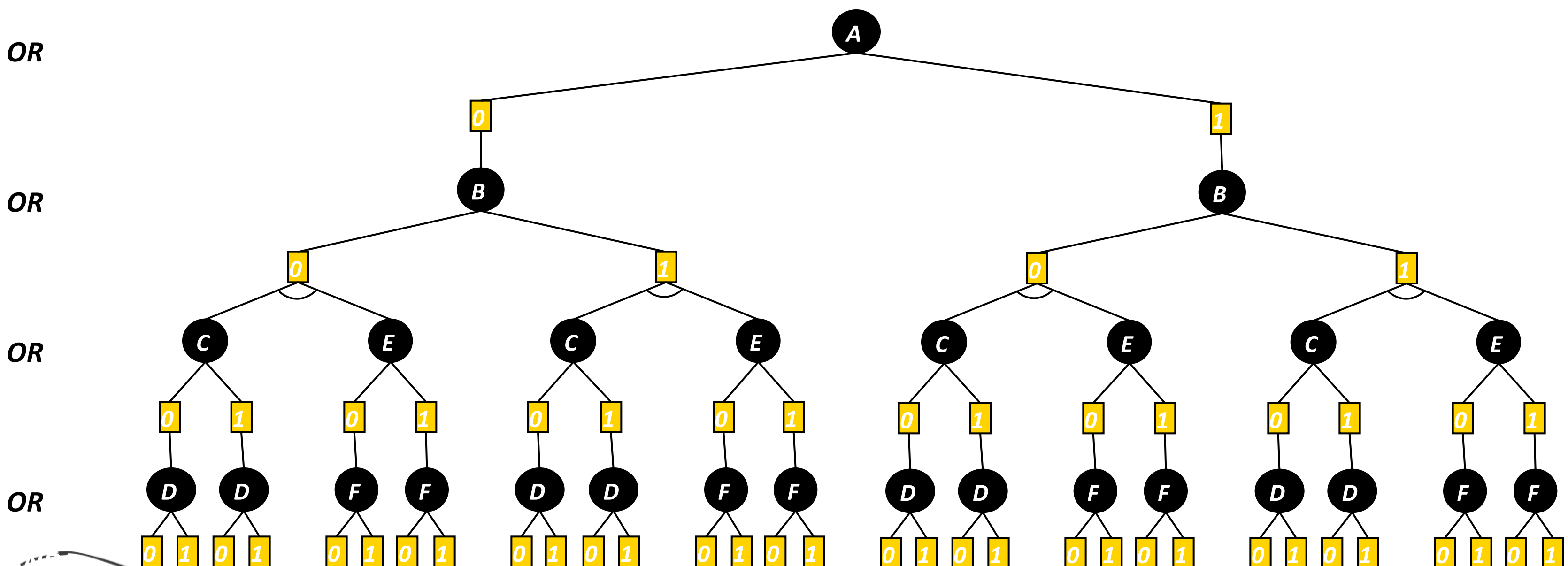
An optimal assignment is A=0, B=1, C=1, D=1, E=0, F=1 with cost 5

The AND/OR Search Tree



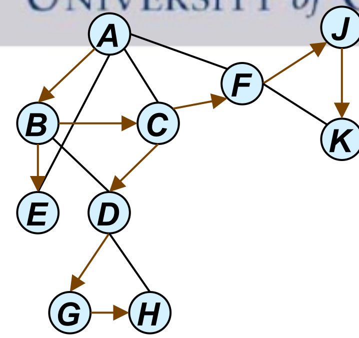


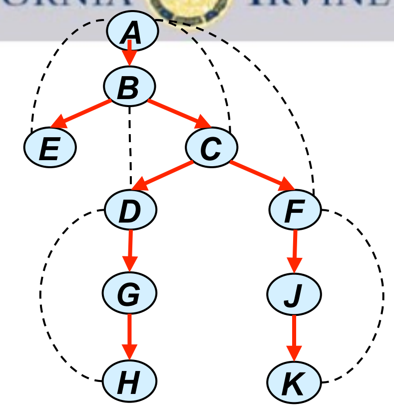
Pseudo tree

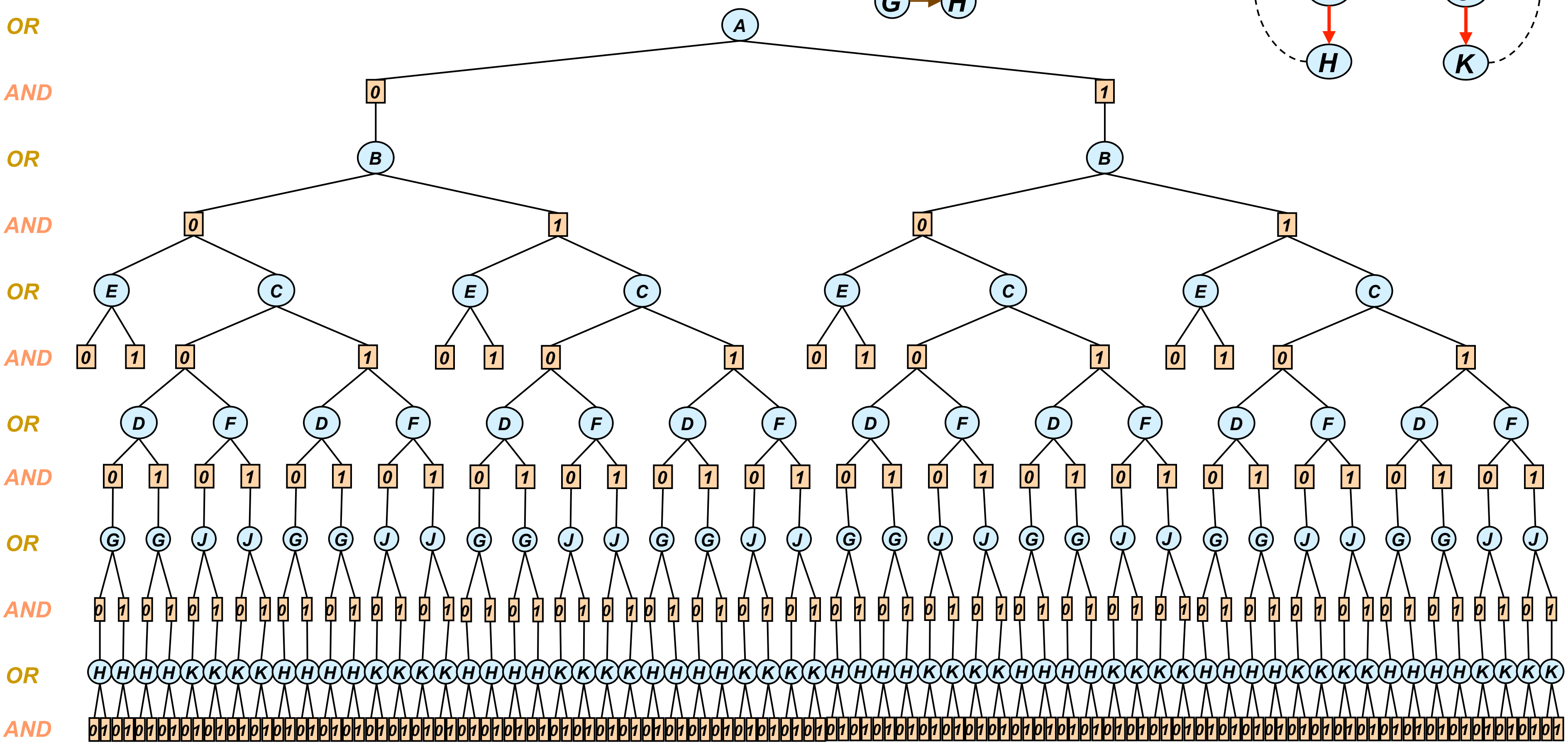


A solution subtree is

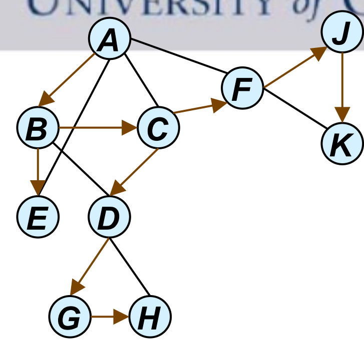
From AND/OR Tree

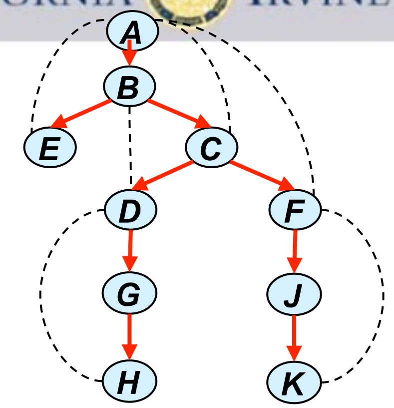


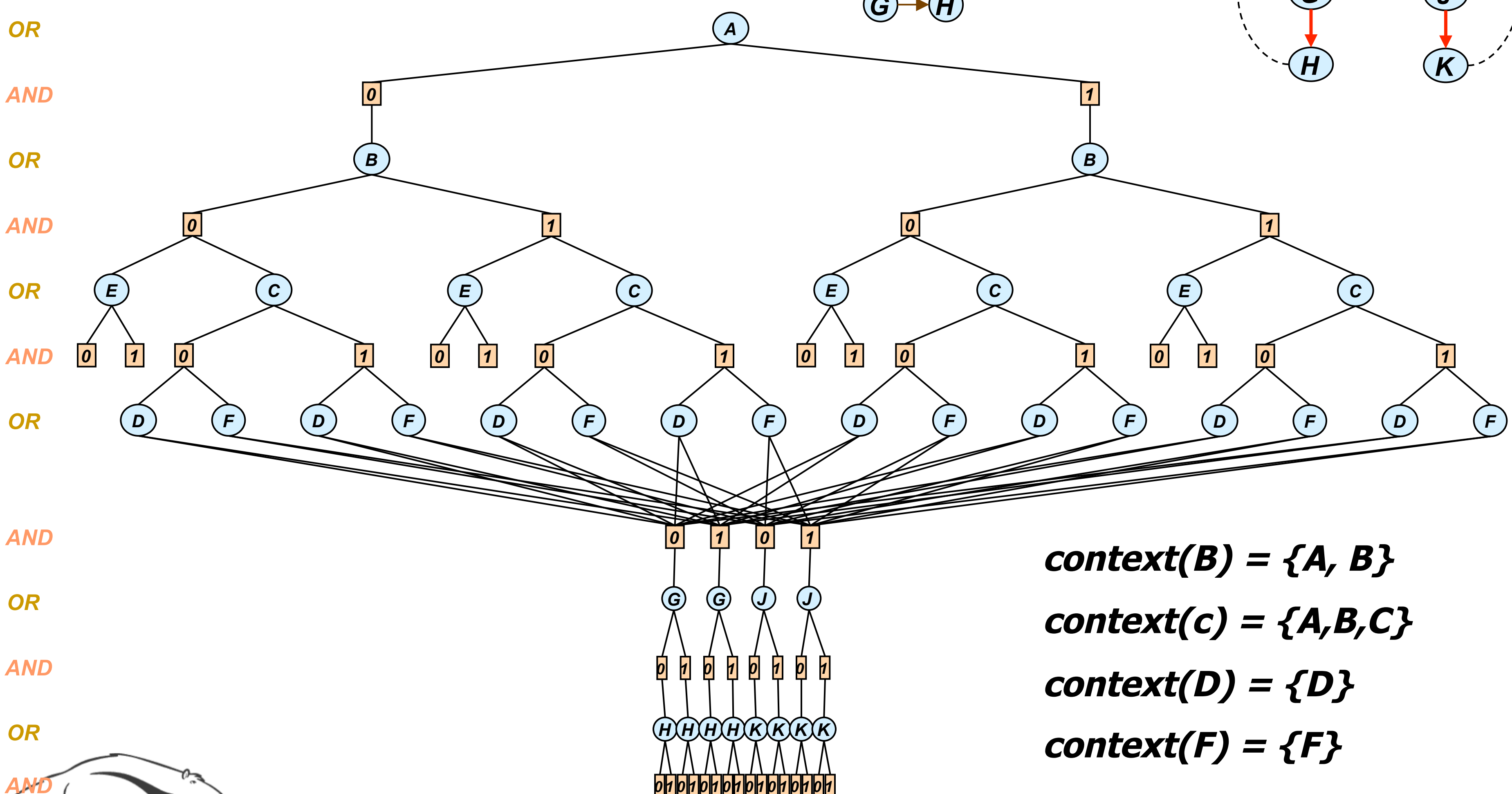




An AND/OR Graph







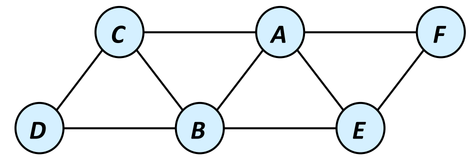
context(B) = {A, B}

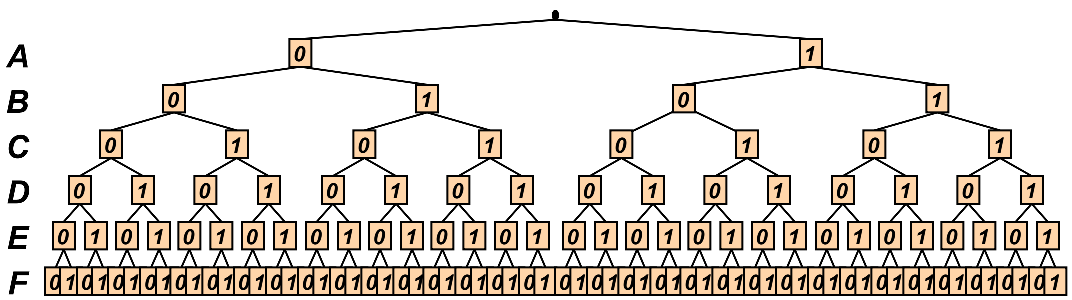
context(c) = {A,B,C}

context(D) = {D}

context(F) = {F}

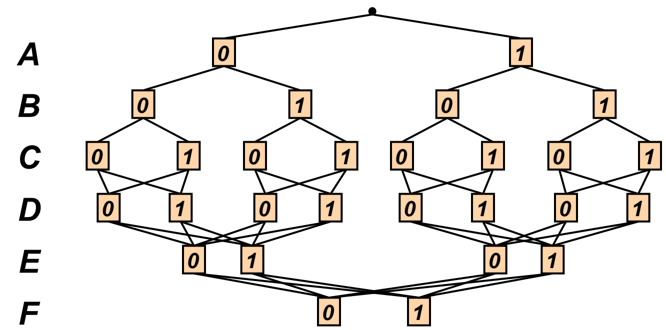
All Four Search Spaces





Full OR search tree

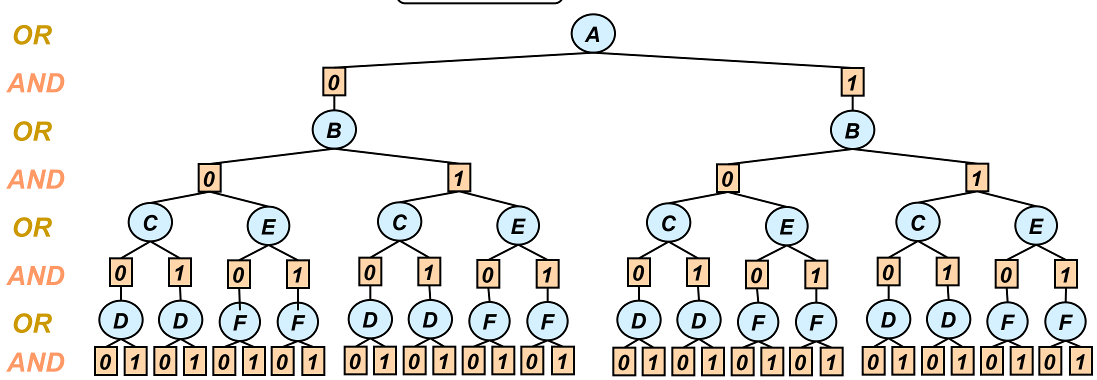
126 nodes



Context minimal OR search graph

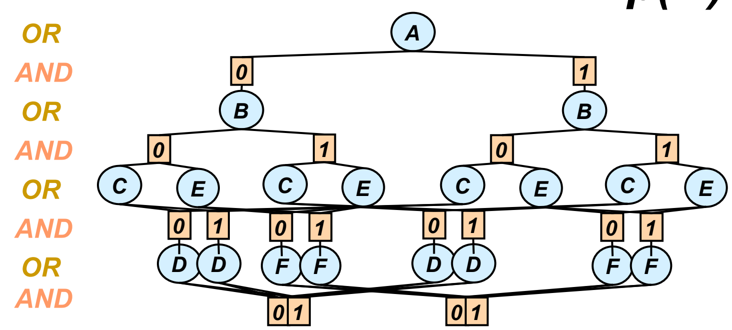
28 nodes

Exp(w)



Full AND/OR search tree

54 AND nodes



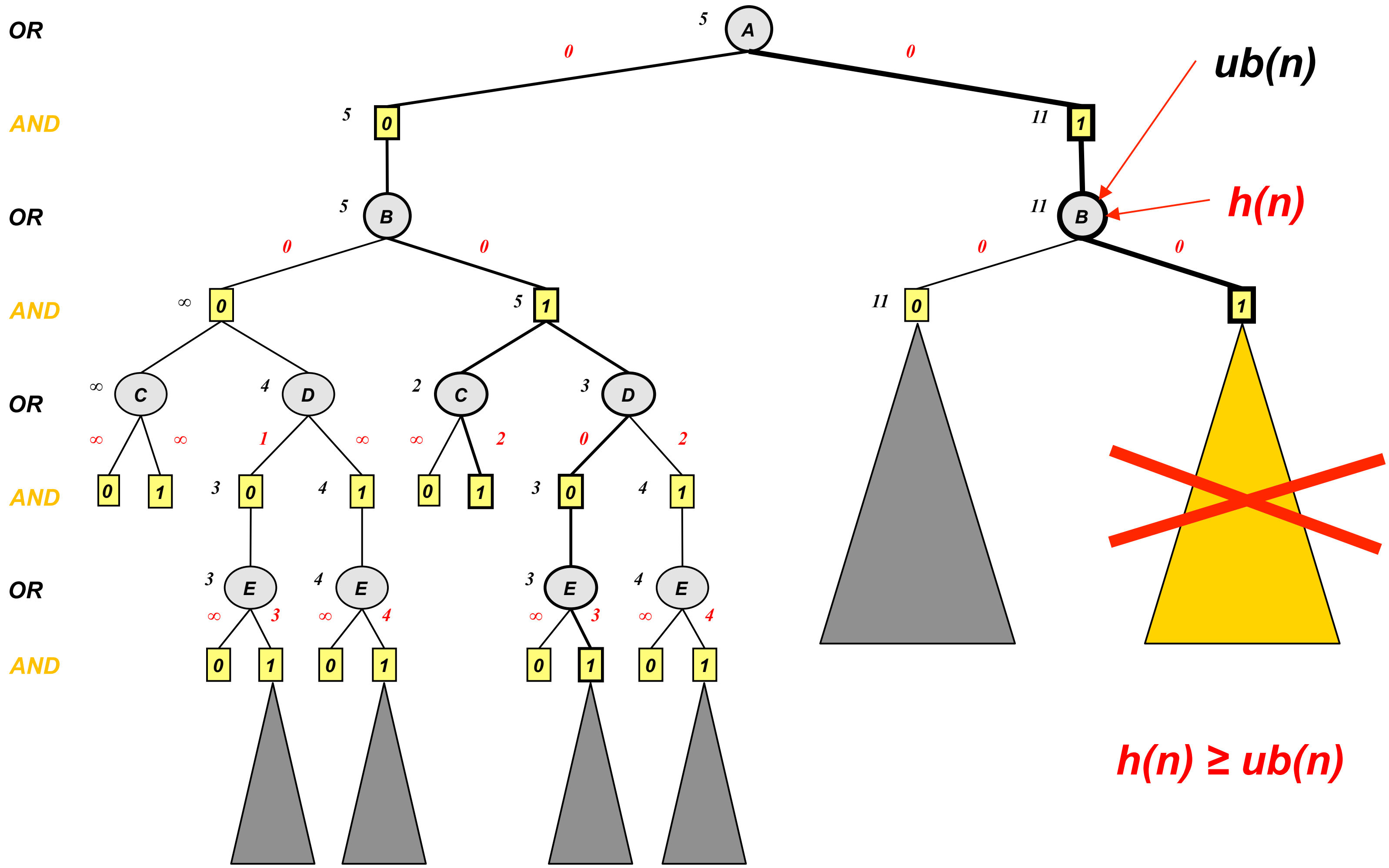
Context minimal AND/OR search graph

18 AND nodes

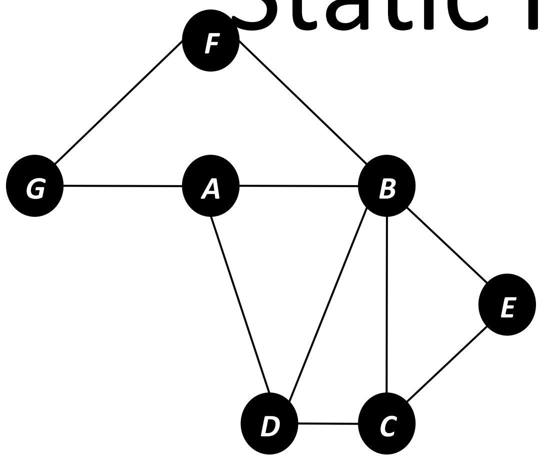
Any query is best computed
Over the c-minimal AO space

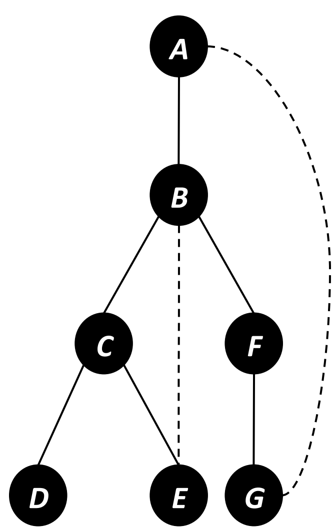
AND/OR Branch-and-Bound

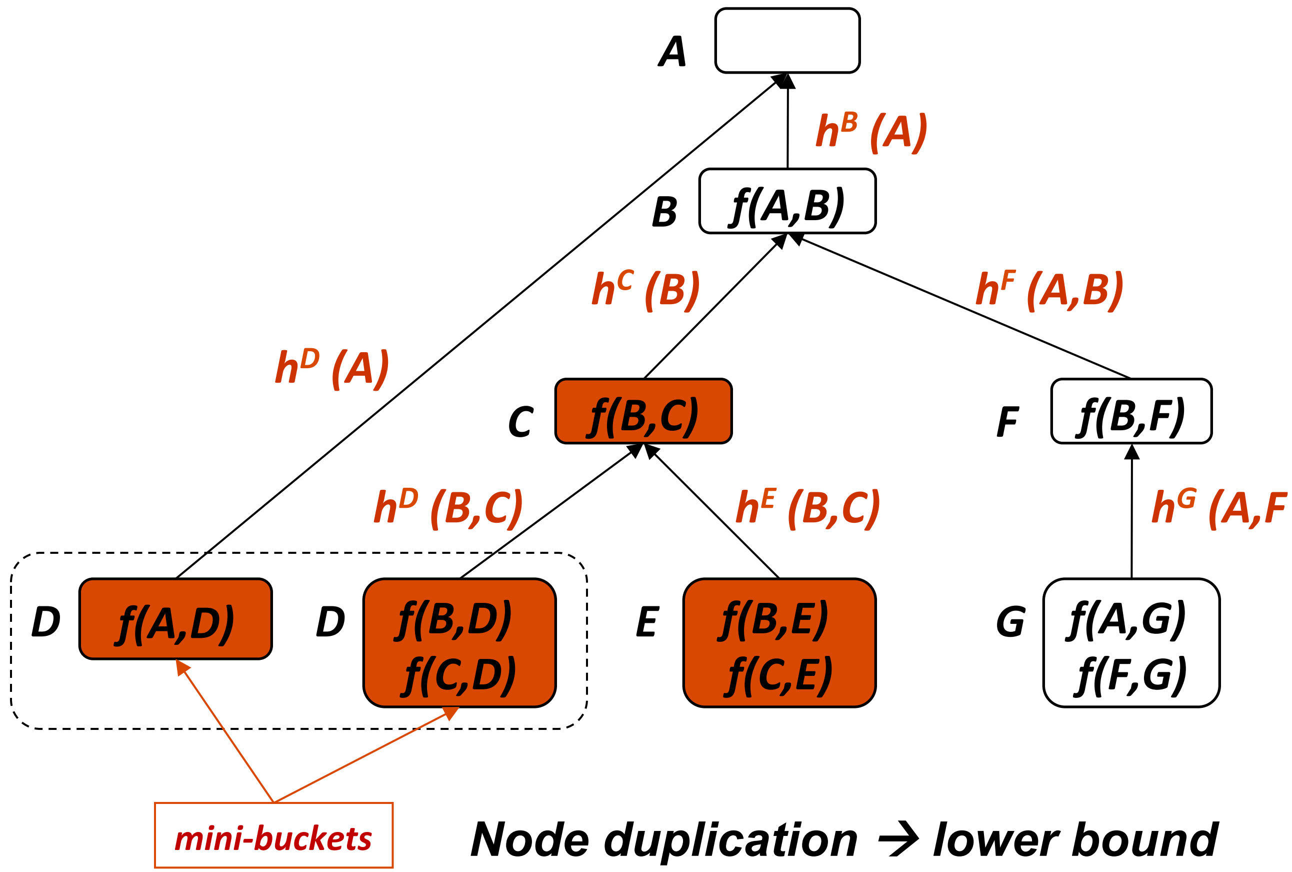
Kask and Dechter 2001
Marinescue and Dechter, 2005-2009



Static Mini-Bucket Heuristics







Node duplication → lower bound

$$h(a, b, c) = h^D(a) + h^D(b, c) + h^E(b, c) \leq h^*(a, b, c)$$

Ordering: (A, B, C, D, E, F, G)

Empirical evaluation

Experimental settings:

- **4 benchmarks:**
  - Pedigrees (10 instances)
  - Type4 (10 instances)
  - LargeFam (40 instances)
  - n-by-n grid networks (32 instances)

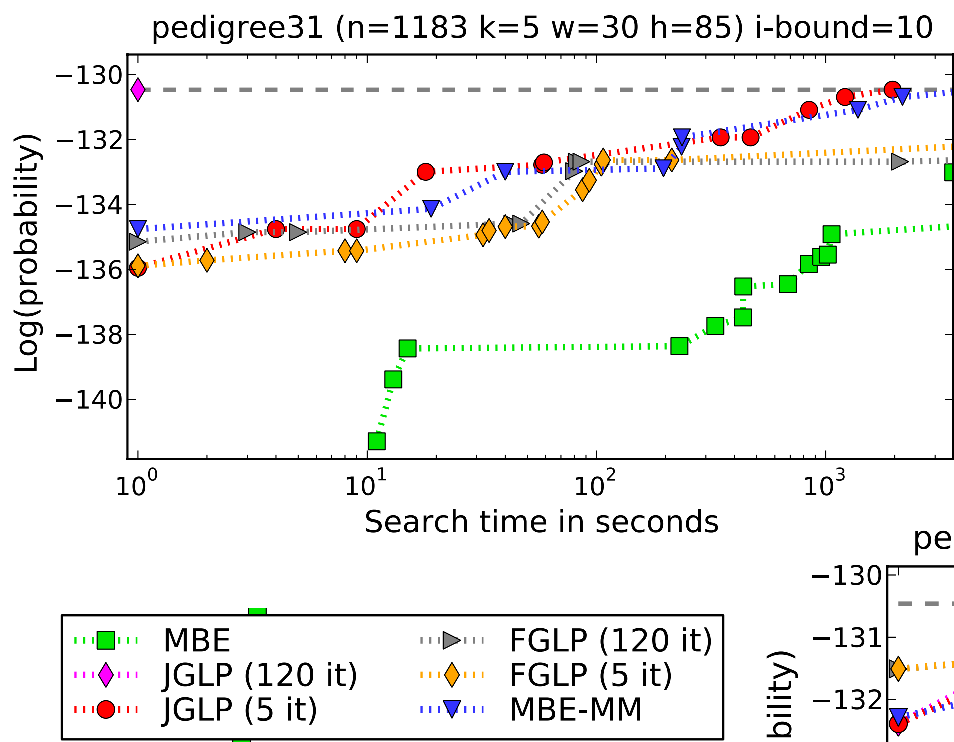
  genetic linkage analysis networks (Pedigrees, Type4, LargeFam)
- Algorithms as bounding schemes
- Algorithms as heuristic generators

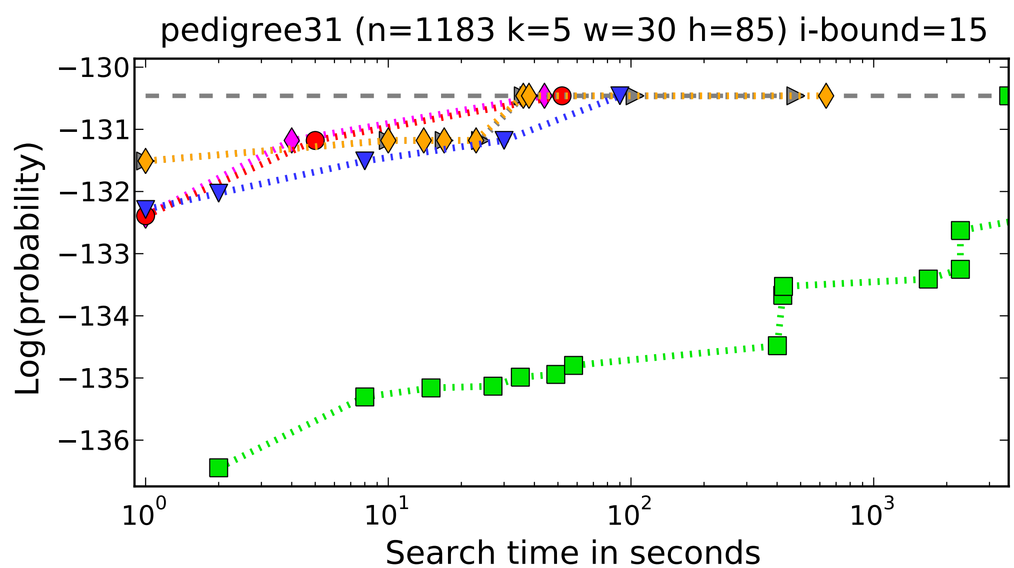
Iterative tightening as heuristic generators

- 4 schemes used:
  - AOBB guided by pure MBE heuristics (**AOBB-MBE**)
  - AOBB guided by MBE and max- marginal matching heuristics (**AOBB-MBE-MM**)
  - AOBB whose heuristics are generated from FGLP followed by MBE (**AOBB-FGLP+MBE**)
  - AOBB guided by JGLP-produced heuristics (**AOBB-JGLP**)
- FGLP, JGLP ran for 30 seconds
- Total search time bound 24 h
- Memory limit 3 Gb
- Mini-bucket z-bounds={10,15,20}

Empirical Evaluation: Haplotype problems

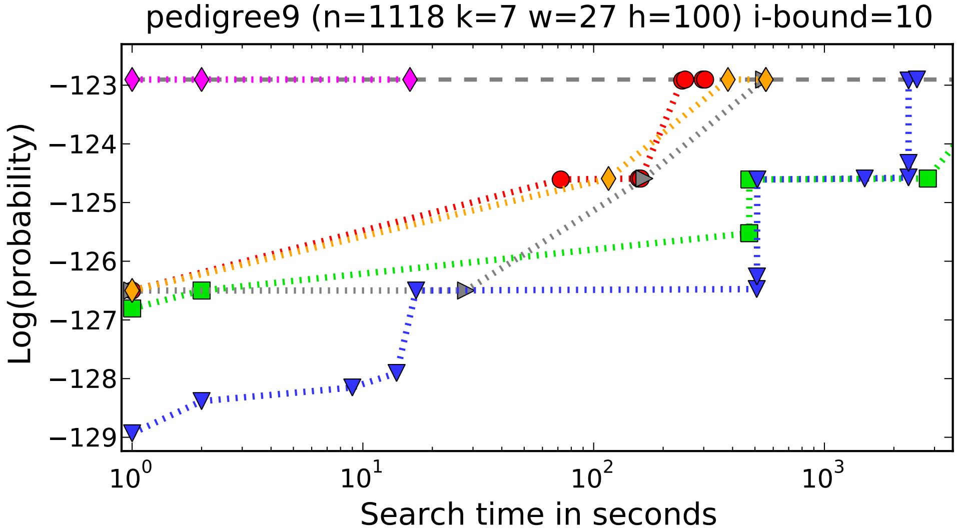
Time bound – 24 h

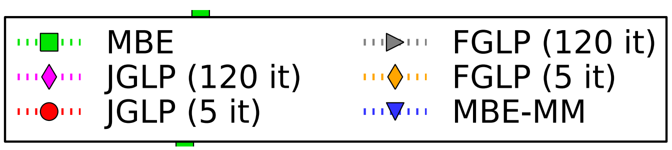


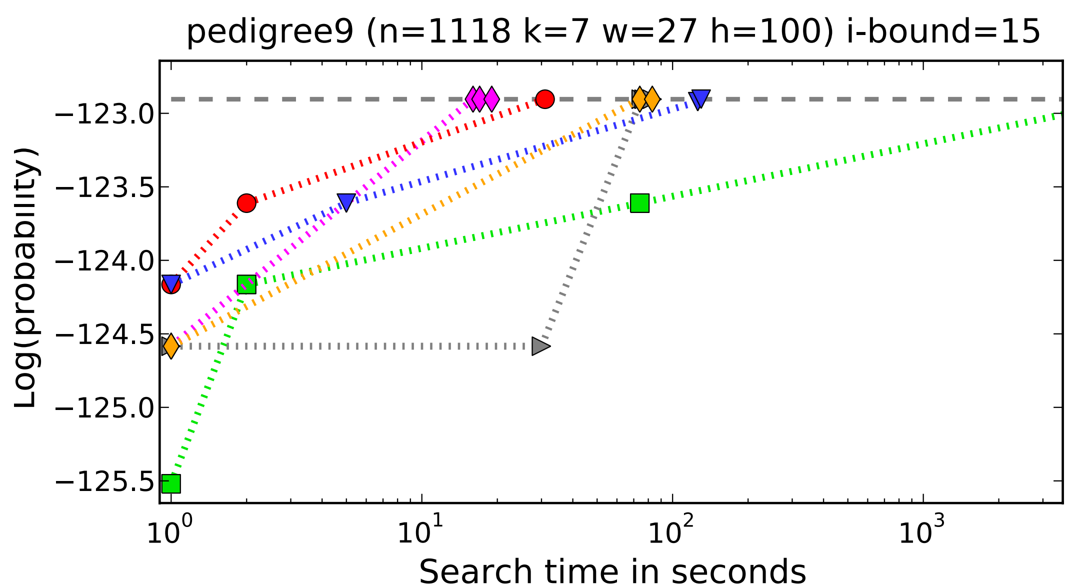


Empirical Evaluation: Haplotype problems

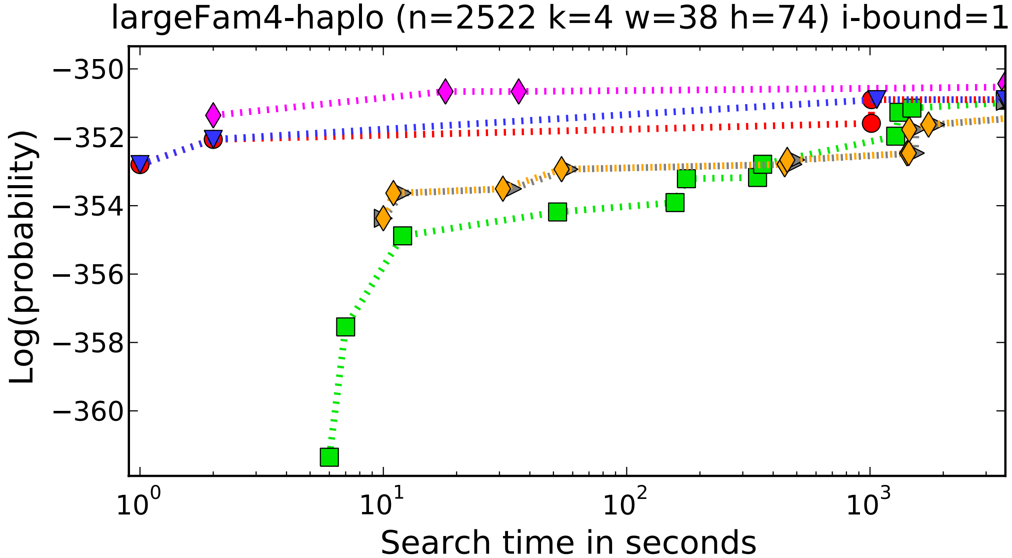
Time bound – 24 h



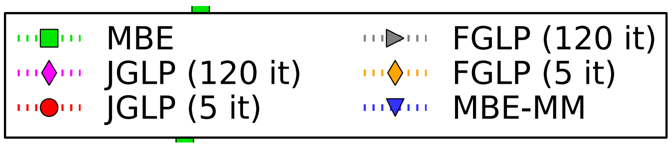


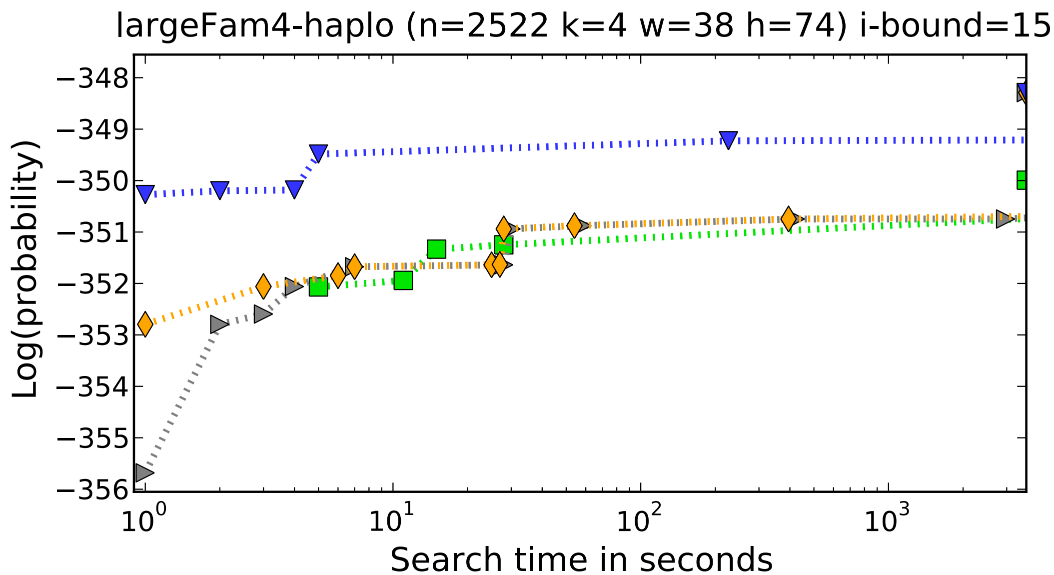


Empirical Evaluation: Large families

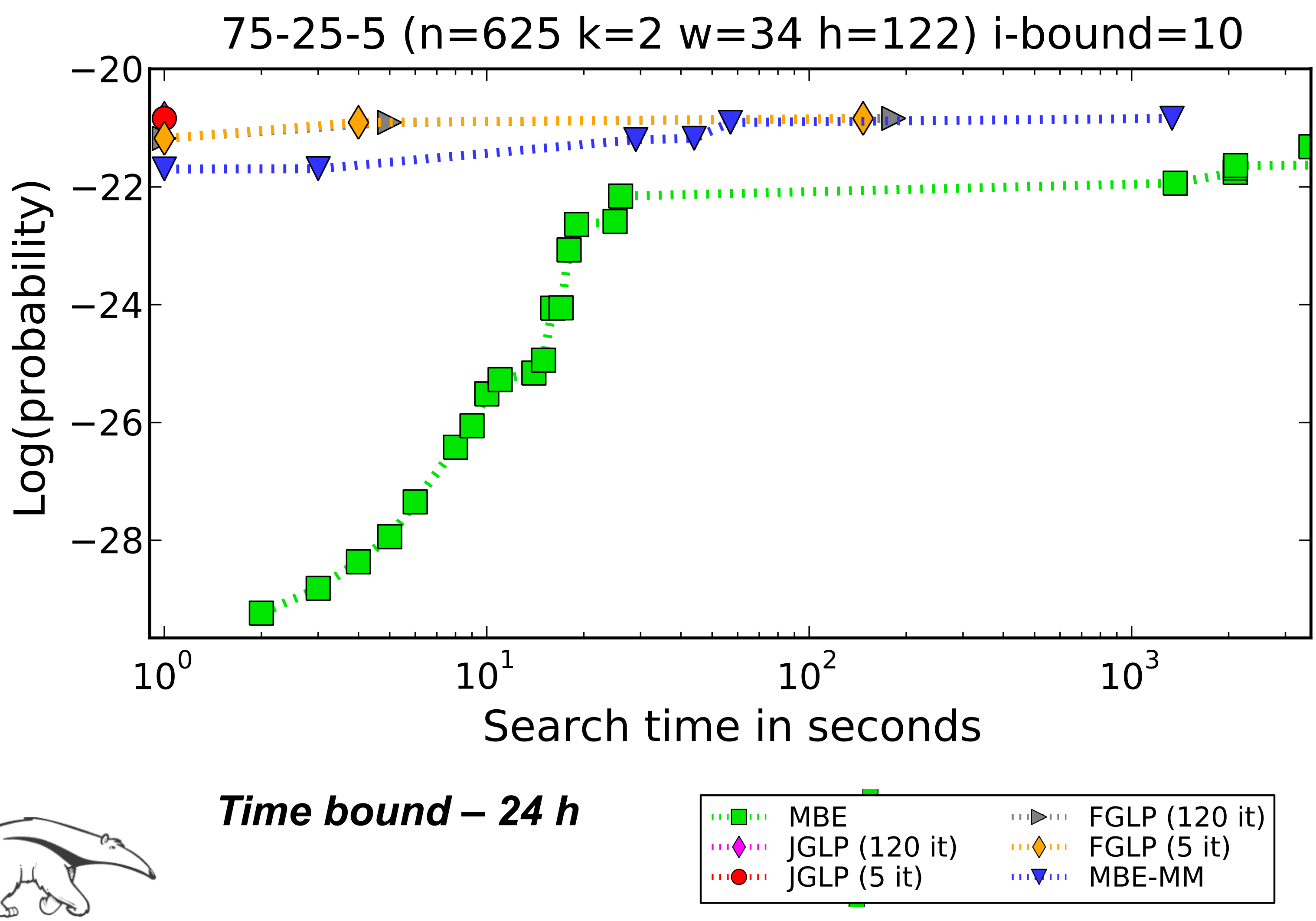


Time bound – 24 h



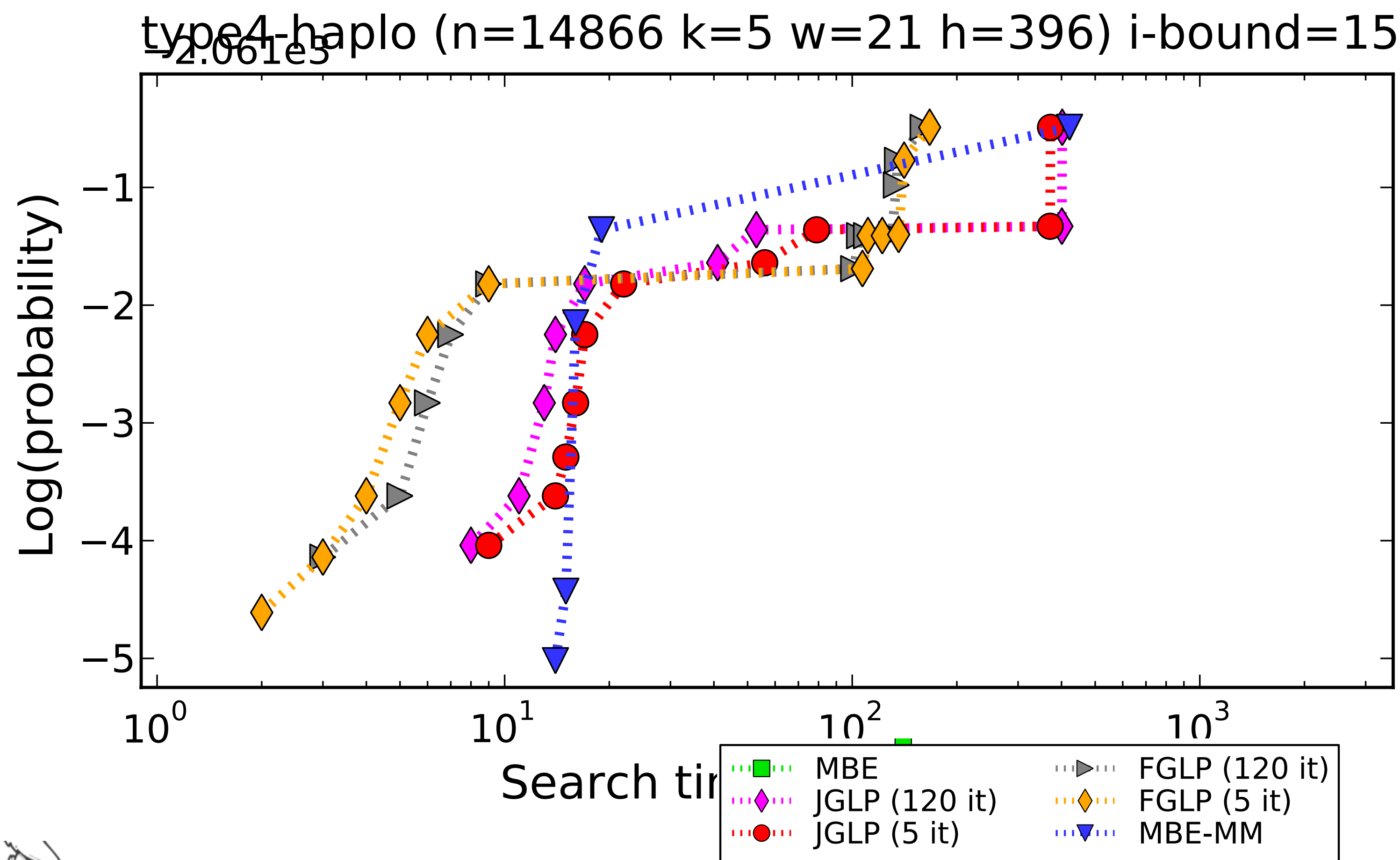


Empiricial Evaluation; Grid networks



Time bound – 24 h

Iterative tightening as heuristic generators



BREN:ICS INFORMATION AND COMPUTER SCIENCES

UNIVERSITY of CALIFORNIA IRVINE

PASCAL 2012 Inference Challenge

DAOOPT: Improving AND/OR Branch-and-Bound for Graphical Models

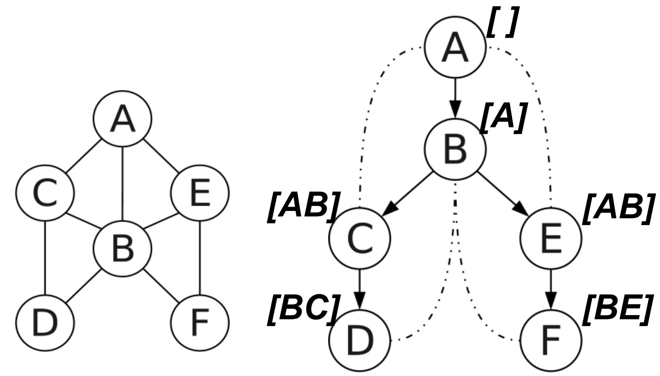
Lars Otten, Alexander Ihler,
Kalev Kask, Rina Dechter

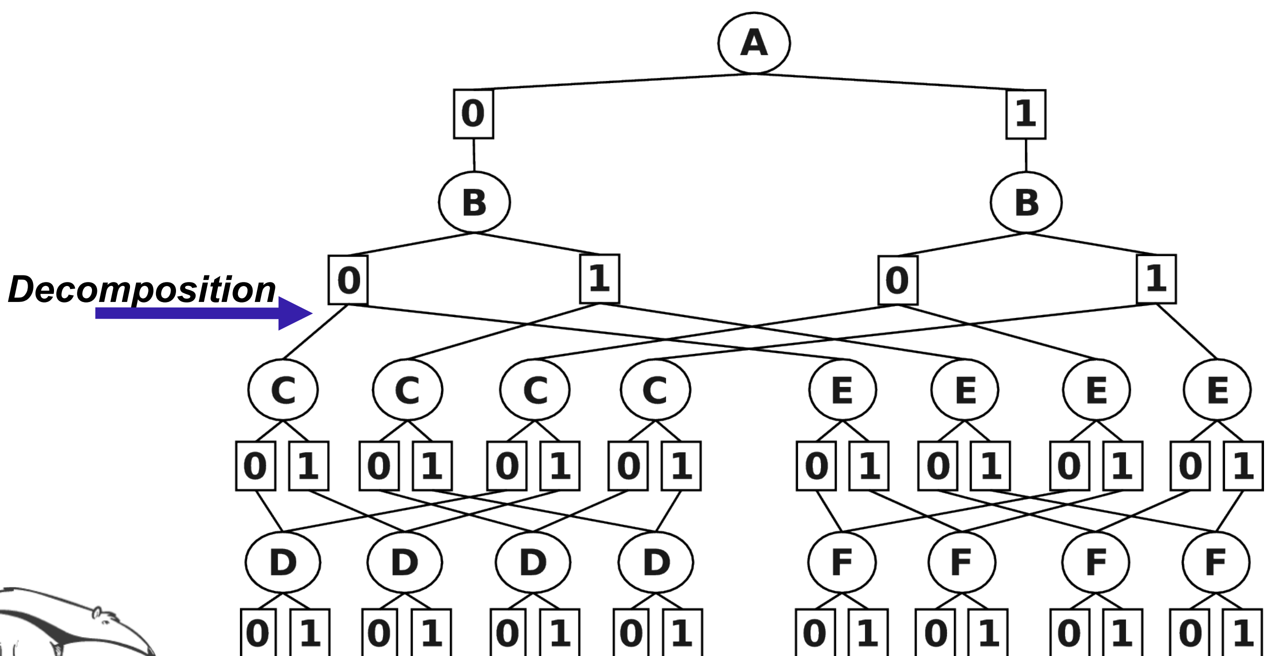
Dept. of Computer Science
University of California, Irvine

Harvard, 10/12

AND/OR Branch and Bound

- Guided by pseudo tree:
  - Subproblem decomposition.

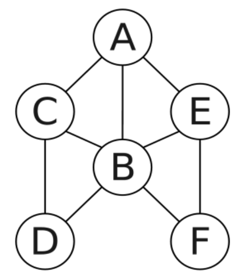


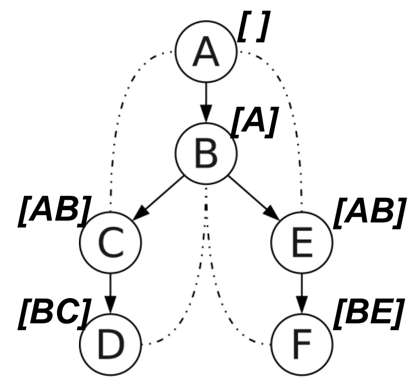


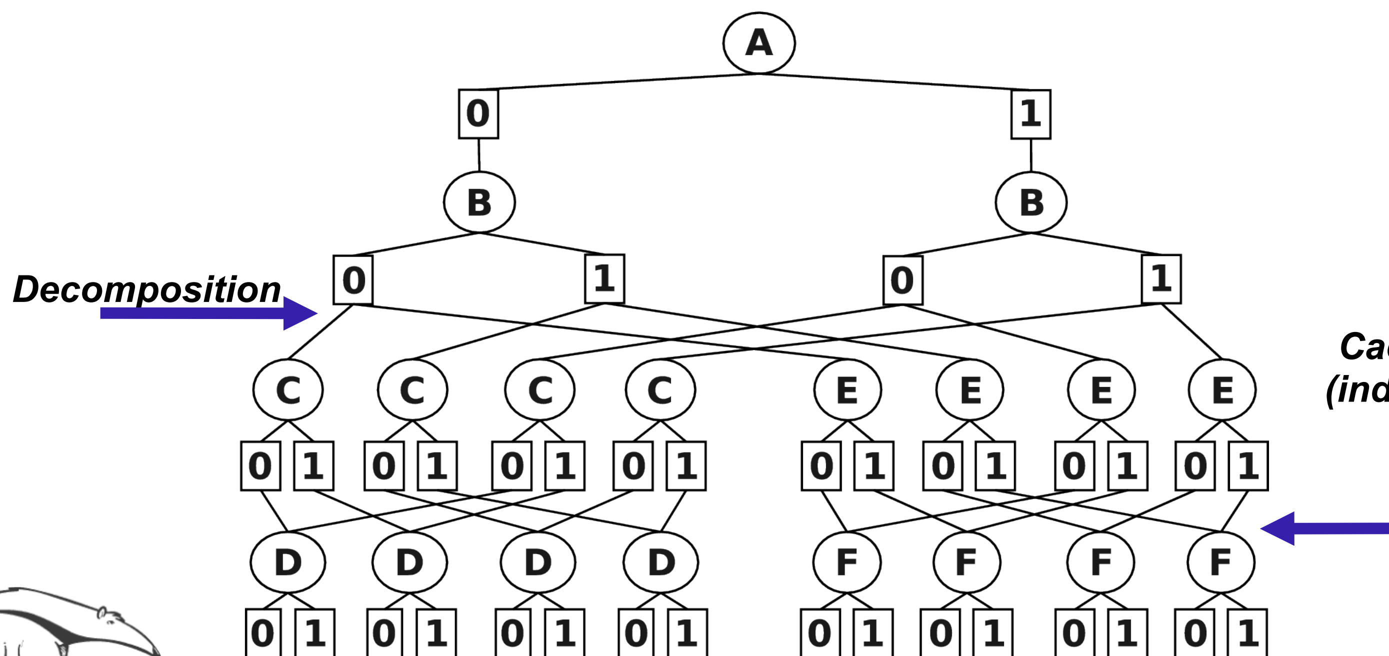
[Marinescu & Dechter, AIJ'09]

AND/OR Branch and Bound

- Guided by pseudo tree:
  - Subproblem decomposition.
  - Merge unifiable subproblems.







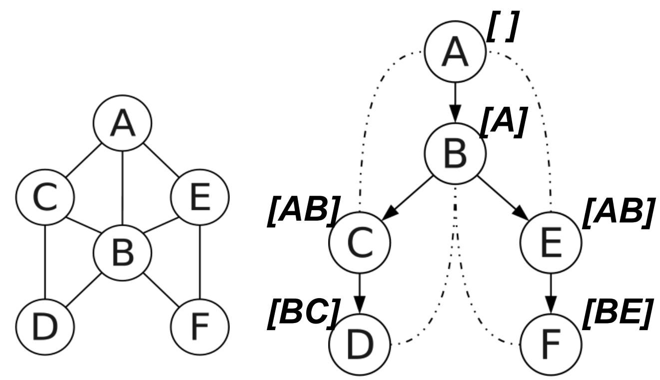
Cache table for F (independent of A)

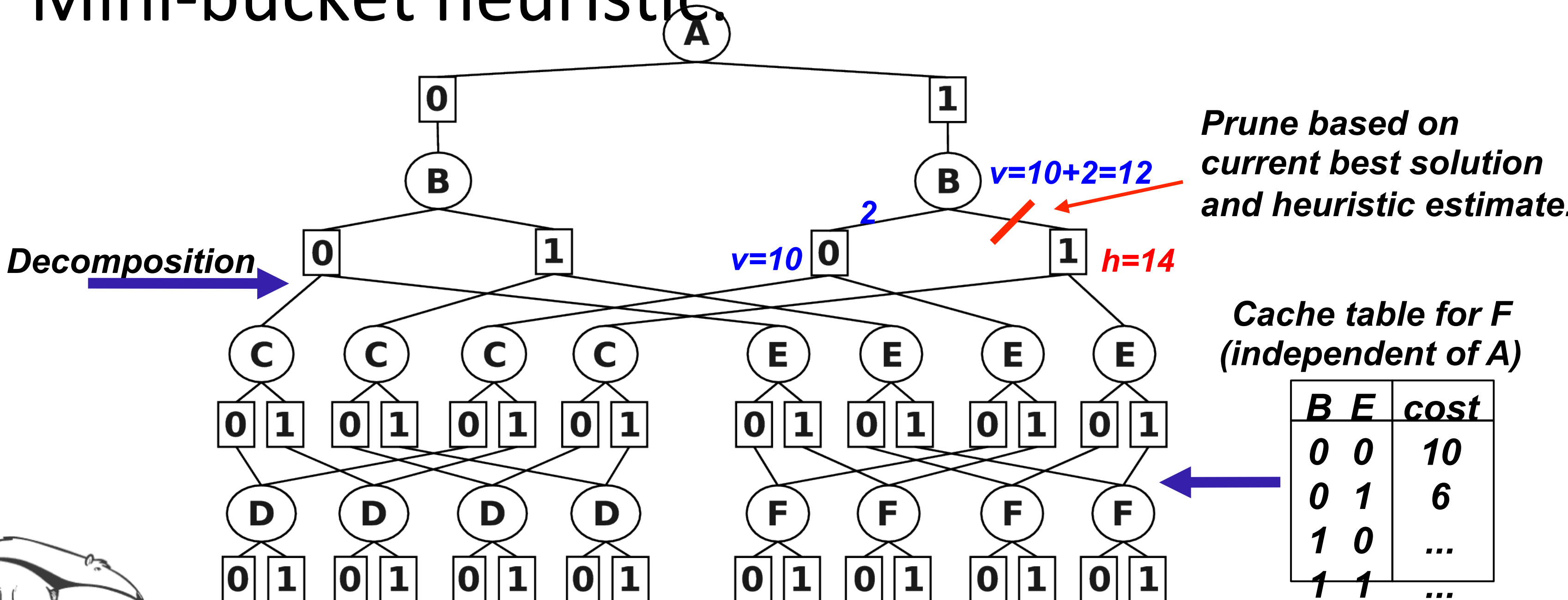
B	E	cost
0	0	10
0	1	6
1	0	...
1	1	...

[Marinescu & Dechter, AIJ'09]

AND/OR Branch and Bound

- Guided by pseudo tree:
  - Subproblem decomposition.
  - Merge unifiable subproblems.
- Mini-bucket heuristic.





B	E	cost
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[Marinescu & Dechter, AIJ'09]

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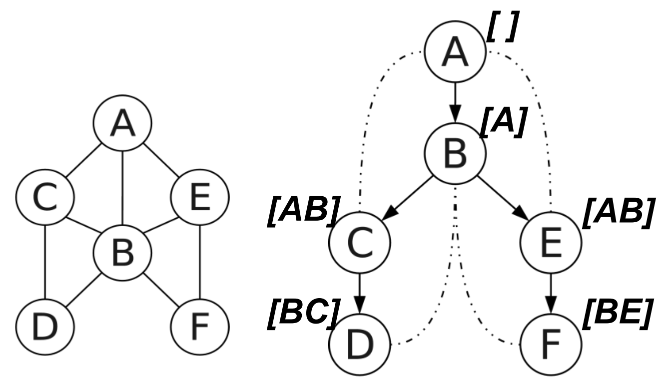
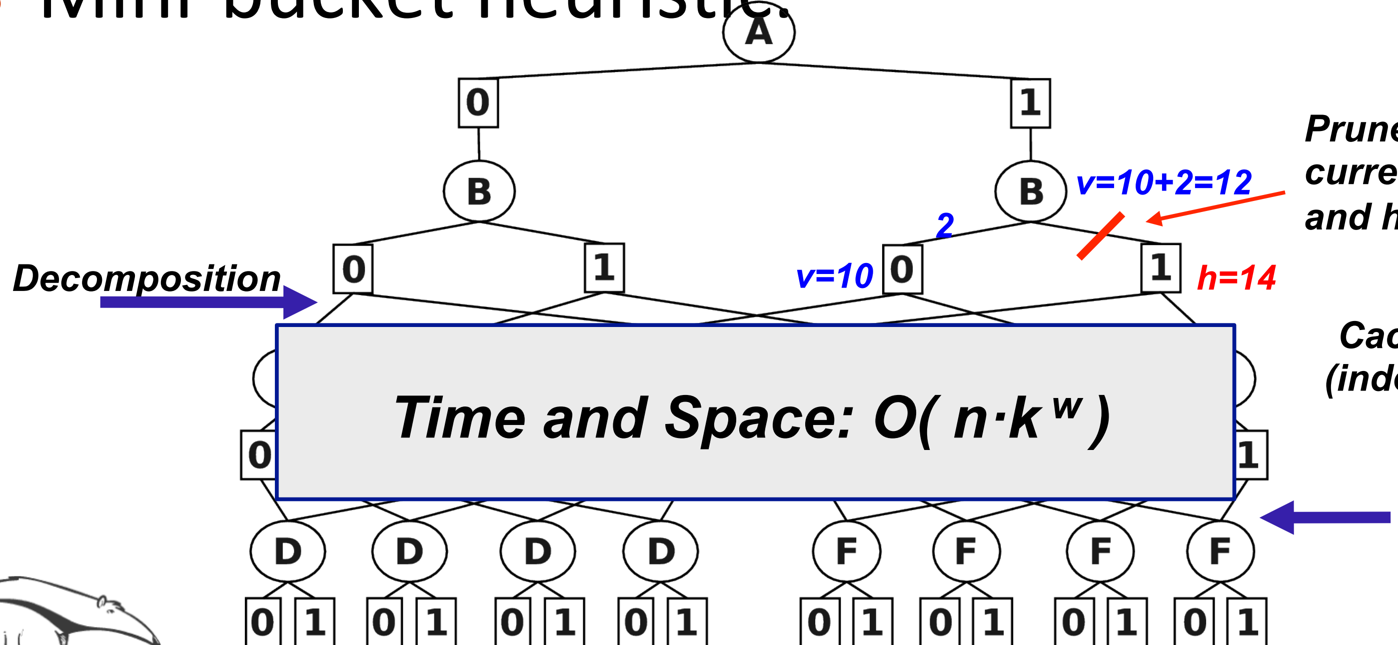


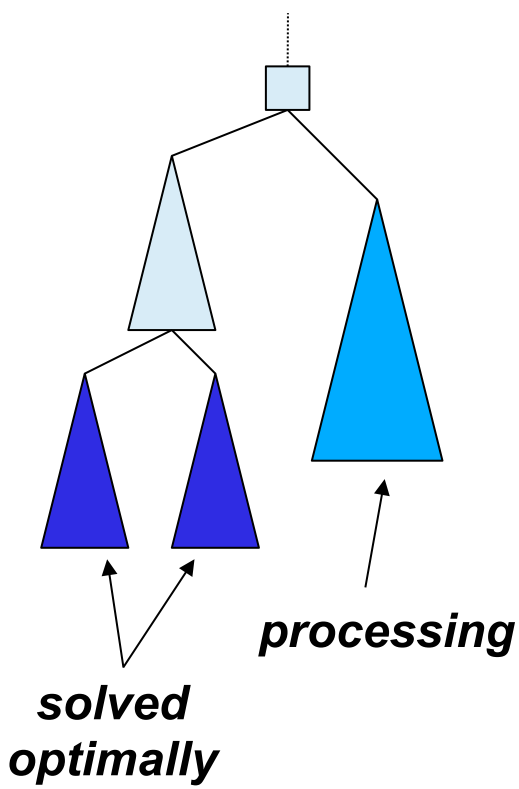


B	E	cost
0	0	10
0	1	6
1	0	...
1	1	...

[Marinescu & Dechter, AIJ'09]

Anytime Performance

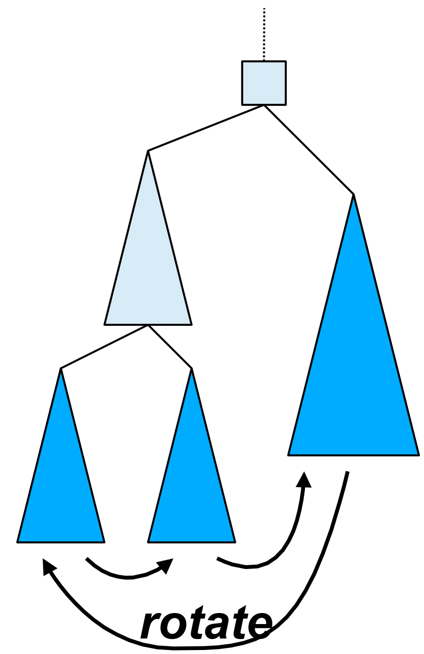
- OR Branch-and-Bound is anytime.
- But AND/OR breaks anytime behavior of depth-first scheme:
  - First anytime solution delayed until last subproblem starts processing.



[Otten & Dechter, AICom '12]

Anytime Performance

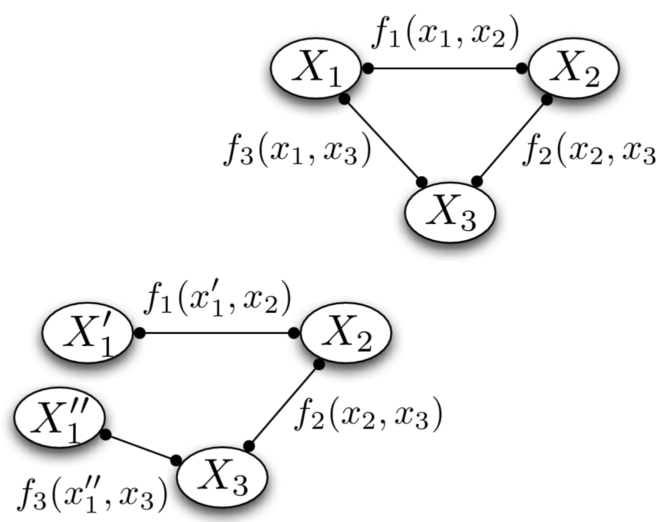
- OR Branch-and-Bound is anytime.
- But AND/OR breaks anytime behavior of depth-first scheme:
  - First anytime solution delayed until last subproblem starts processing.
- **Breadth-Rotating AOBB:**
  - Take turns processing subproblems.
    - Limit number of expansions per visit.
  - Solve each subproblem depth-first.
    - Maintain favorable complexity bounds.

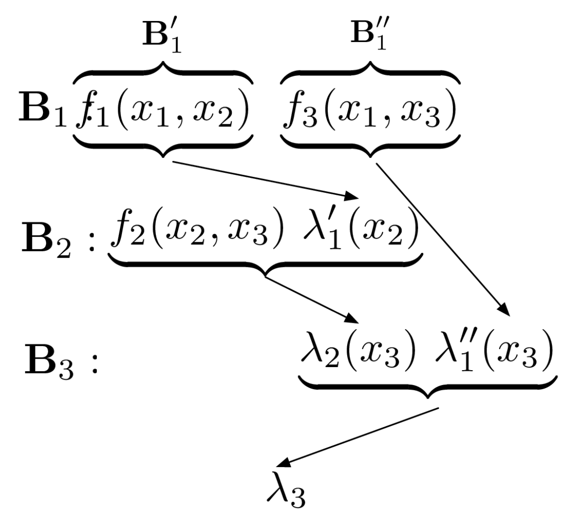


[Otten & Dechter, AICom '12]

Mini-buckets with Moment Matching

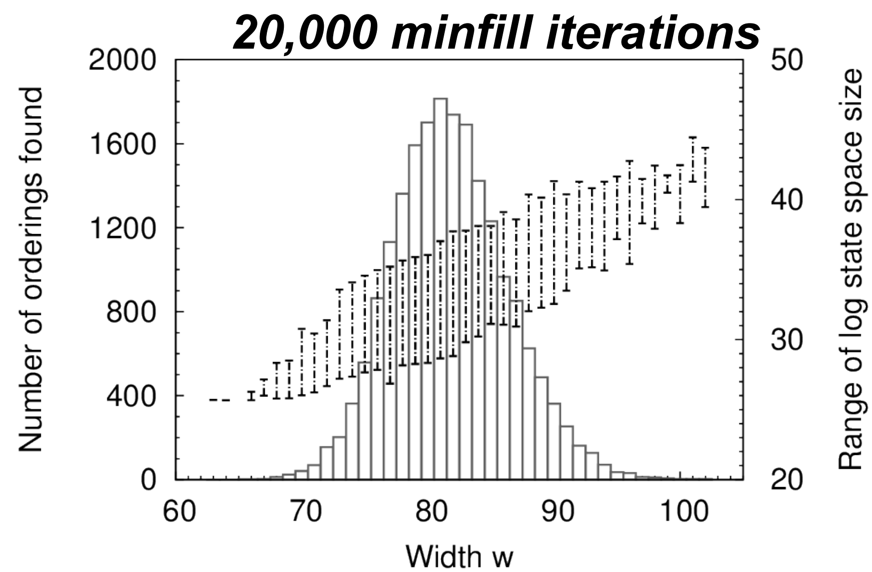
- Heuristic function generated from mini buckets.
  - Apply exact variable elimi-nation to relaxed problem.
  - *i*-bound parameter controls accuracy / complexity.
- Augmented with MPLP cost-shifting.
  - Max-marginal matching per bucket for tighter bounds.





Stochastic Variable Orderings

- AOBB complexity: $O(nk^w)$
  - High variance in width of orderings.
- Our implementation:
  - Minfill heuristic.
  - Random tie-breaking.
  - Allow deviation from heuristic optimum.
  - Highly optimized data structures, early termination.
    - Can do many thousands of iterations.



[Kask, Gelfand, Otten, Dechter, AAAI '11]

Putting It All Together

- Steps in competition entry for 1 hour track:
  1. MPLP cost-shifting on original graph (1 min / 2,000 iter).
  2. Stochastic local search (3 min).
  3. Find variable ordering (3 min / 30,000 iter).
  4. Join-graph MPLP cost-shifting (1 min / 1,000 iter).
  5. Compute mini-buckets with moment matching.
     - Highest possible *i*-bound for given memory limit.
  6. Run limited discrepancy search ($d_{max} = 2$).
  7. Run complete Breadth-Rotating AOBB.
- Final result in 1 hour category:
  - Daoopt: **-8.3214**, ficolofo: **-8.3196** ($\delta = 0.0018$)

BREN:ICS INFORMATION AND COMPUTER SCIENCES

UNIVERSITY of CALIFORNIA IRVINE

Thank you!

For publication see:

http://www.ics.uci.edu/~dechter/publications.html

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