

# Outline

- **Introduction**
- **Inference**
- **Bounds and heuristics**
- **AND/OR search**
  - AND/OR search spaces
  - Depth-first AND/OR branch and bound
  - Best-first AND/OR search
  - Advanced searches and tasks
- **Exploiting parallelism**
- **Software**

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# Solution Techniques

## AND/OR search

*Time:  $\exp(\text{treewidth} * \log n)$*

*Space: linear*

*Space:  $\exp(\text{treewidth})$*

*Time:  $\exp(\text{treewidth})$*

Complete

DFS search

Branch-and-Bound

A\*

*Time:  $\exp(\text{treewidth})$*

*Space:  $\exp(\text{treewidth})$*

## Search: Conditioning

*Time:  $\exp(n)$*

*Space: linear*

*Time:  $\exp(\text{pathwidth})$*

*Space:  $\exp(\text{pathwidth})$*

Incomplete

Simulated Annealing

Gradient Descent

Stochastic Local Search

Incomplete

Local Consistency

Unit Resolution

Mini-bucket(i)

Hybrids

Complete

Adaptive Consistency

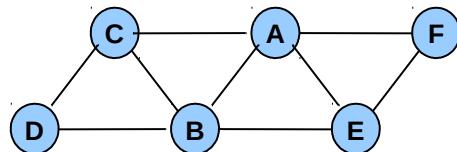
Tree Clustering

Variable Elimination

Resolution

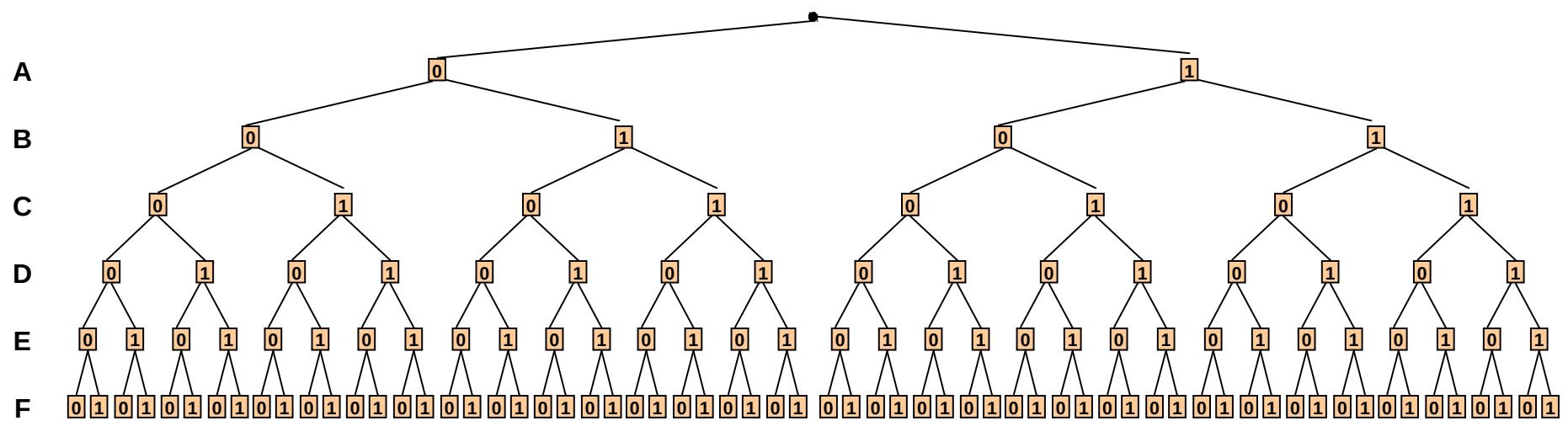
## Inference: Elimination

# Classic OR Search Space

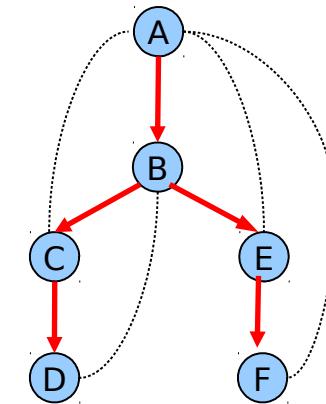
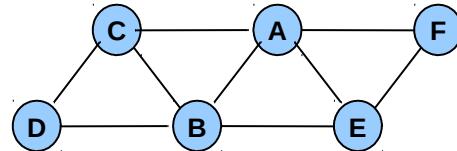


A	B	f <sub>1</sub>	A	C	f <sub>2</sub>	A	E	f <sub>3</sub>	A	F	f <sub>4</sub>	B	C	f <sub>5</sub>	B	D	f <sub>6</sub>	B	E	f <sub>7</sub>	C	D	f <sub>8</sub>	E	F	f <sub>9</sub>
0	0	<b>2</b>	0	0	<b>3</b>	0	0	<b>0</b>	0	0	<b>2</b>	0	0	<b>0</b>	0	0	<b>4</b>	0	0	<b>3</b>	0	0	<b>1</b>	0	0	<b>1</b>
0	1	<b>0</b>	0	1	<b>0</b>	0	1	<b>3</b>	0	1	<b>0</b>	0	1	<b>1</b>	0	1	<b>2</b>	0	1	<b>2</b>	0	1	<b>4</b>	0	1	<b>0</b>
1	0	<b>1</b>	1	0	<b>0</b>	1	0	<b>2</b>	1	0	<b>0</b>	1	0	<b>2</b>	1	0	<b>1</b>	1	0	<b>1</b>	1	0	<b>0</b>	1	0	<b>0</b>
1	1	<b>4</b>	1	1	<b>1</b>	1	1	<b>0</b>	1	1	<b>2</b>	1	1	<b>4</b>	1	1	<b>0</b>	1	1	<b>0</b>	1	1	<b>0</b>	1	1	<b>2</b>

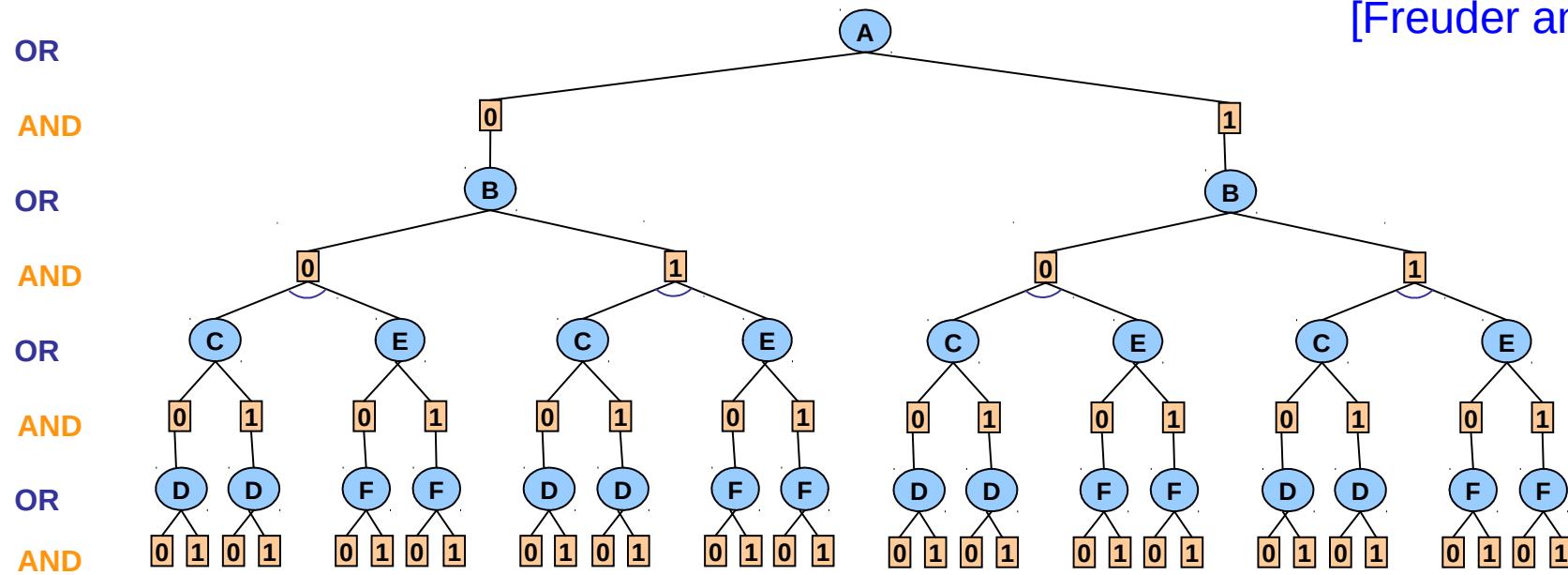
**Objective function:**  $F^* = \min_X \sum_i f_i(X)$



# The AND/OR Search Tree



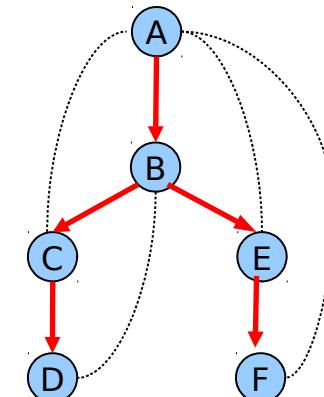
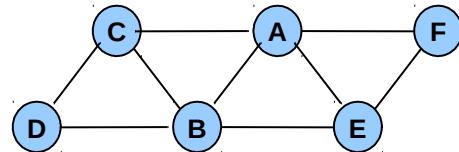
Pseudo tree  
[Freuder and Quinn, 1985]



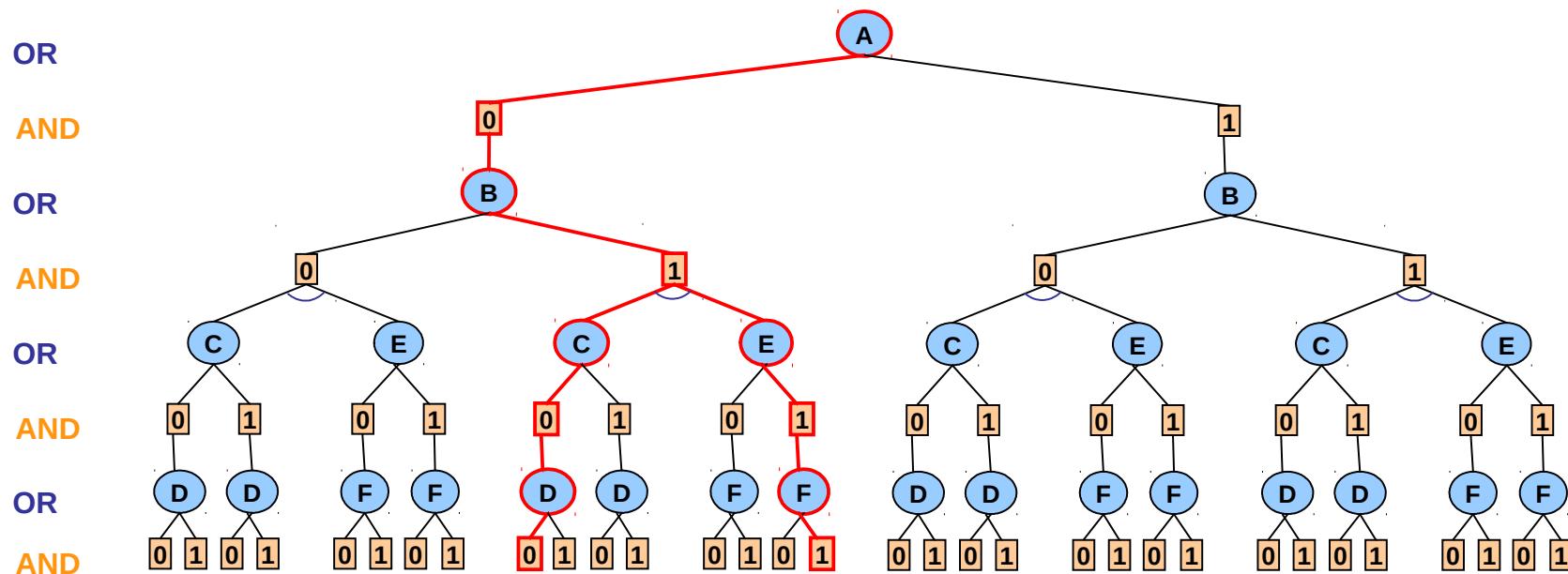
[Dechter and Mateescu, 2007]

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# The AND/OR Search Tree

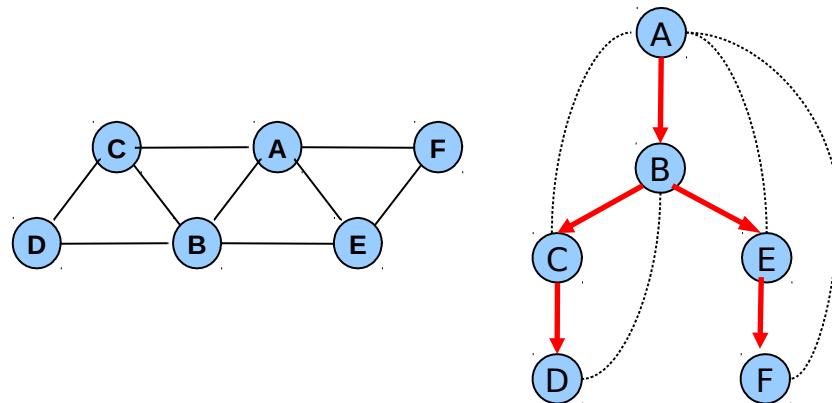


Pseudo tree



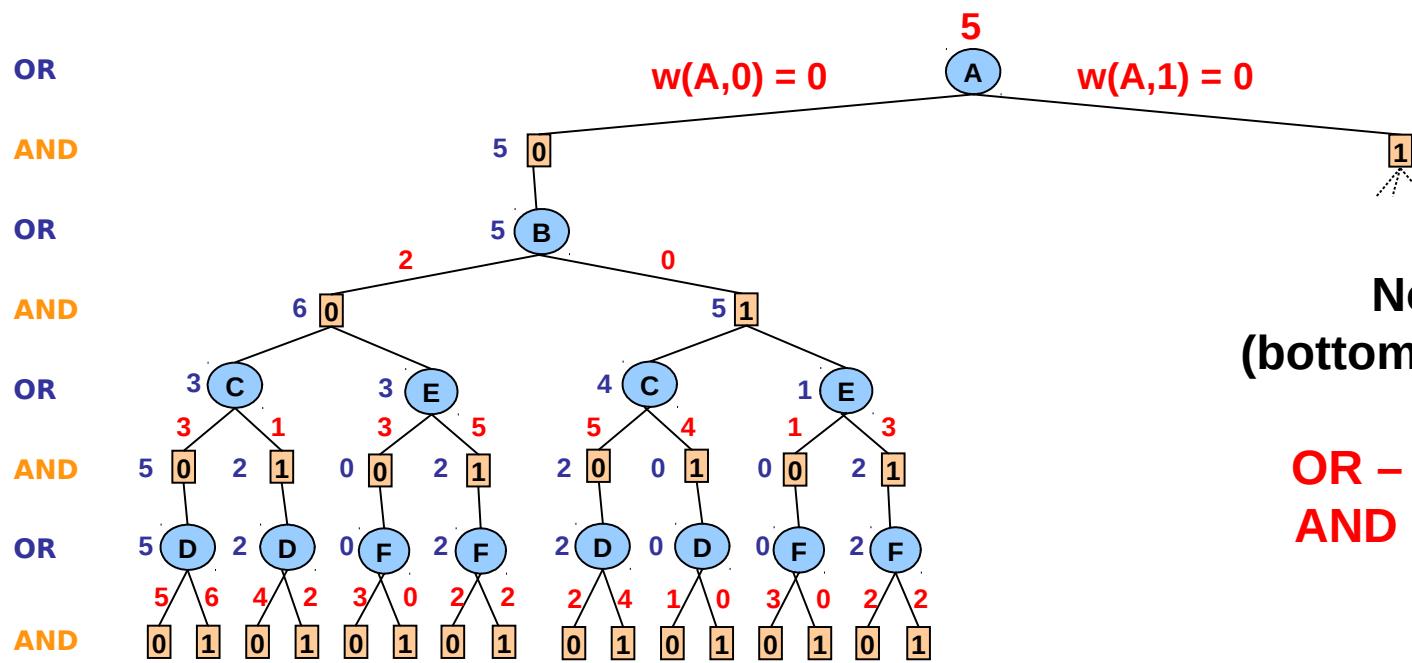
A solution subtree is (A=0, B=1, C=0, D=0, E=1, F=1)

# Weighted AND/OR Search Tree



A	B	$f_1$	A	C	$f_2$	A	E	$f_3$	A	F	$f_4$	B	C	$f_5$	B	D	$f_6$	B	E	$f_7$	C	D	$f_8$	E	F	$f_9$
0	0	2	0	0	3	0	0	0	0	0	2	0	0	0	0	0	4	0	0	3	0	0	1	0	0	
0	1	0	0	1	0	0	1	3	0	1	0	0	1	0	1	1	2	0	1	2	0	1	4	0	1	0
1	0	1	1	0	0	1	0	2	1	0	0	0	1	0	2	1	0	1	0	1	0	1	0	0	1	0
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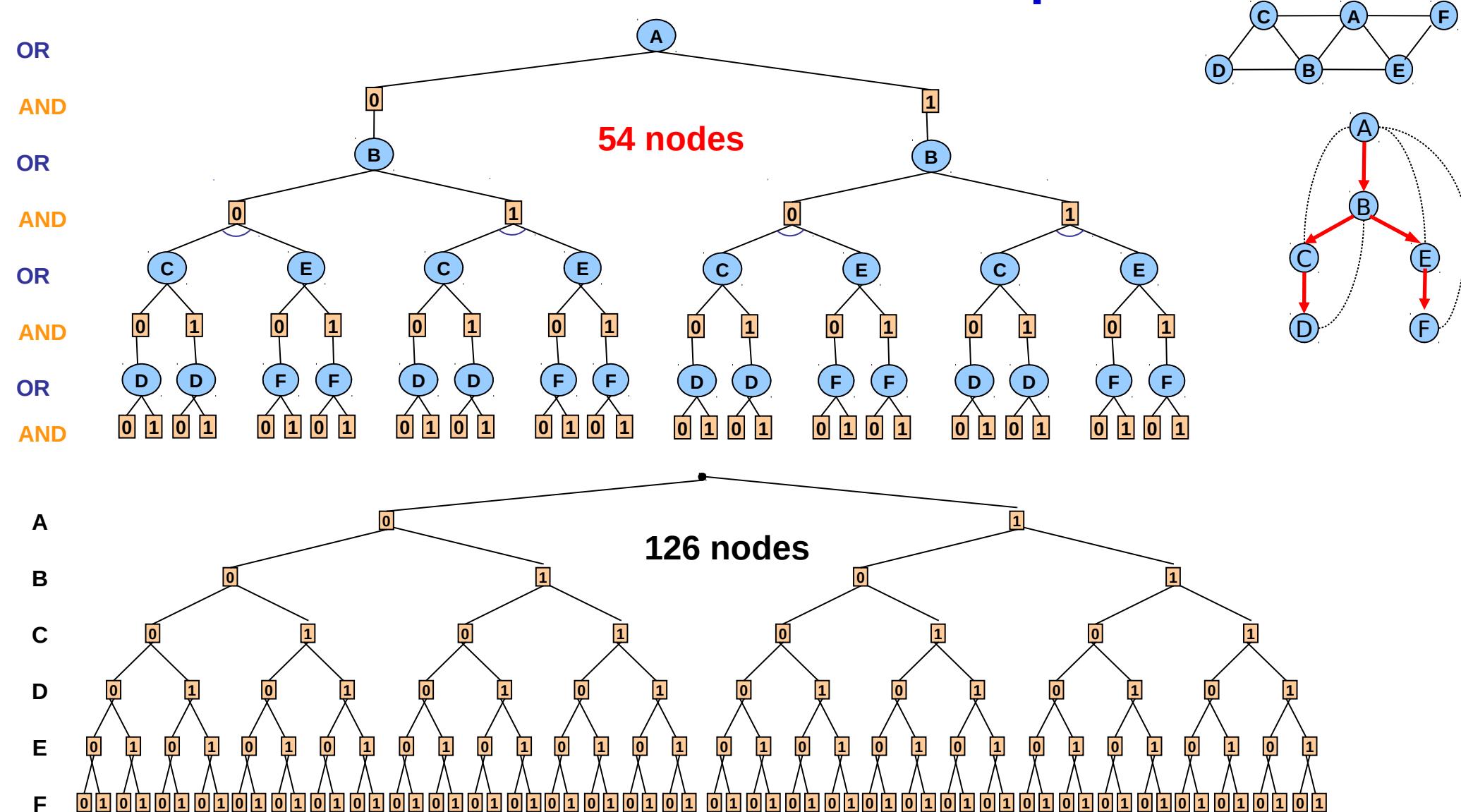
Objective function:  $F^* = \min_X \sum_i f_i(X)$



**Node Value  
(bottom-up evaluation)**

**OR – minimization  
AND – summation**

# AND/OR versus OR Spaces



# Complexity of AND/OR Tree Search

	AND/OR tree	OR tree
Space	$O(n)$	$O(n)$
Time	$O(n d^t)$ $O(n d^{(w^* \log n)})$	$O(d^n)$

[Freuder & Quinn85], [Collin, Dechter & Katz91],  
[Bayardo & Miranker95], [Darwiche01]

$d$  = domain size

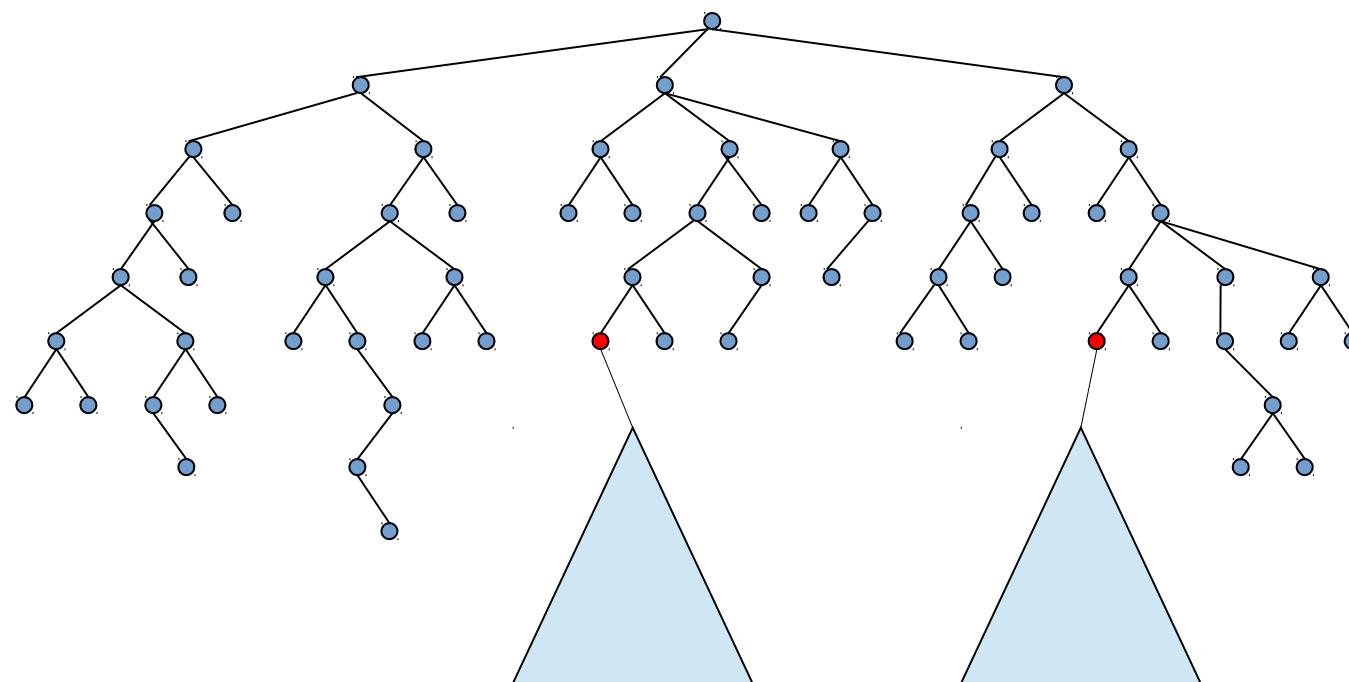
$t$  = depth of pseudo tree

$n$  = number of variables

$w^*$  = induced width

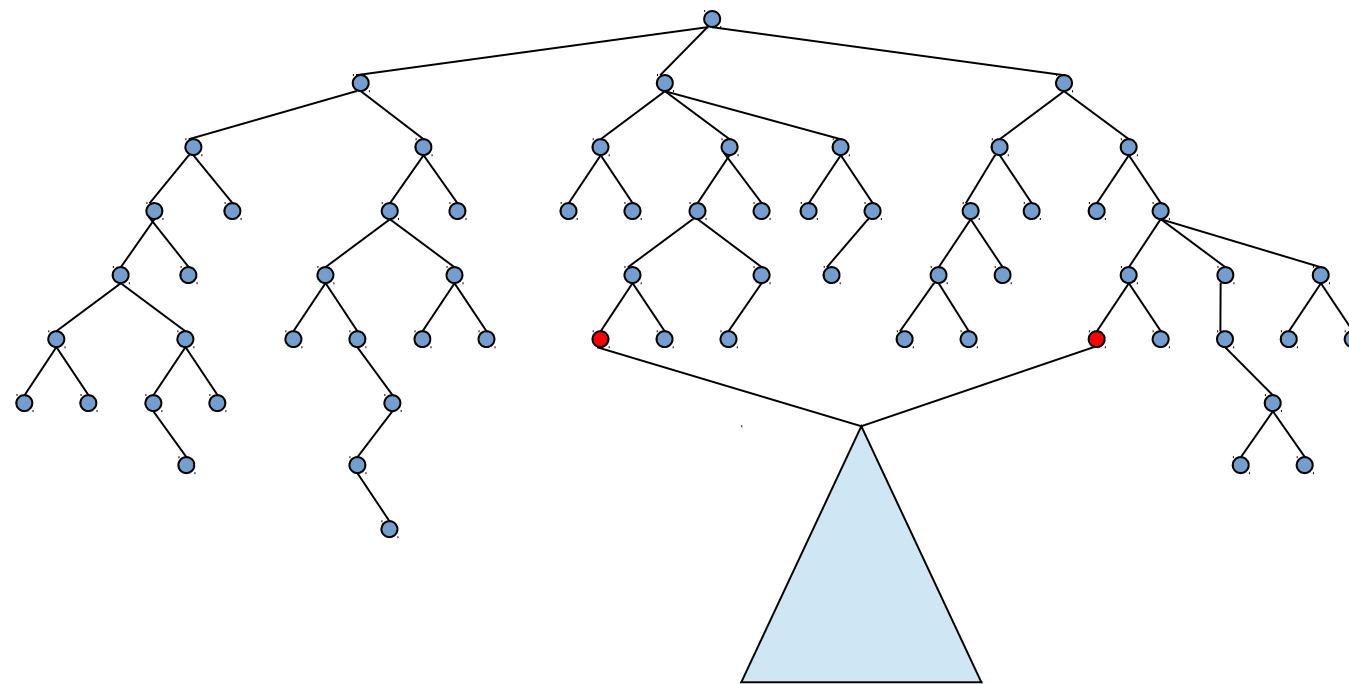
# From Search Trees to Search Graphs

- Any two nodes that root **identical** sub-trees or sub-graphs can be **merged**



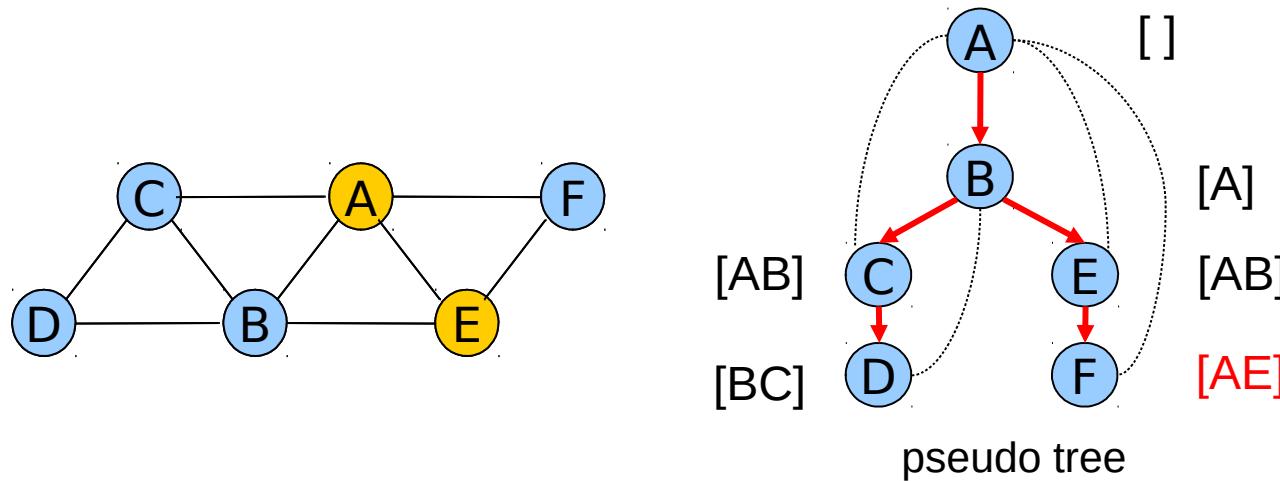
# From Search Trees to Search Graphs

- Any two nodes that root **identical** subtrees or subgraphs can be **merged**

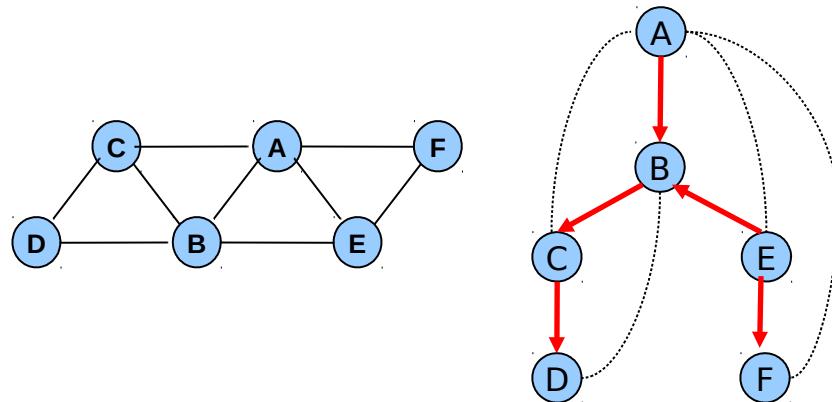


# Merging Based on Contexts

- One way of recognizing nodes that can be merged (based on the graph structure)
  - **context(X)** = ancestors of X in the pseudo tree that are connected to X or to descendants of X

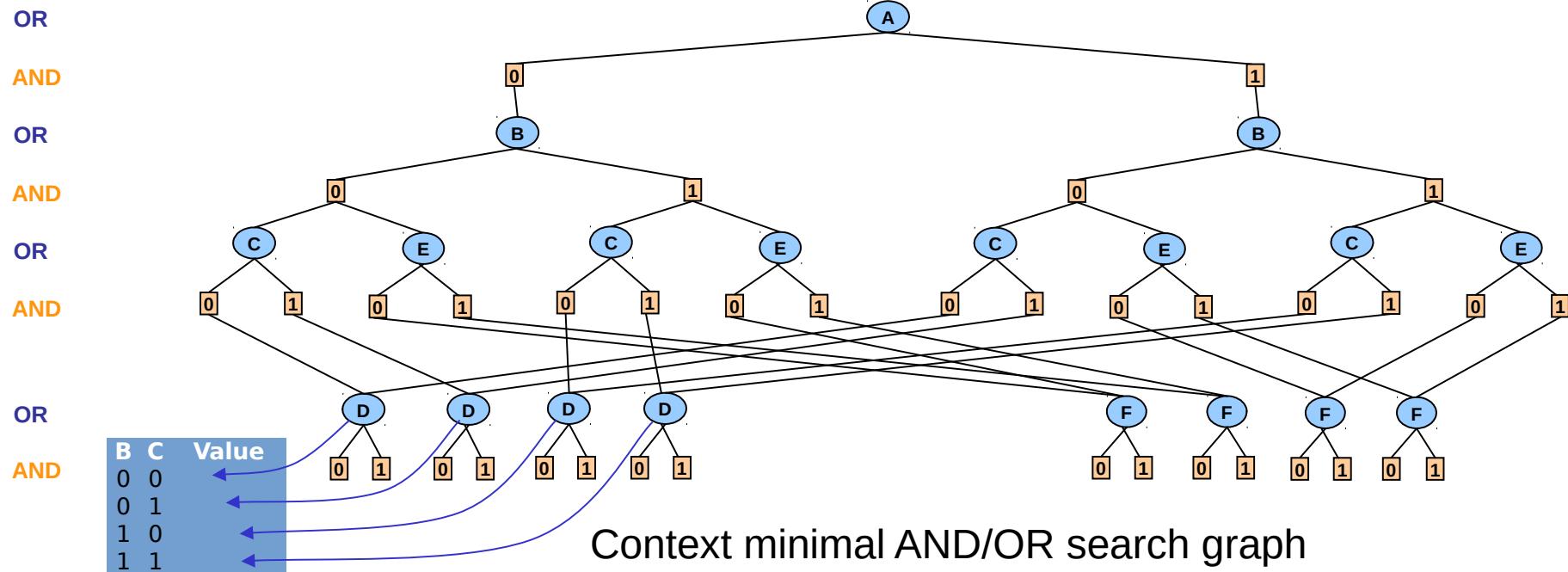


# AND/OR Search Graph



A	B	$f_{ab}$	A	C	$f_{ac}$	A	E	$f_{ae}$	A	F	$f_{af}$	B	C	$f_{bc}$	B	D	$f_{bd}$	B	E	$f_{be}$	C	D	$f_{cd}$	E	F	$f_{ef}$
0	0	2	0	0	3	0	0	0	0	0	2	0	0	0	0	0	4	0	0	3	0	0	1	0	0	
0	1	0	0	1	0	0	1	3	0	1	0	0	1	1	0	1	2	0	1	2	0	1	4	1	0	
1	0	1	1	0	0	1	0	2	1	0	0	1	0	2	1	0	1	4	1	0	1	1	0	0	1	
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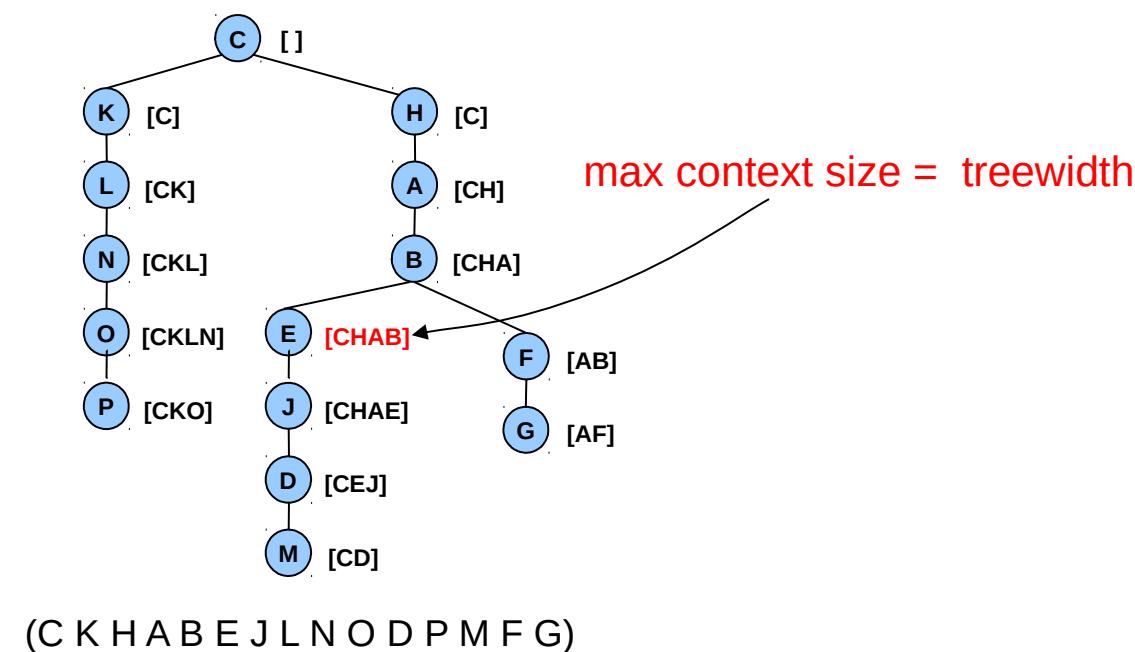
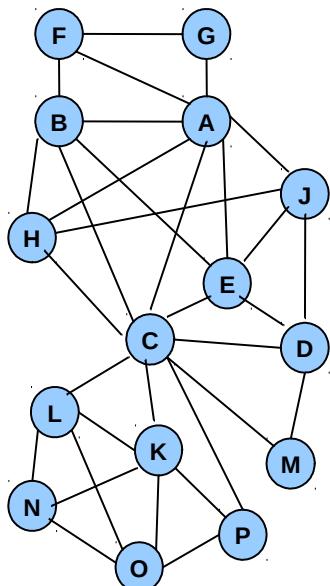
Objective function:  $F^* = \min_x \sum_{\alpha} f_{\alpha}(x_{\alpha})$



Cache table for **D**

# How Big Is The Context?

- **Theorem:** The maximum context size for a pseudo tree **is equal** to the **treewidth** of the graph along the pseudo tree.



# Complexity of AND/OR Graph Search

	AND/OR graph	OR graph
Space	$O(n d^{w^*})$	$O(n d^{pw^*})$
Time	$O(n d^{w^*})$	$O(n d^{pw^*})$

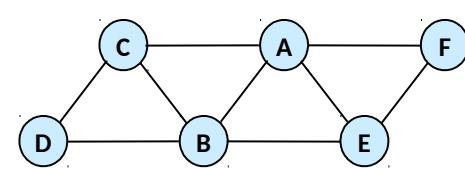
$d$  = domain size

$w^*$  = induced width

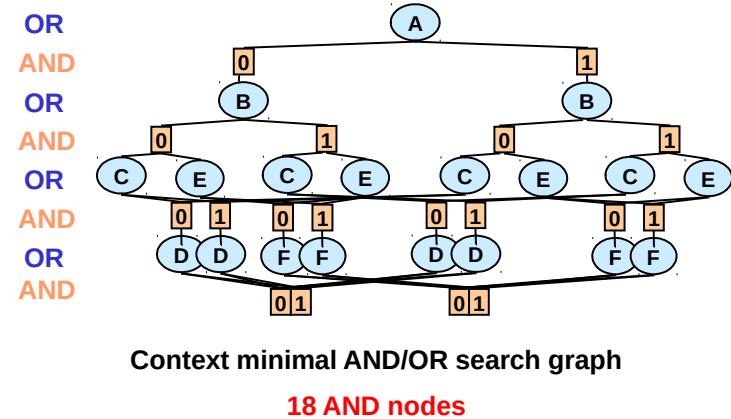
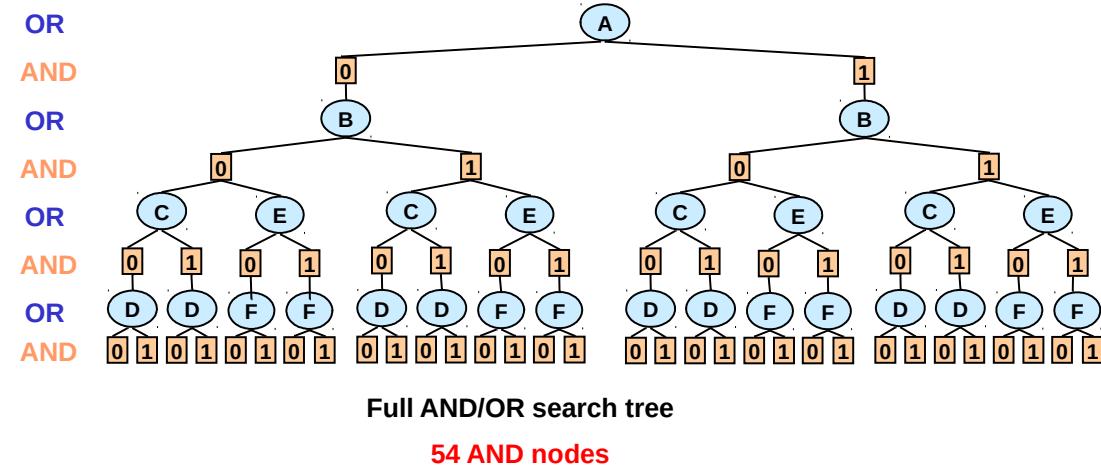
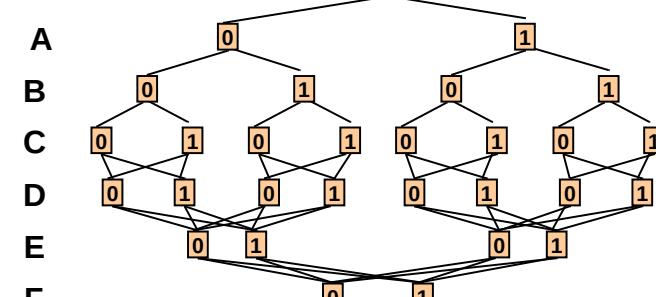
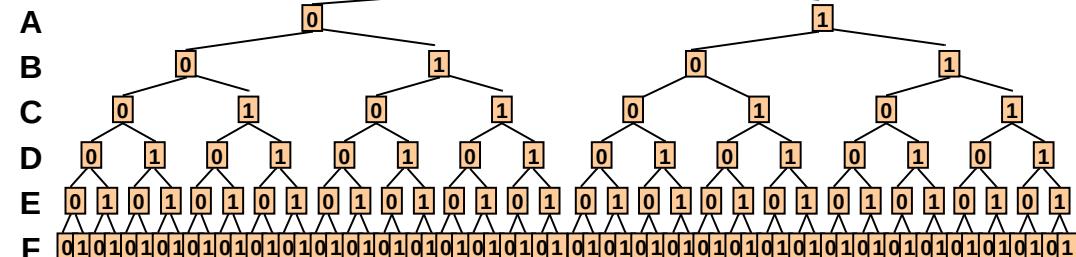
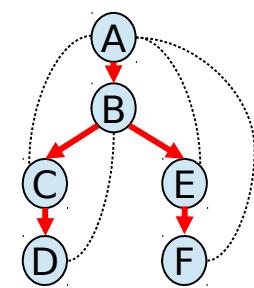
$n$  = number of variables

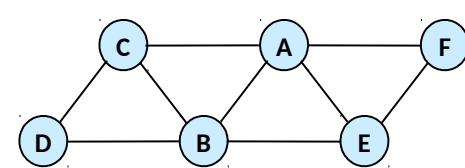
$pw^*$  = pathwidth

$$w^* \leq pw^* \leq w^* \log n$$

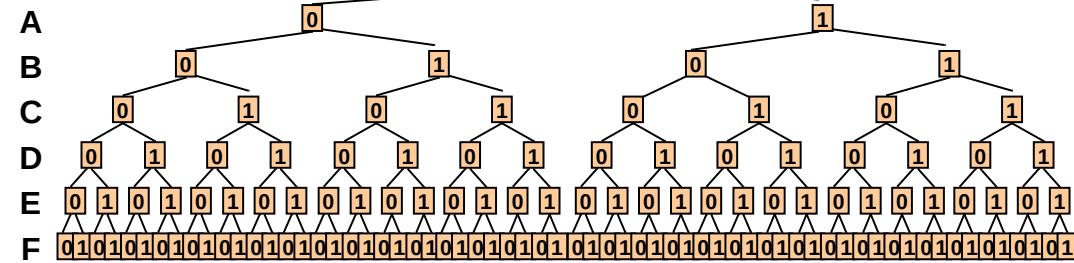
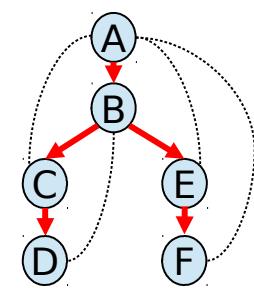


# All Four Search Spaces



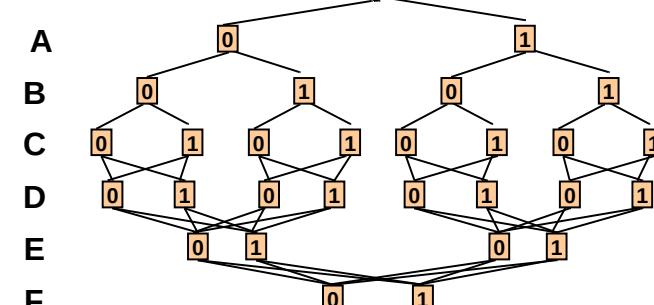


# All Four Search Spaces



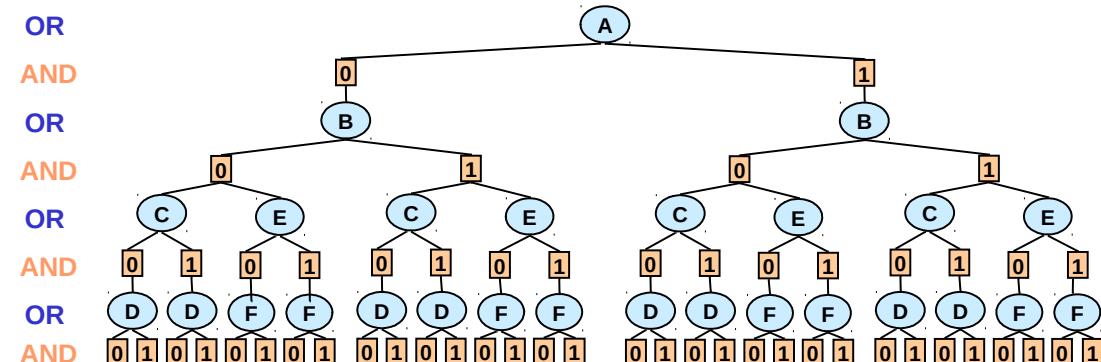
Full OR search tree

**126 nodes**



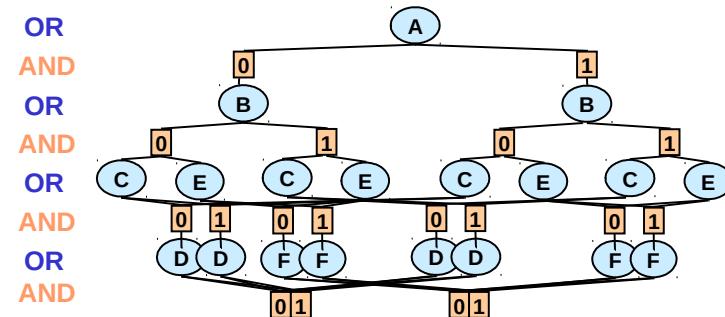
Context minimal OR search graph

**28 nodes**



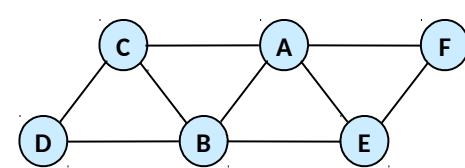
Full AND/OR search tree

**54 AND nodes**

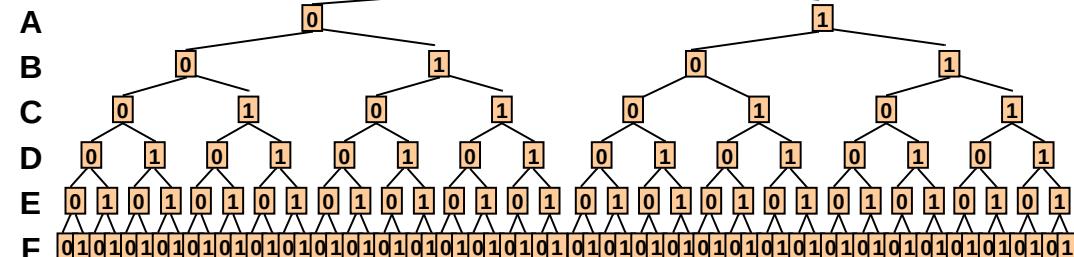
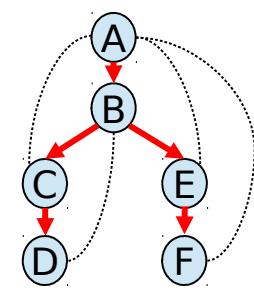


Context minimal AND/OR search graph

**18 AND nodes**

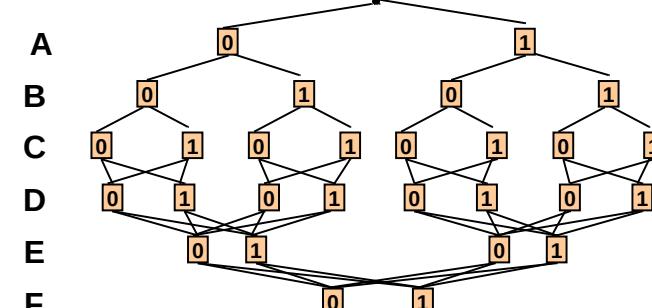


# All Four Search Spaces



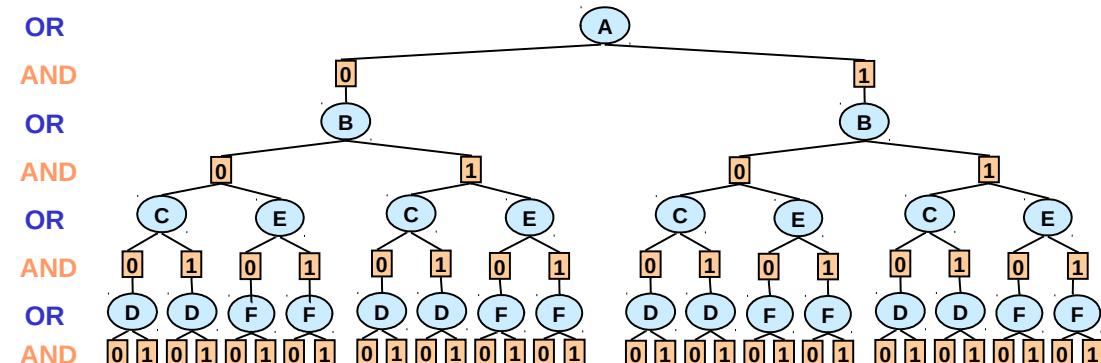
Full OR search tree

126 nodes



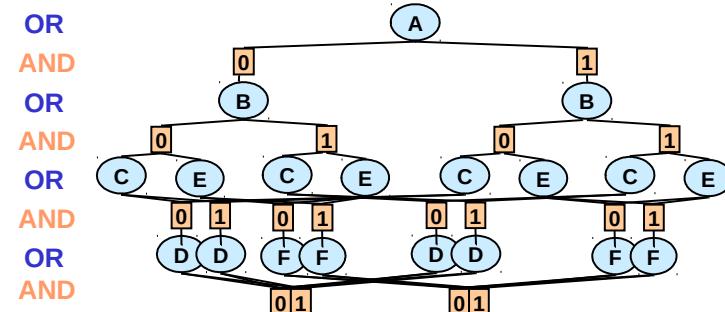
Context minimal OR search graph

28 nodes



Full AND/OR search tree

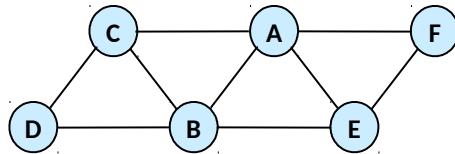
54 AND nodes



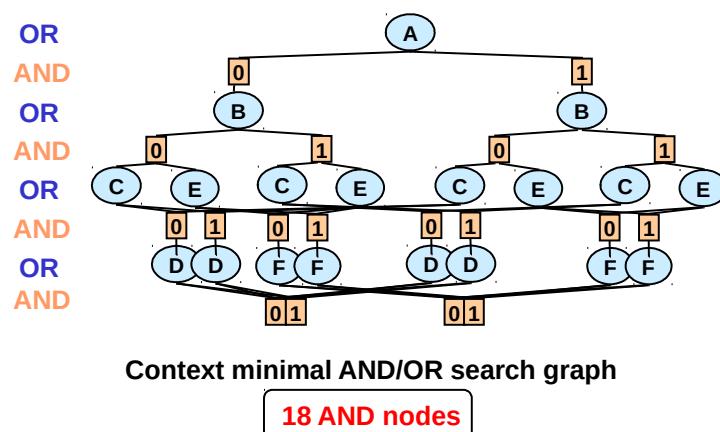
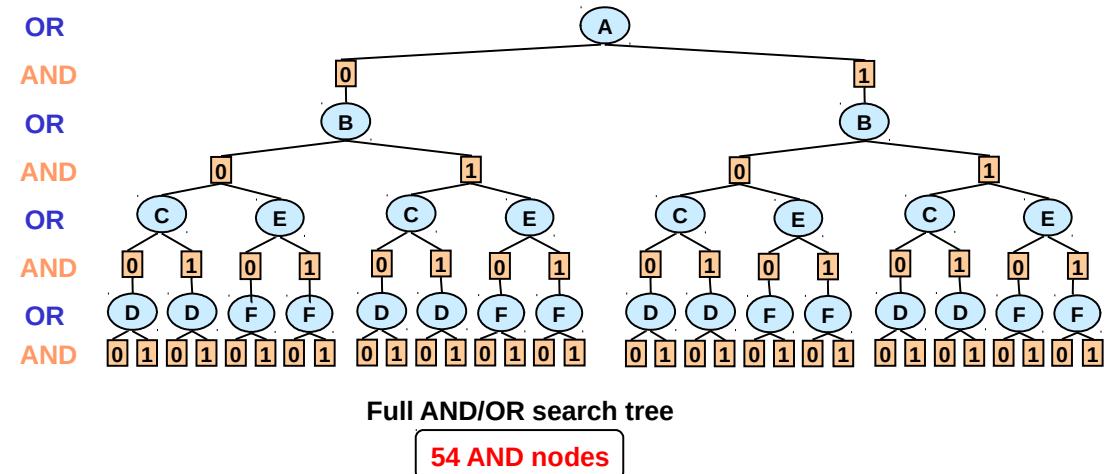
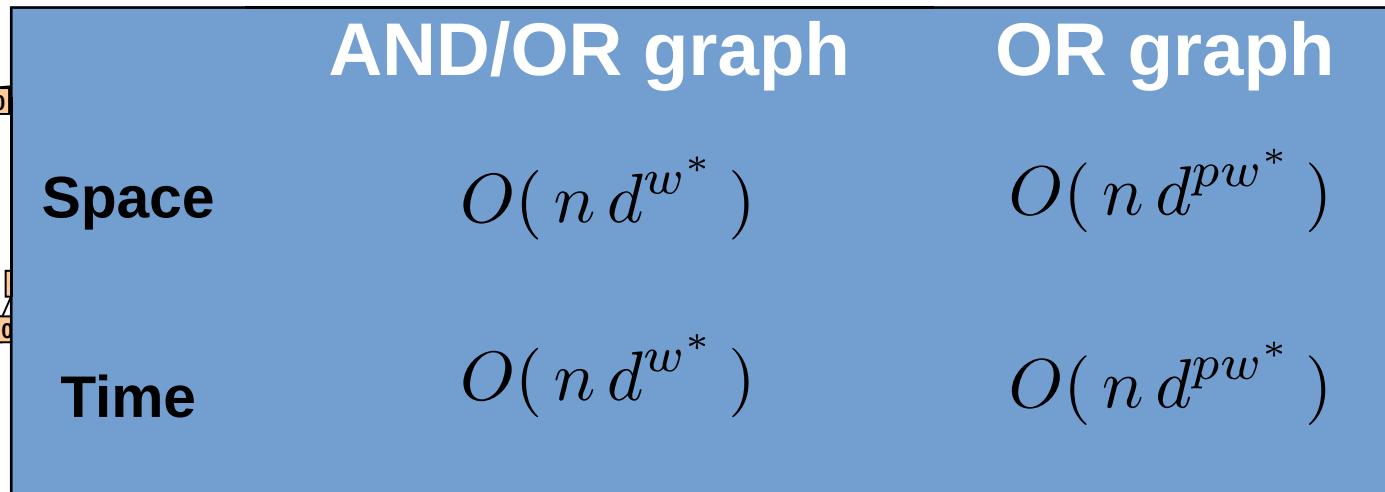
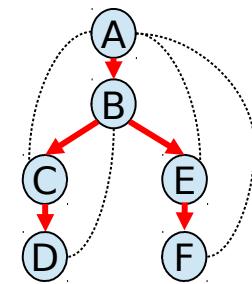
Context minimal AND/OR search graph

18 AND nodes

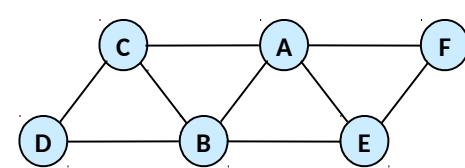
Any query is best computed over the c-minimal AO space



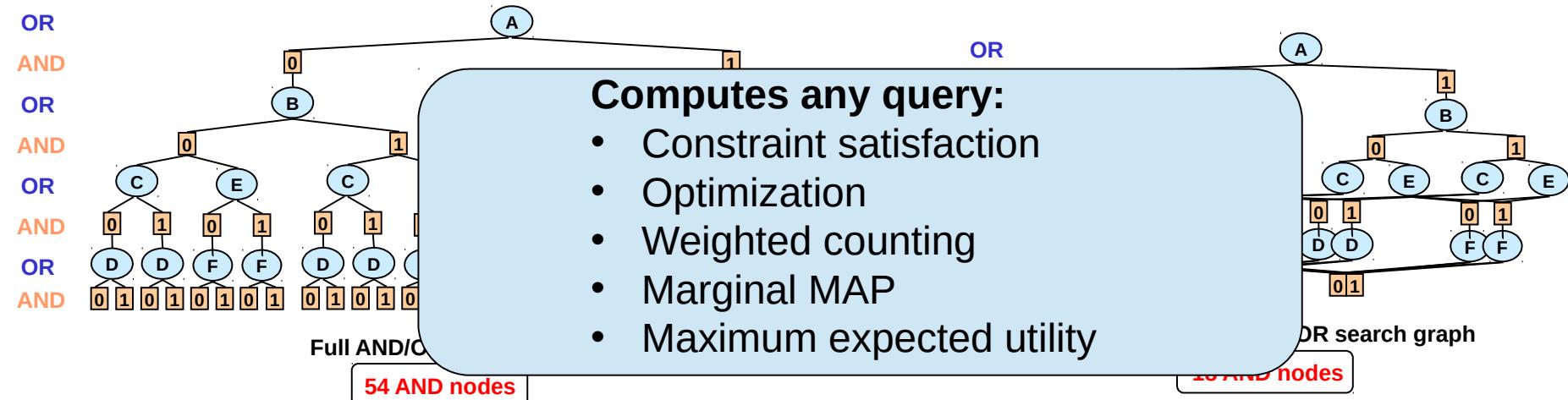
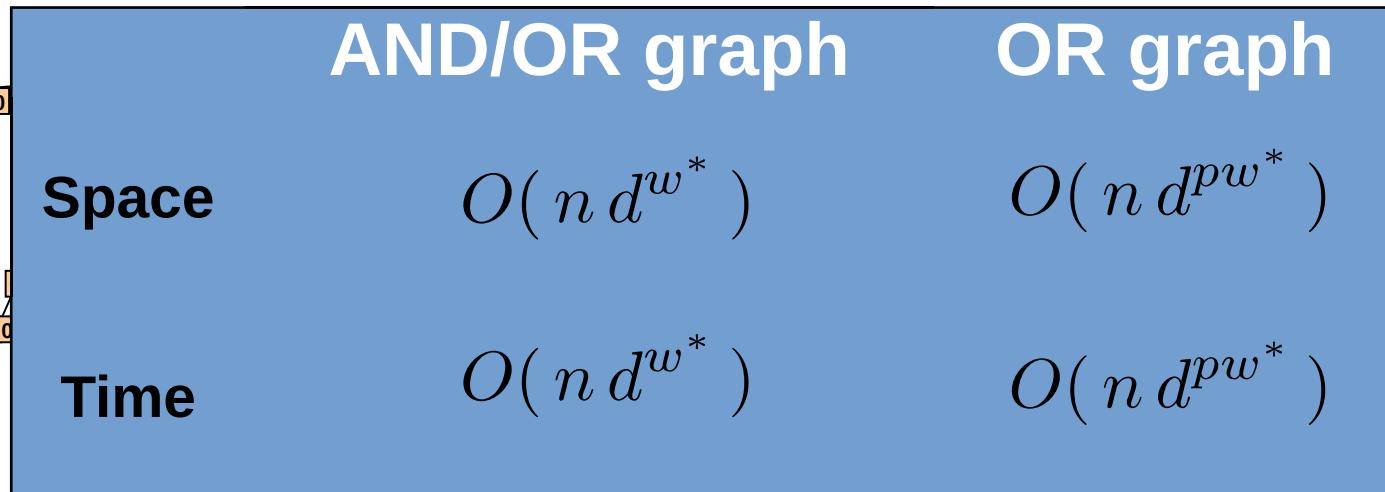
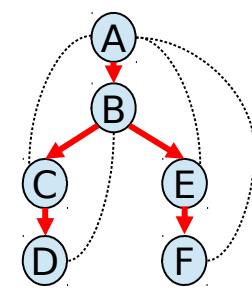
# All Four Search Spaces



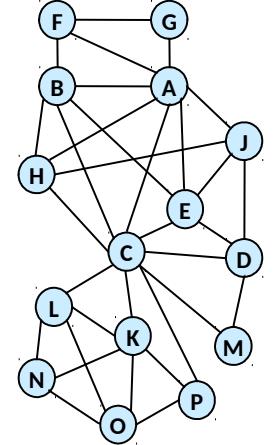
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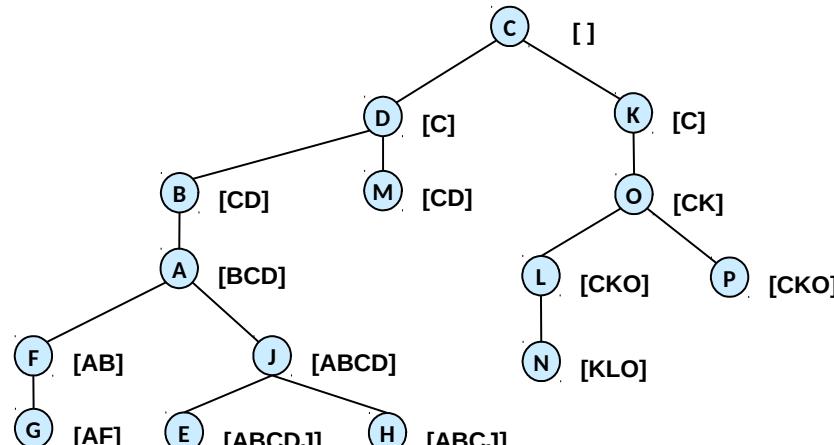
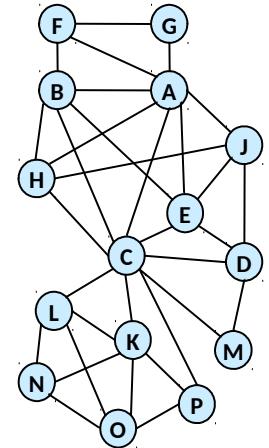
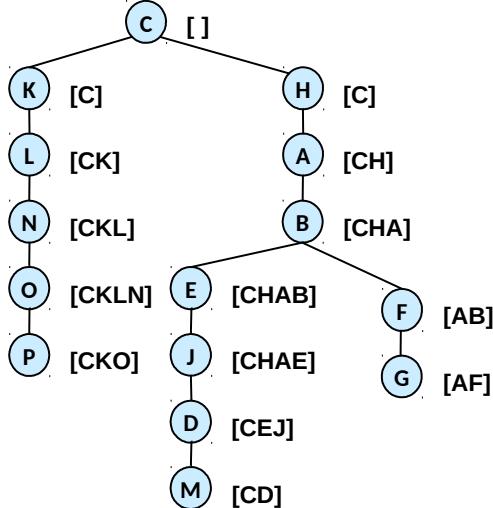
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# The Impact of the Pseudo Tree

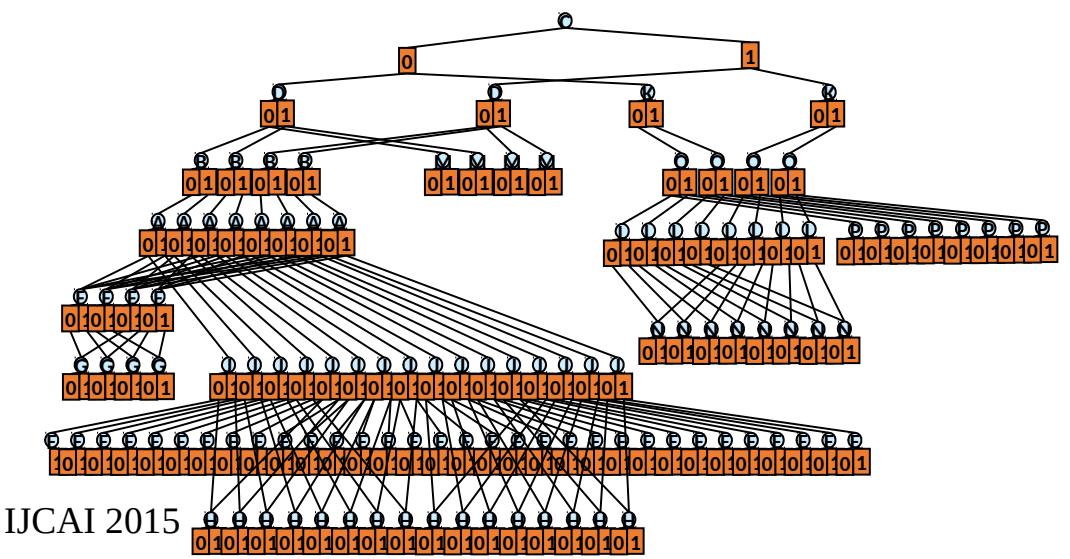
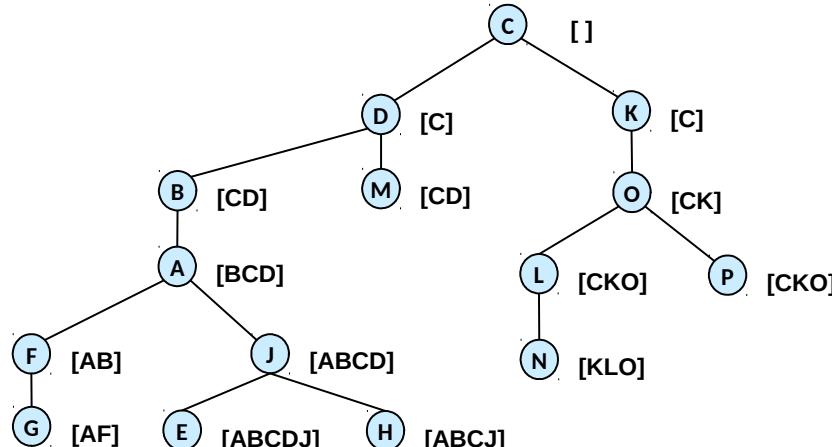
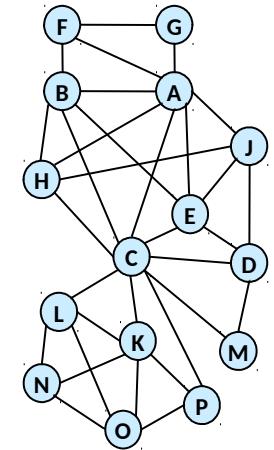
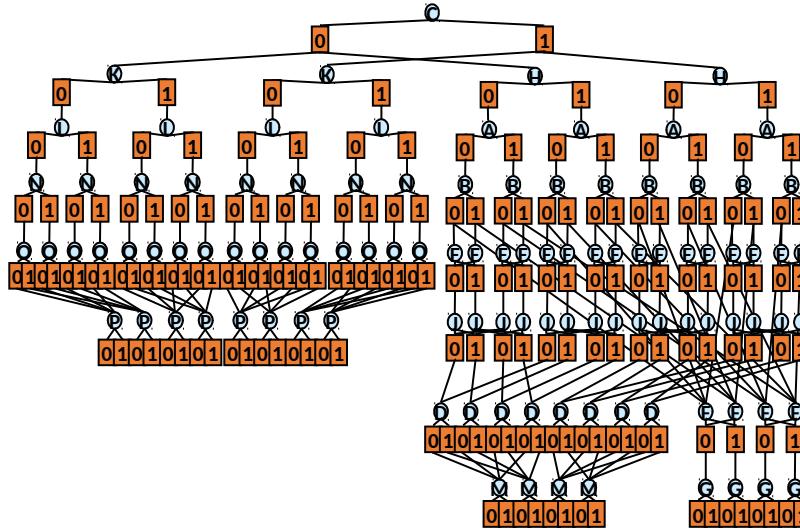
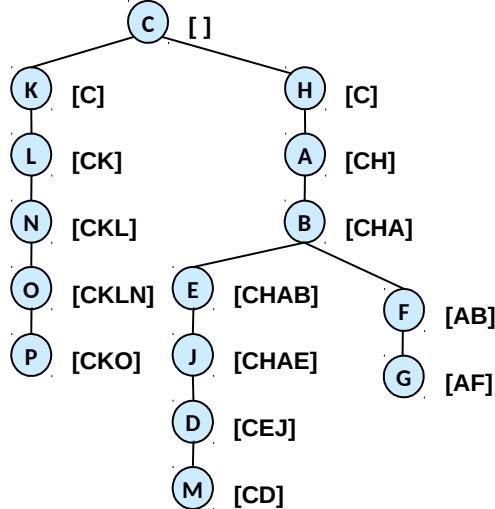


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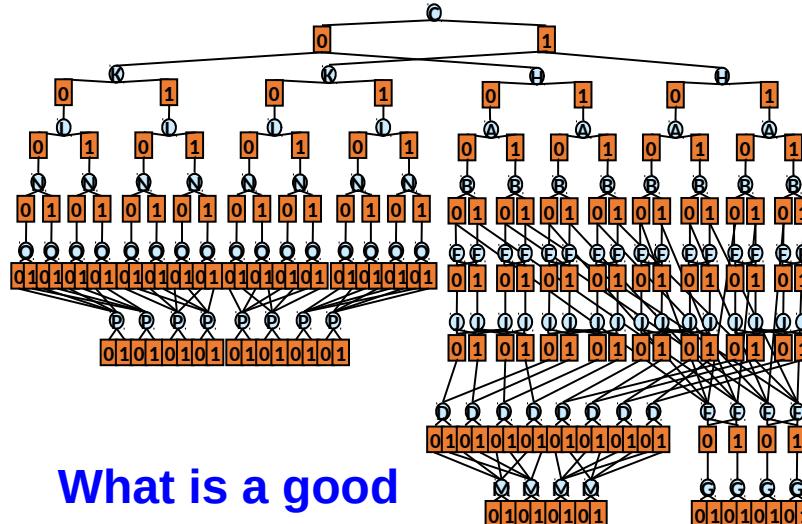
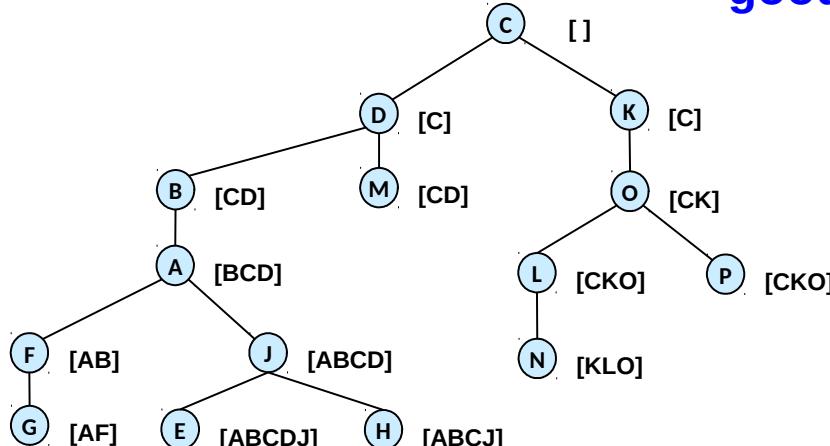
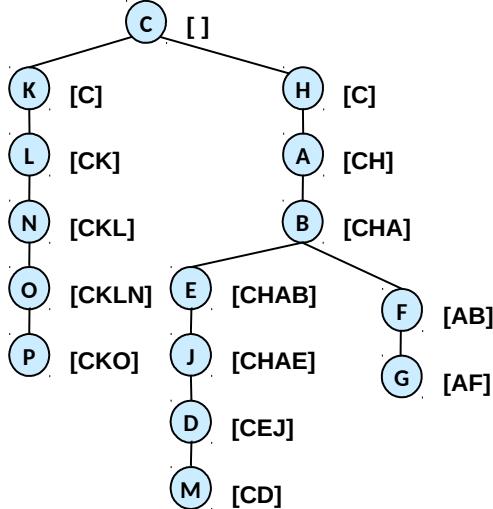


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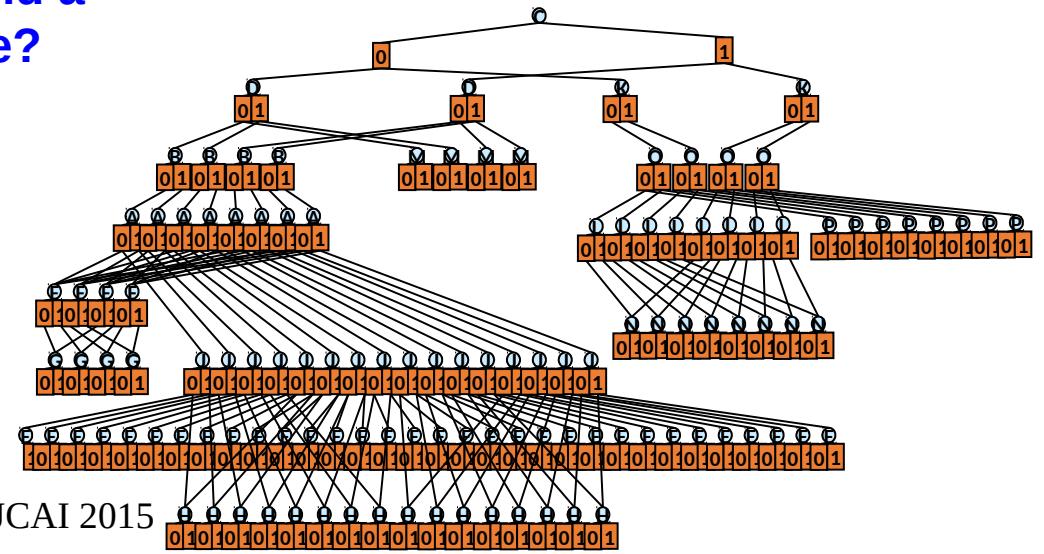


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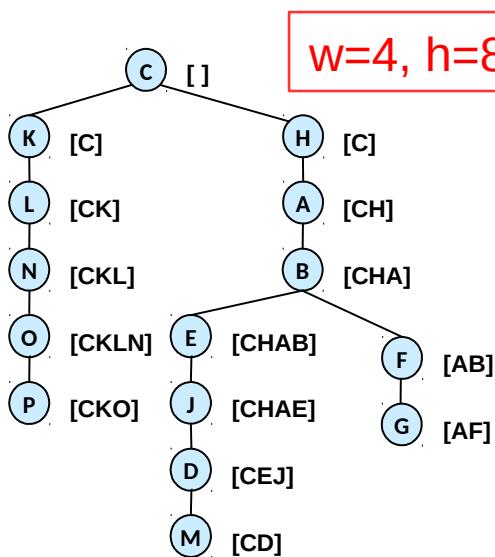


What is a good  
pseudo-tree?

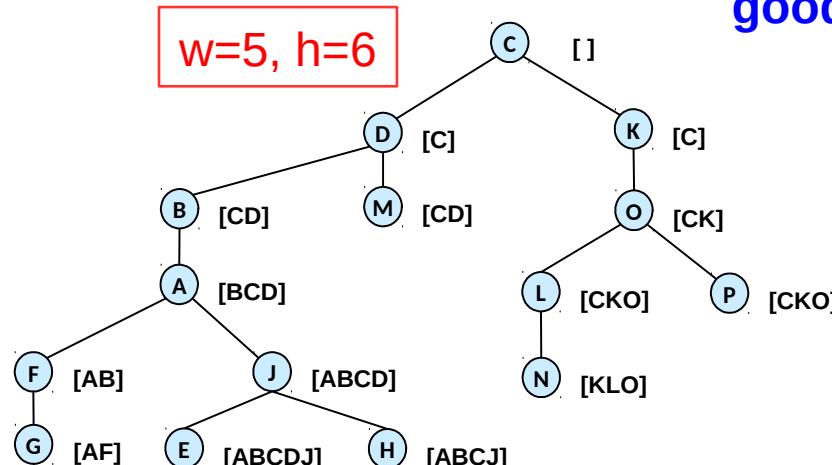
How to find a  
good one?



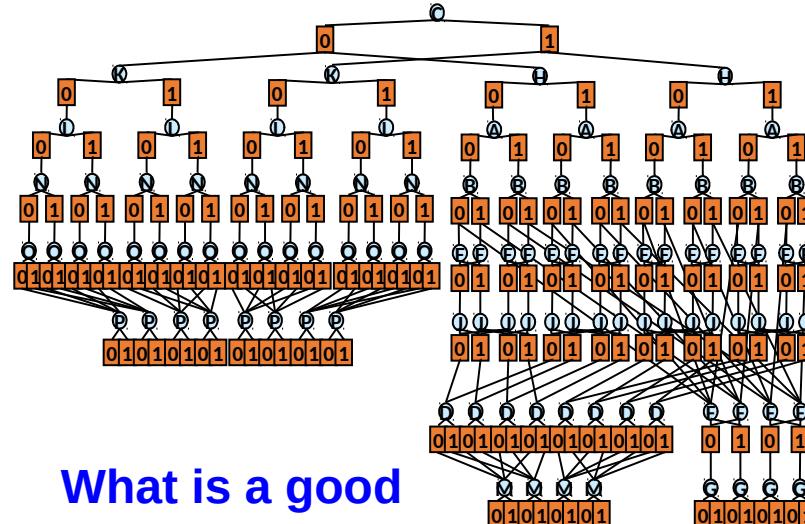
# The Impact of the Pseudo Tree



(C K H A B E J L N O D P M F G)

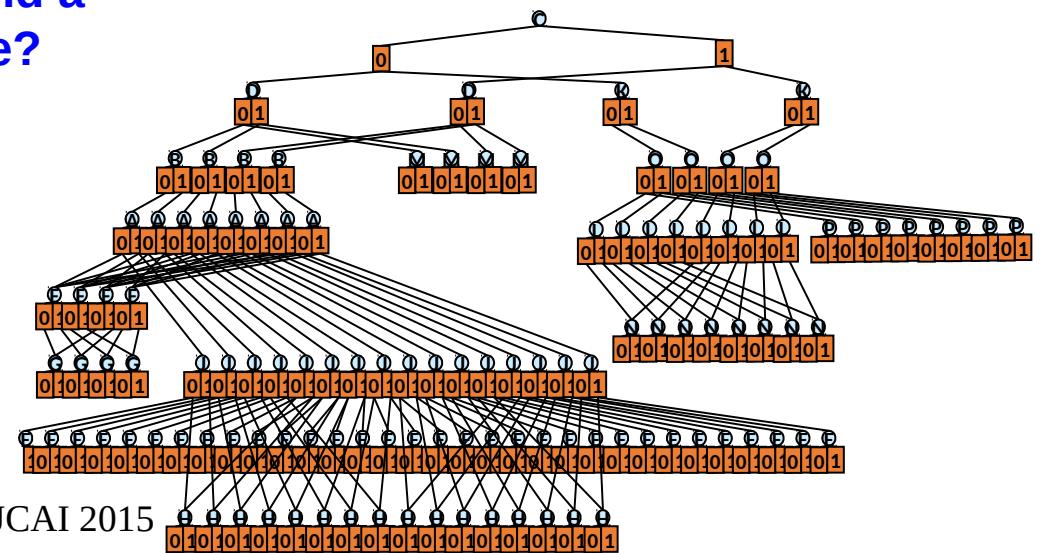


(C D K B A O M L N P J H E F G)

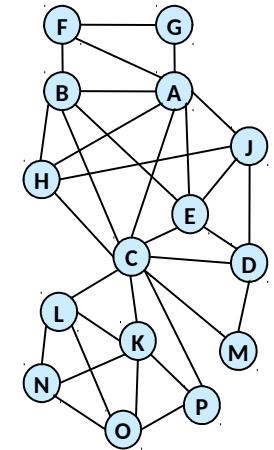


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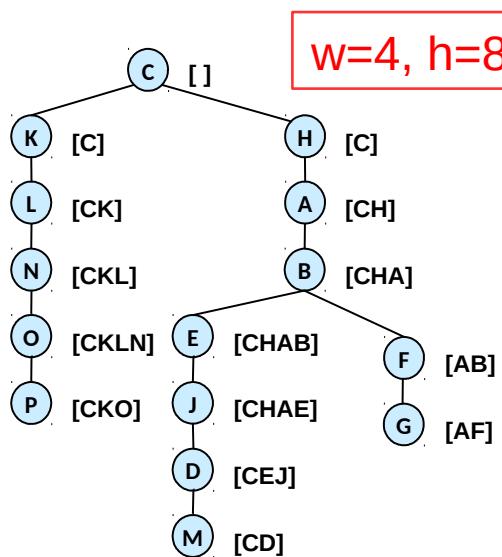
How to find a good one?



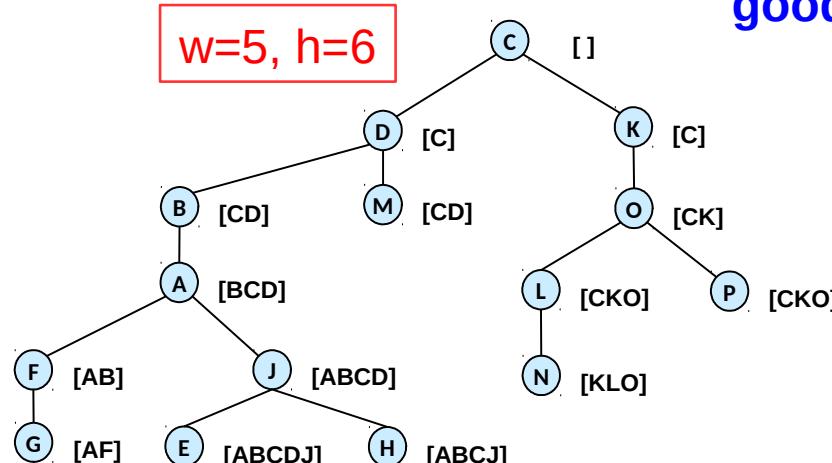
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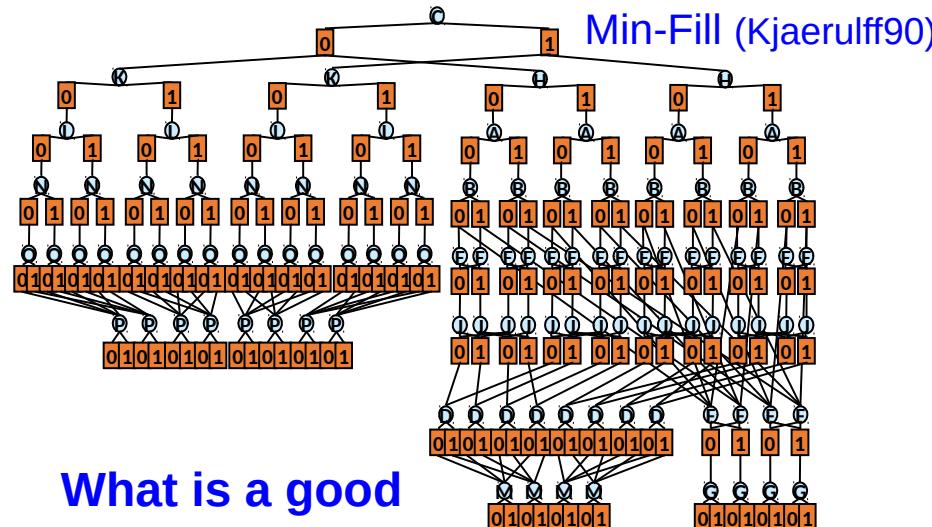
# The Impact of the Pseudo Tree



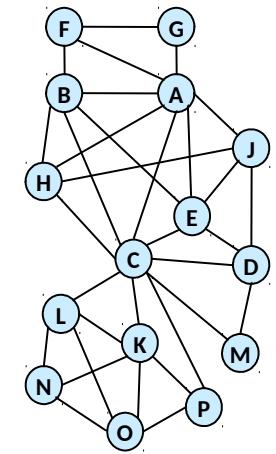
(C K H A B E J L N O D P M F G)



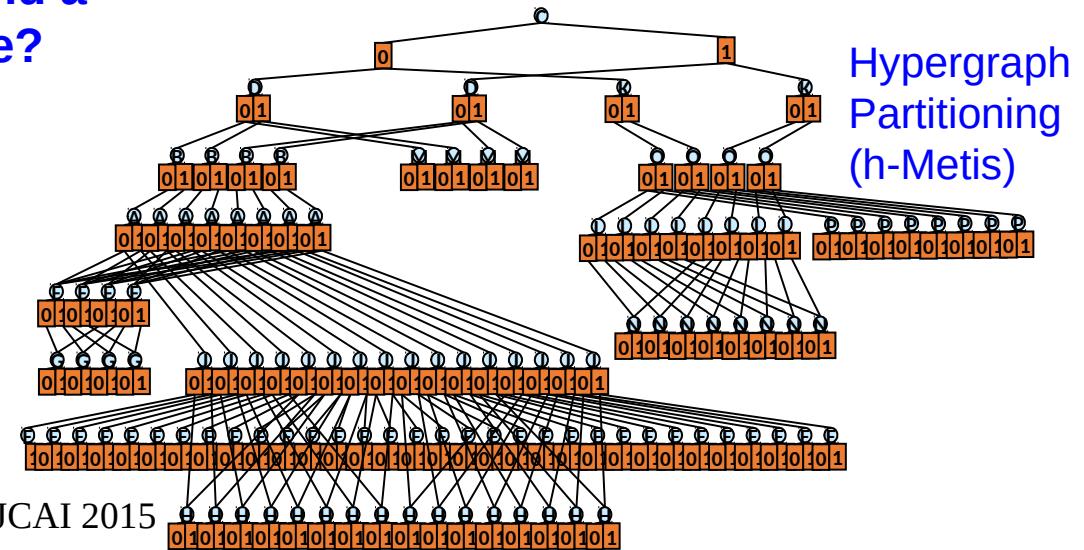
(C D K B A O M L N P J H E F G)



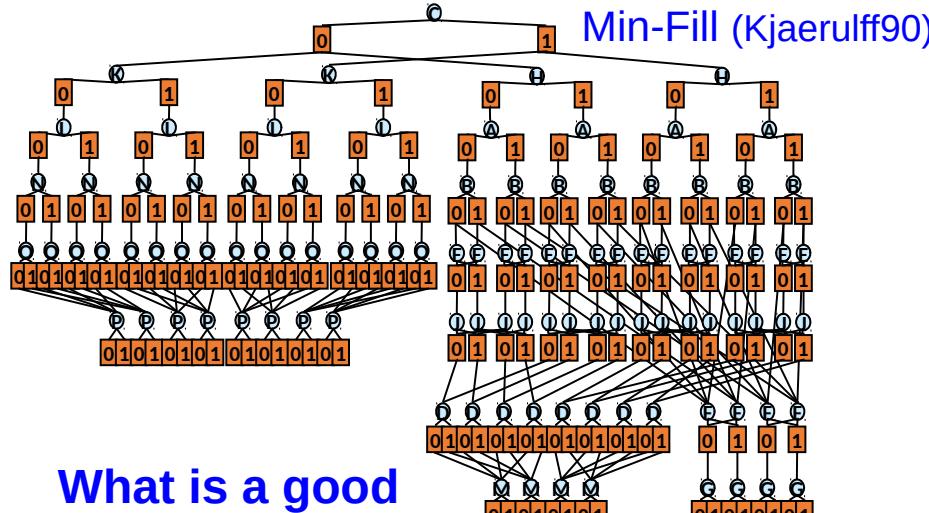
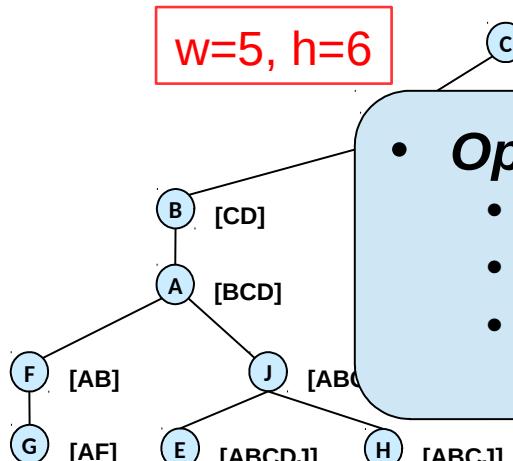
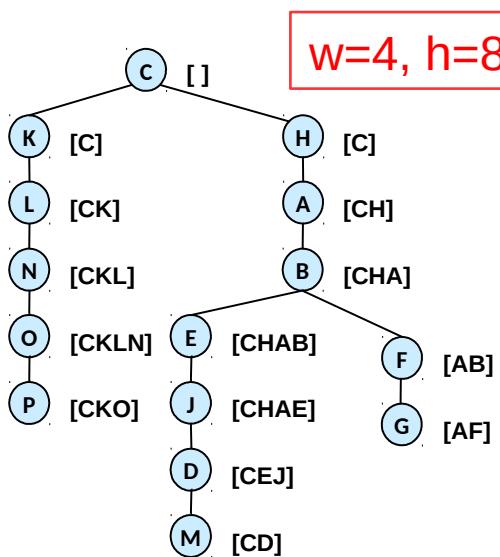
**What is a good pseudo-tree?**



**How to find a good one?**



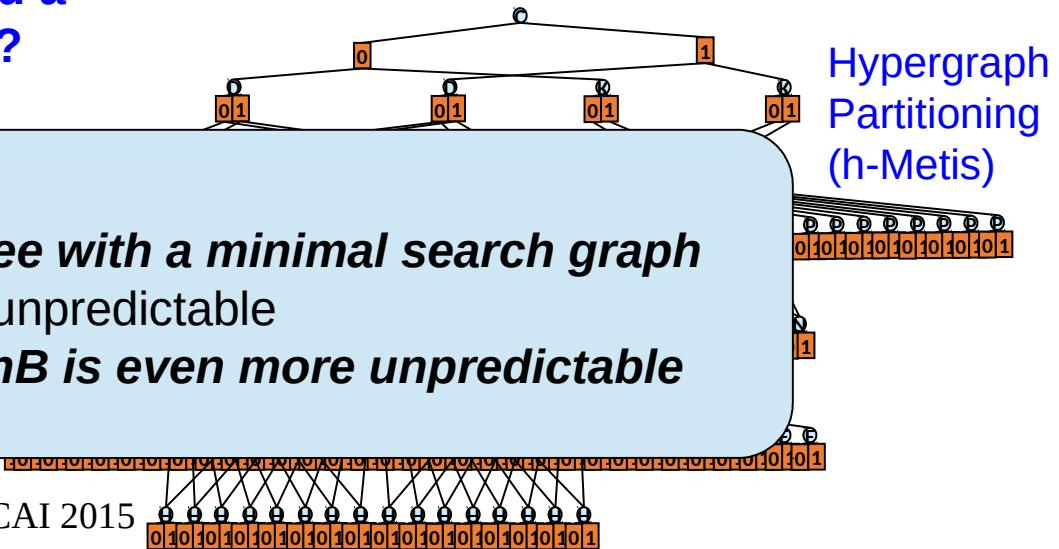
# The Impact of the Pseudo Tree



How to find a good one?

- **Optimization**
  - *Choose pseudo tree with a minimal search graph*
  - But determinism is unpredictable
  - *And pruning by BnB is even more unpredictable*

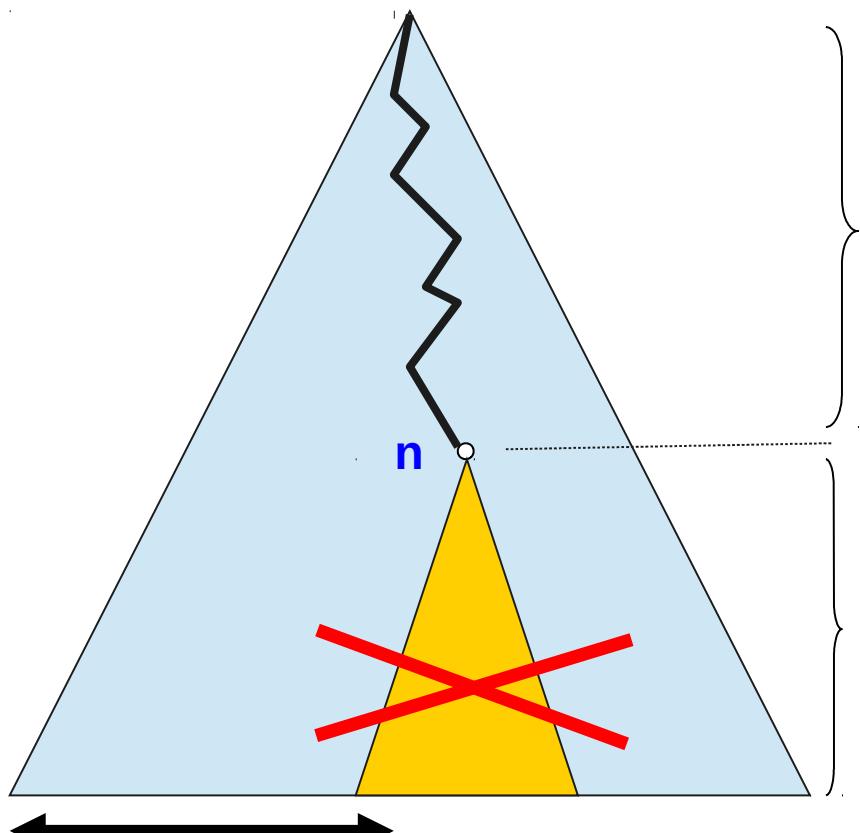
IJCAI 2015



# Outline

- Introduction
- Inference
- Bounds and heuristics
- **AND/OR search**
  - AND/OR search spaces
  - Depth-first AND/OR branch and bound
  - Best-first AND/OR search
  - Advanced searches and tasks
- Exploiting parallelism
- Software

# Classic Depth-First Branch and Bound



Each node is a COP sub-problem  
(defined by current conditioning)

$g(n)$  : cost of the path from root to n

$$\tilde{f}(n) = g(n) + \tilde{h}(n)$$

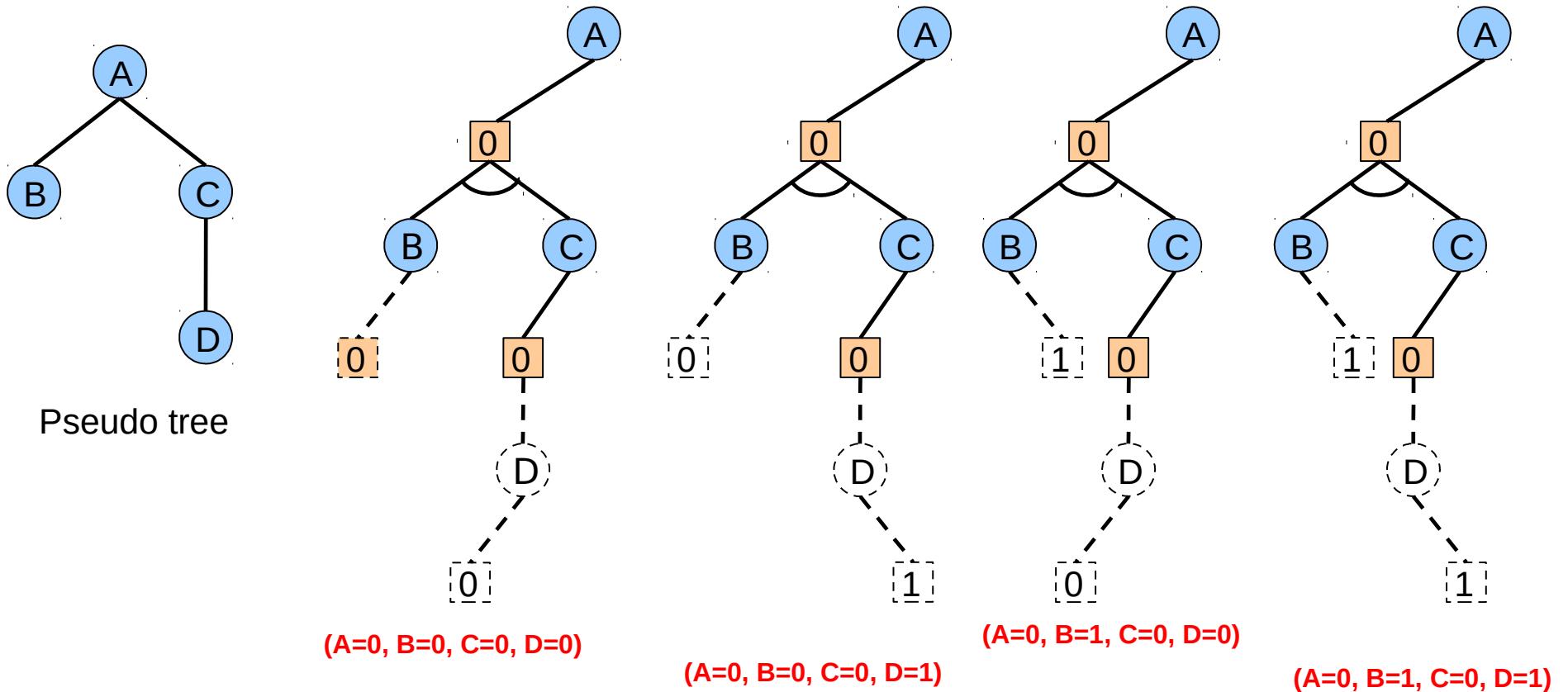
(lower bound)

**Prune if  $\tilde{f}(n) \geq UB$**

$\tilde{h}(n)$  : under-estimates optimal cost below n

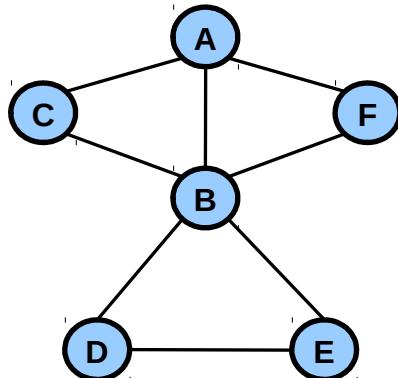
**(UB)** Upper Bound = best solution so far

# Partial Solution Tree



Extension( $T'$ ) – solution trees that extend  $T'$

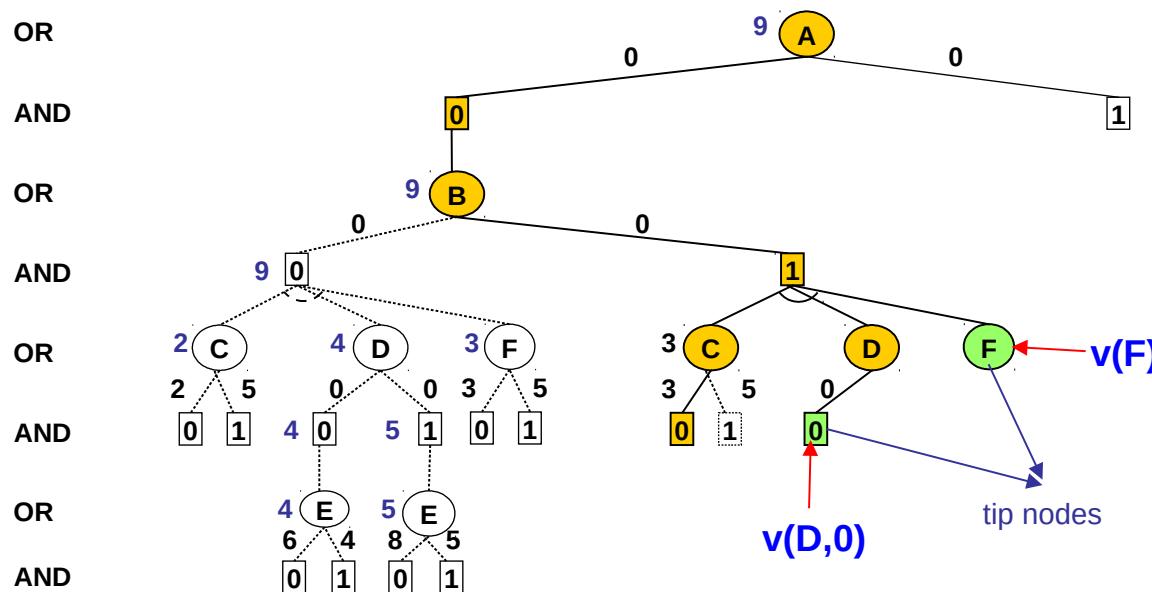
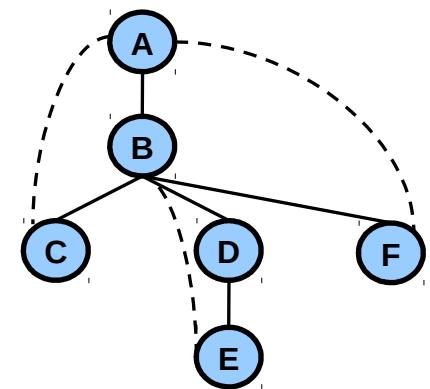
# Exact Evaluation Function



A	B	C	$f_1(ABC)$
0	0	0	2
0	0	1	5
0	1	0	3
0	1	1	5
1	0	0	9
1	0	1	3
1	1	0	7
1	1	1	2

A	B	F	$f_2(ABF)$
0	0	0	3
0	0	1	5
0	1	0	1
0	1	1	4
1	0	0	6
1	0	1	5
1	1	0	6
1	1	1	5

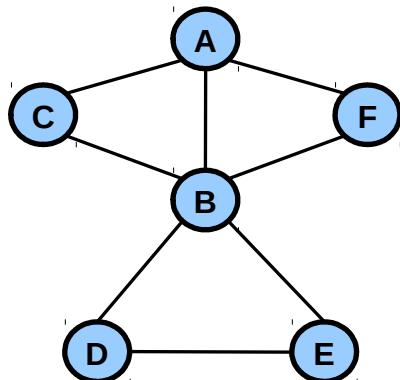
B	D	E	$f_3(BDE)$
0	0	0	6
0	0	1	4
0	1	0	8
0	1	1	5
1	0	0	9
1	0	1	3
1	1	0	7
1	1	1	4



$$f^*(T') = w(A, 0) + w(B, 1) + w(C, 0) + w(D, 0) + v(D, 0) + v(F)$$

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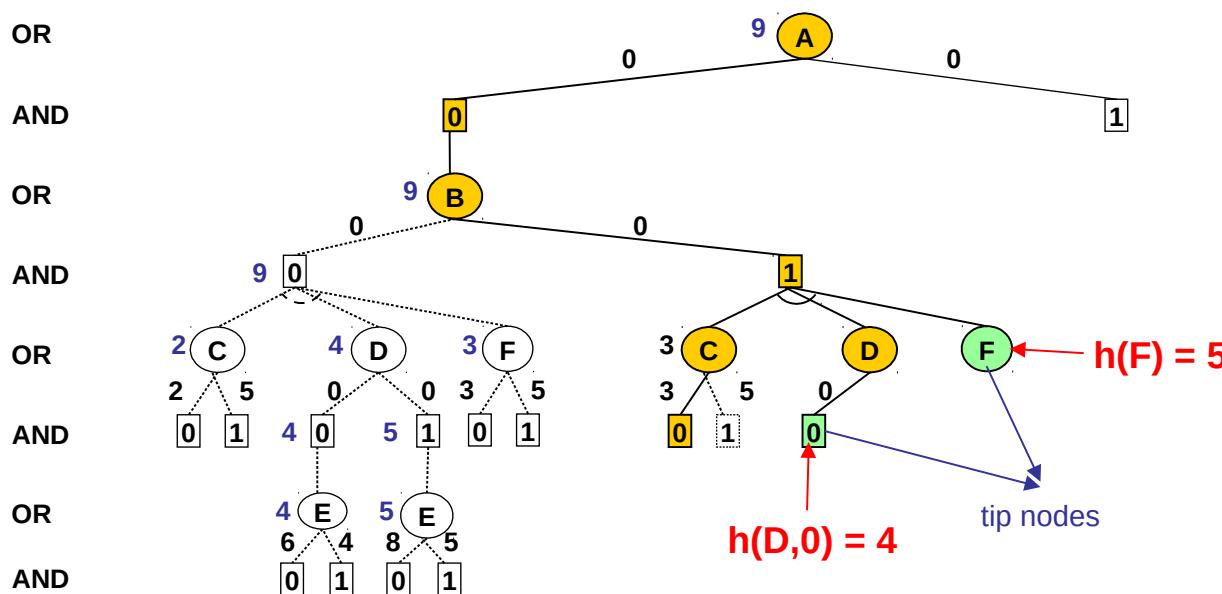
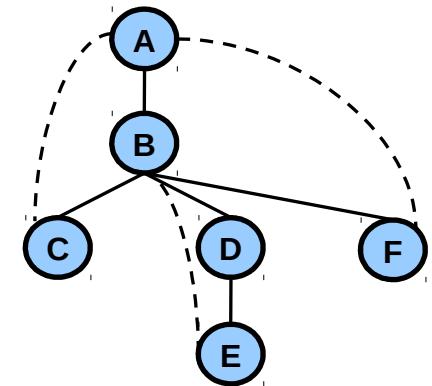
# Heuristic Evaluation Function



A	B	C	$f_1(ABC)$
0	0	0	2
0	0	1	5
0	1	0	3
0	1	1	5
1	0	0	9
1	0	1	3
1	1	0	7
1	1	1	2

A	B	F	$f_2(ABF)$
0	0	0	3
0	0	1	5
0	1	0	1
0	1	1	4
1	0	0	6
1	0	1	5
1	1	0	6
1	1	1	5

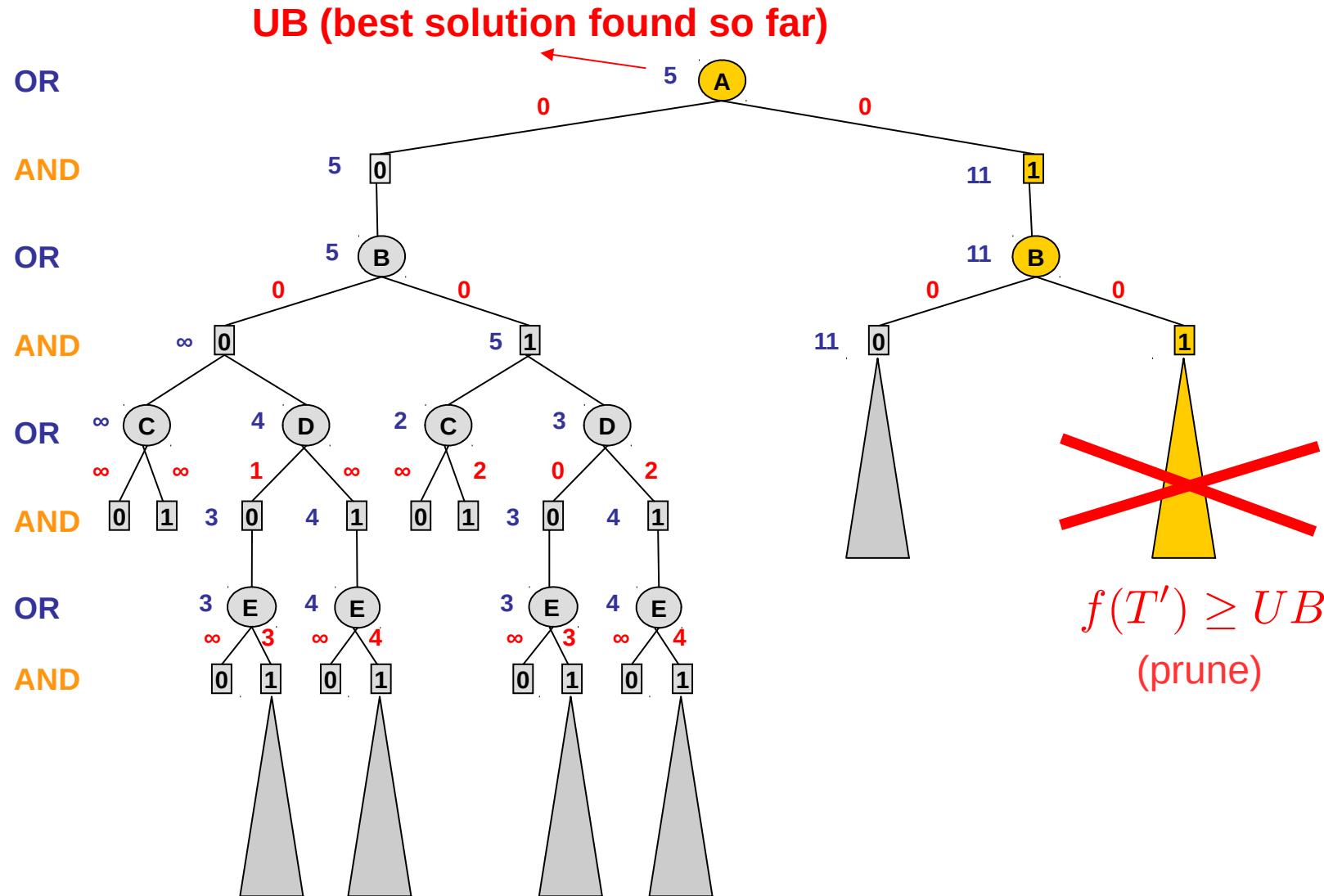
B	D	E	$f_3(BDE)$
0	0	0	6
0	0	1	4
0	1	0	8
0	1	1	5
1	0	0	9
1	0	1	3
1	1	0	7
1	1	1	4



$$f(T') = w(A,0) + w(B,1) + w(C,0) + w(D,0) + h(D,0) + h(F) = 12 \leq f^*(T')$$

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# AND/OR Branch and Bound Search

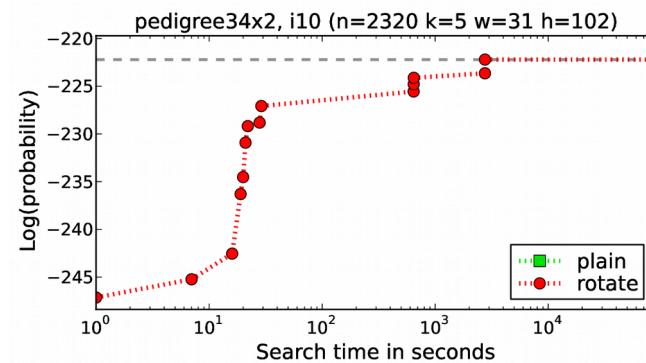
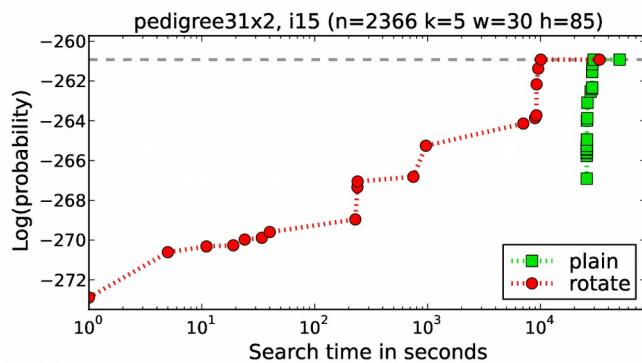
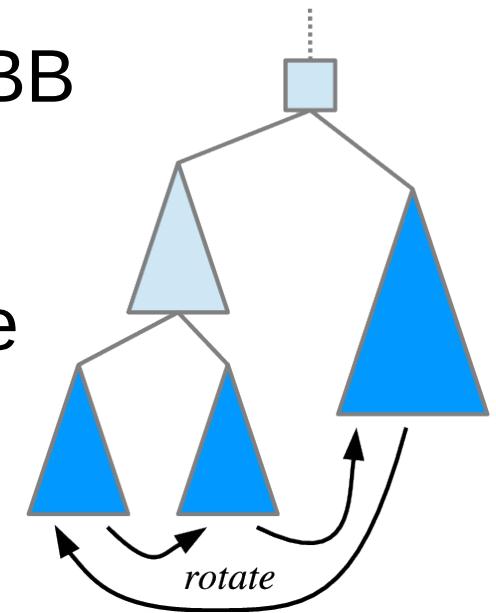


# AND/OR Branch and Bound (AOBB)

- Each node  $n$ : heuristic lower bound  $h(n)$  on  $v(n)$
- **EXPAND** (top-down)
  - Evaluate  $f(T')$  and prune search if  $f(T') \geq UB$
  - If not in cache, generate successors of the tip node  $n$
- **UPDATE** (bottom-up)
  - Update value of the parent  $p$  of  $n$ 
    - OR nodes: **minimization**
    - AND nodes: **summation**
  - Cache value of  $n$  based on context

# Breadth-Rotating AOBB

- AND/OR decomposition vs. depth-first search:
  - Compromises anytime property of AOBB
- **Breadth-Rotating AOBB:**
  - Combined breadth/depth-first schedule
  - Maintains depth-first complexity
  - Superior experimental results



# Mini-Bucket Heuristics for AND/OR Search

- The depth-first and best-first AND/OR search algorithms use  $h(n)$  that can be computed:
  - **Static Mini-Bucket Heuristics**
    - Pre-compiled
    - Reduced computational overhead
    - Less accurate
    - Static variable ordering
  - **Dynamic Mini-Bucket Heuristics**
    - Computed dynamically, during search
    - Higher computational overhead
    - High accuracy
    - Dynamic variable ordering

# Outline

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# Basic Heuristic Search Schemes

Heuristic function  $\tilde{f}(\hat{x}_p)$  computes a lower bound on the best extension of partial configuration  $\hat{x}_p$  and can be used to guide heuristic search.

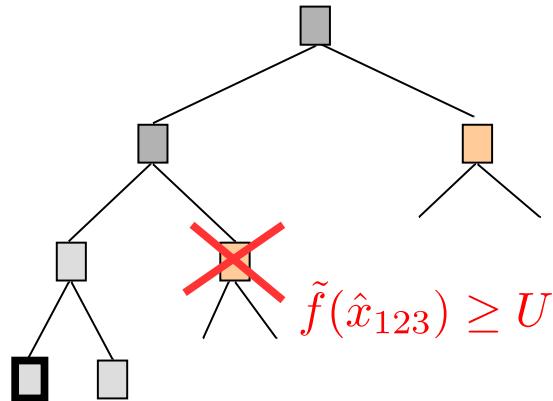
We focus on:

## 1. Branch-and-Bound

Use heuristic function  $\tilde{f}(\hat{x}_p)$  to prune the depth-first search tree

Linear space

- *improve U from “above”*

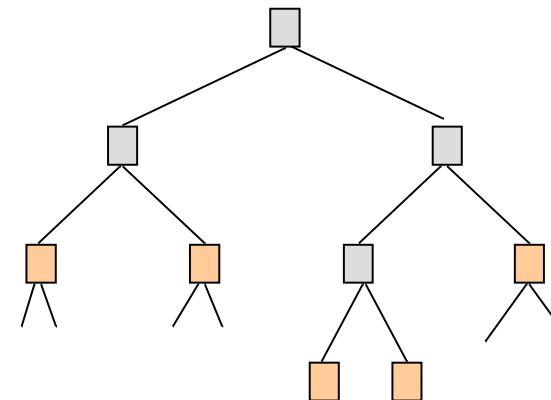


## 2. Best-First Search

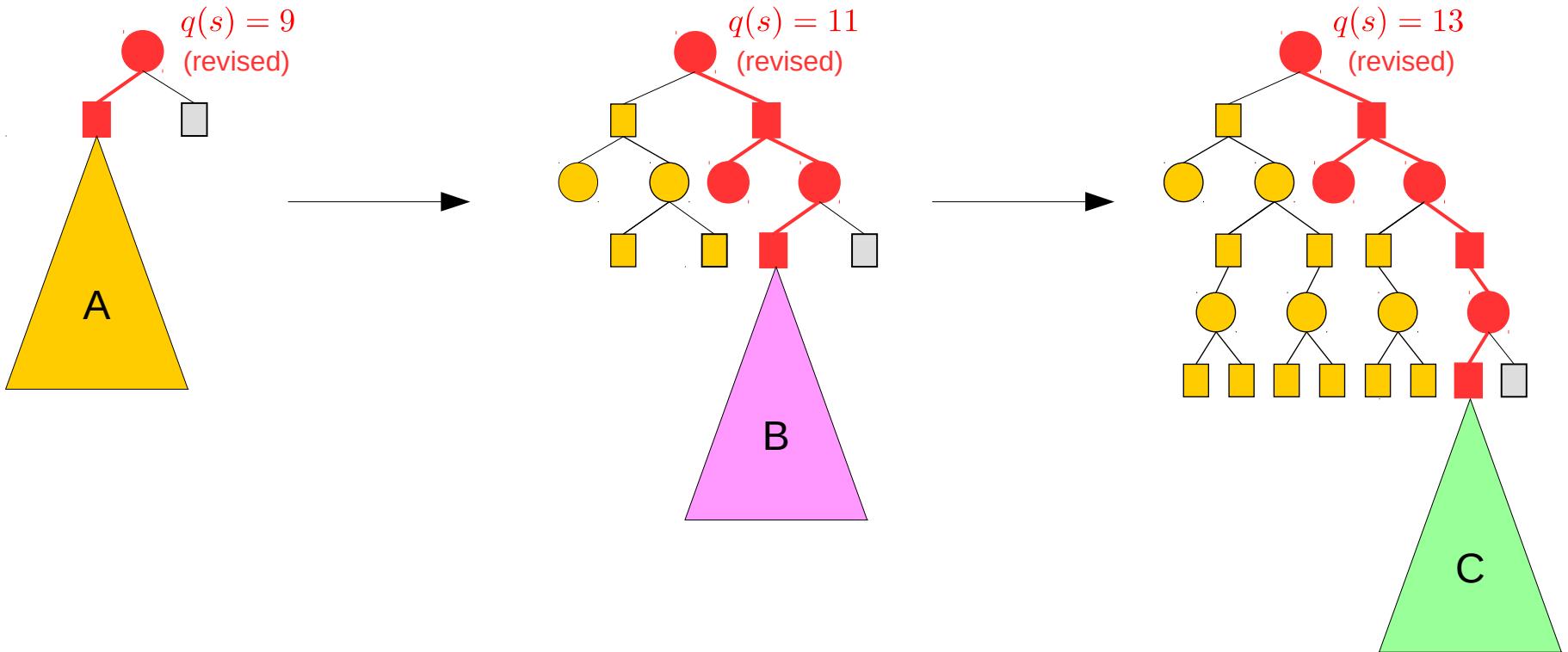
Always expand the node with the lowest heuristic value  $\tilde{f}(\hat{x}_p)$

Needs lots of memory

- *improve L from “below”*



# AOBF: Best-First AND/OR Search



- Each node maintains a q-value  $q(n)$  initially  $q(n) = h(n)$
- Node q-values revised bottom-up after each node expansion
- Update current best partial solution subtree (a tip node expanded next)
- All expanded nodes are stored in memory
- Search terminates with optimal solution

[Marinescu and Dechter, 2006; 2009]

# AOBF: Best-First AND/OR Search

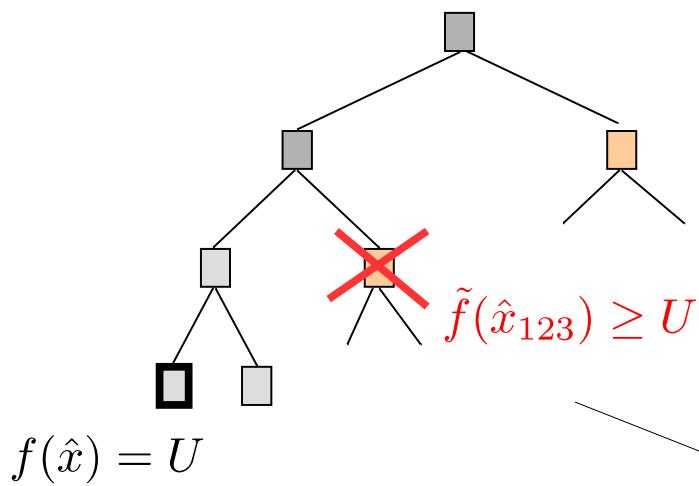
- AO\*-like traversal of the context minimal AND/OR graph
  - All nodes expanded are stored in memory
  - Each node maintains a q-value:  $q(n)$ 
    - best lower bound on optimal cost below n
- Node q-values are revised bottom-up after each expansion
  - OR: minimization:  $q(n) = \min_{n' \in \text{succ}(n)} (w(n, n') + q(n'))$
  - AND: summation:  $q(n) = \sum_{n' \in \text{succ}(n)} q(n')$ 
    - (initially,  $q(n) = h(n)$  – heuristic lower bound on cost below n)
- Current best partial solution tree updated using efficient arc-marking mechanism
  - OR nodes mark best AND successor, following cost revision
  - Any of its tip nodes will be expanded at the next iteration

# Recursive Best-First Search

## 1. Branch-and-Bound (AOBB)

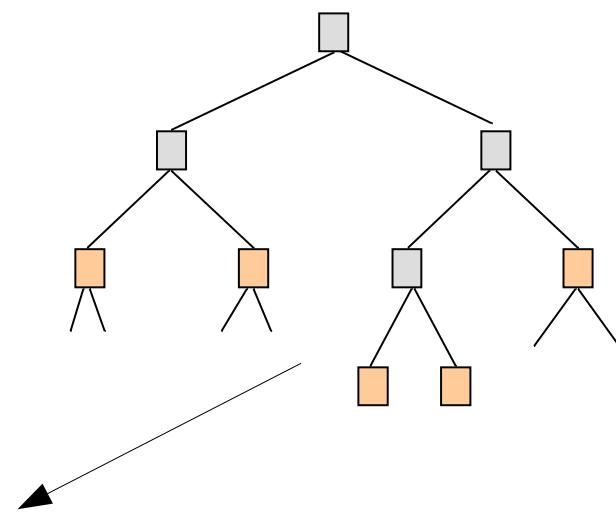
Use heuristic function  $\tilde{f}(\hat{x}_p)$  to prune the depth-first search tree

Linear space



## 2. Best-First Search (AOBF)

Always expand the node with the lowest heuristic value  $\tilde{f}(\hat{x}_p)$   
Needs lots of memory

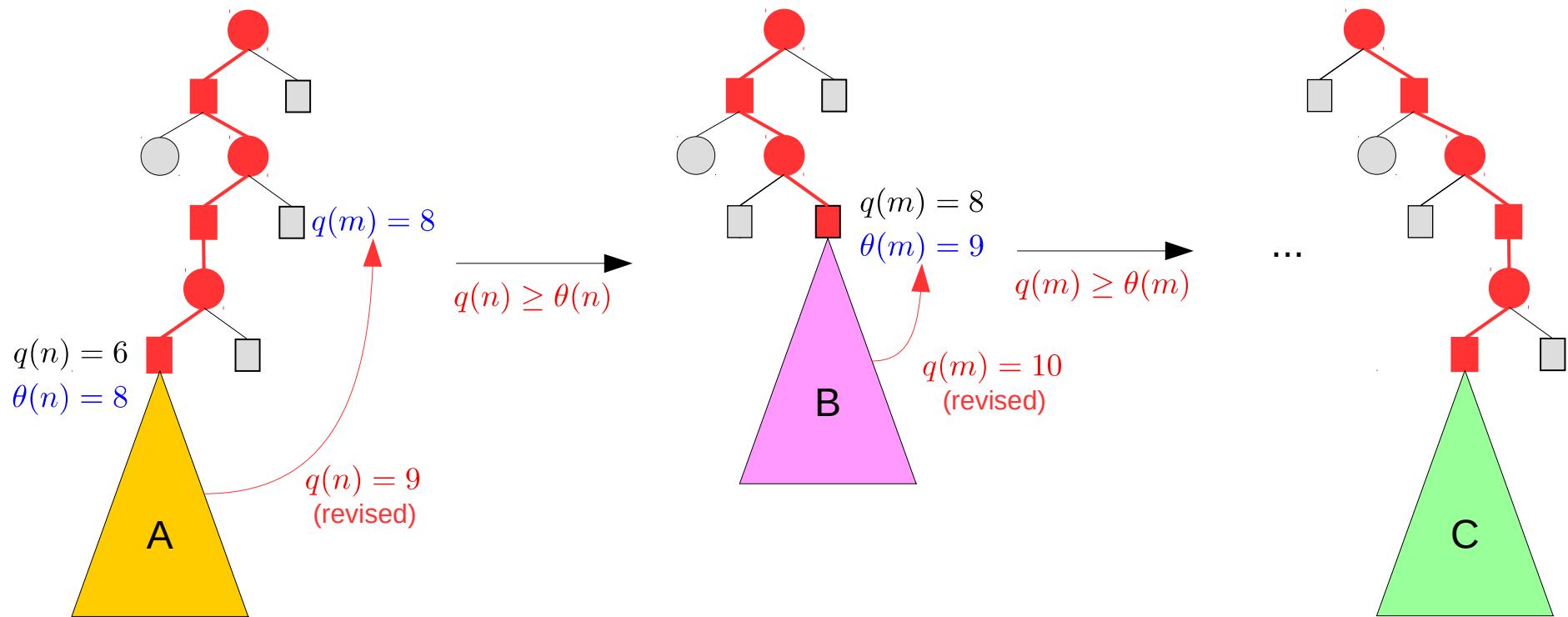


## 3. Recursive Best-First Search

Expand nodes best-first with the lowest heuristic value  $\tilde{f}(\hat{x}_p)$

Linear or Bounded memory

# RBFAOO: Recursive Best-First AND/OR Search

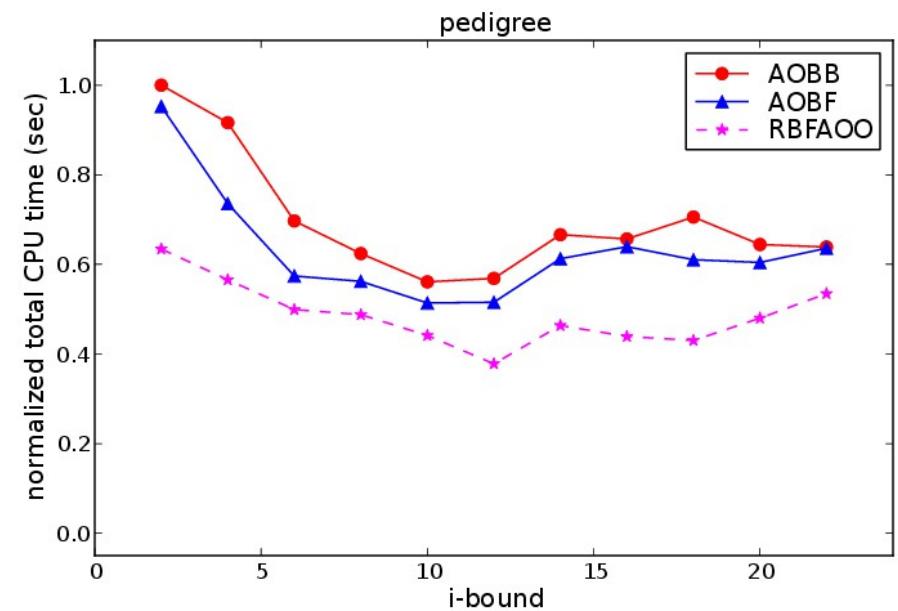
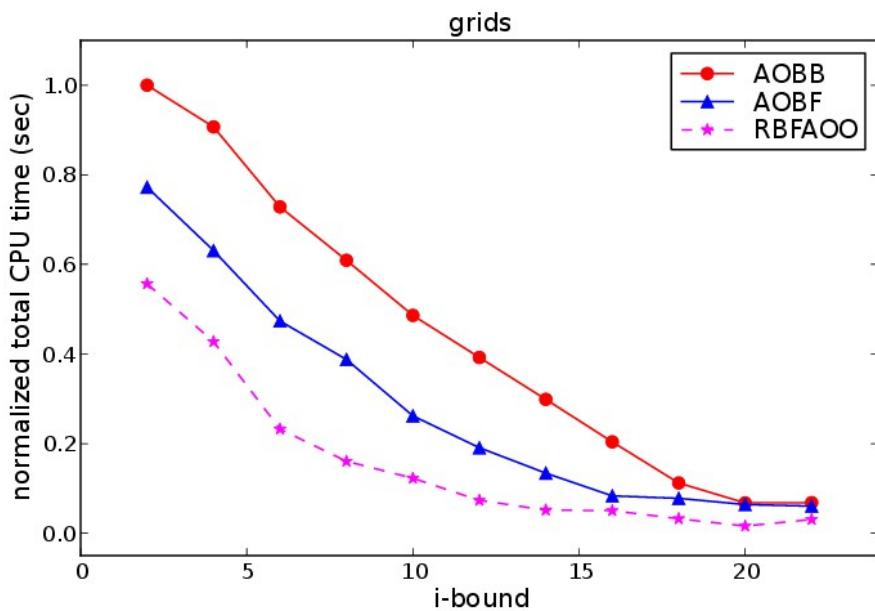


- Threshold  $\theta(n)$  of node  $n$  is the second best cost to  $q(n)$
- Search sub-problem A (below  $n$ ) until revised  $q(n)$  exceeds threshold
- Backtrack to node  $m$  and discard (or cache) nodes just expanded
- Backup revised  $q(n)$  as new threshold for  $m$  and search sub-problem B
- ...

# RBFAOO: Recursive Best-First AND/OR Search

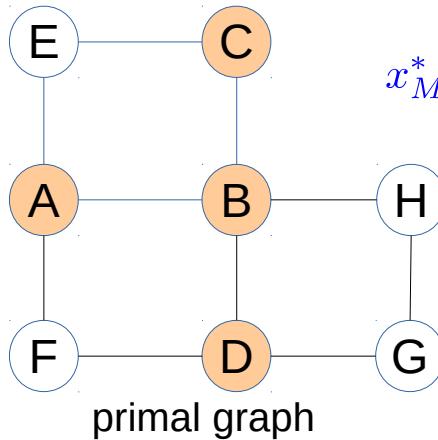
- AO\*-like best-first search is transformed into depth-first search with a threshold
  - Backtrack whenever:  $q(n) \geq \theta(n)$
- Node q-values are updated in the usual manner
  - OR: minimization:  $q(n) = \min_{n' \in \text{succ}(n)} (w(n, n') + q(n'))$
  - AND: summation:  $q(n) = \sum_{n' \in \text{succ}(n)} q(n')$
  - (initially,  $q(n) = h(n)$  – heuristic lower bound on cost below  $n$ )
- Context-based caching is used for efficiency
- Overestimation is used to avoid frequent node re-expansions

# Empirical Evaluation



Exact MAP inference. Grid and Pedigree benchmarks. Time limit 1 hour.

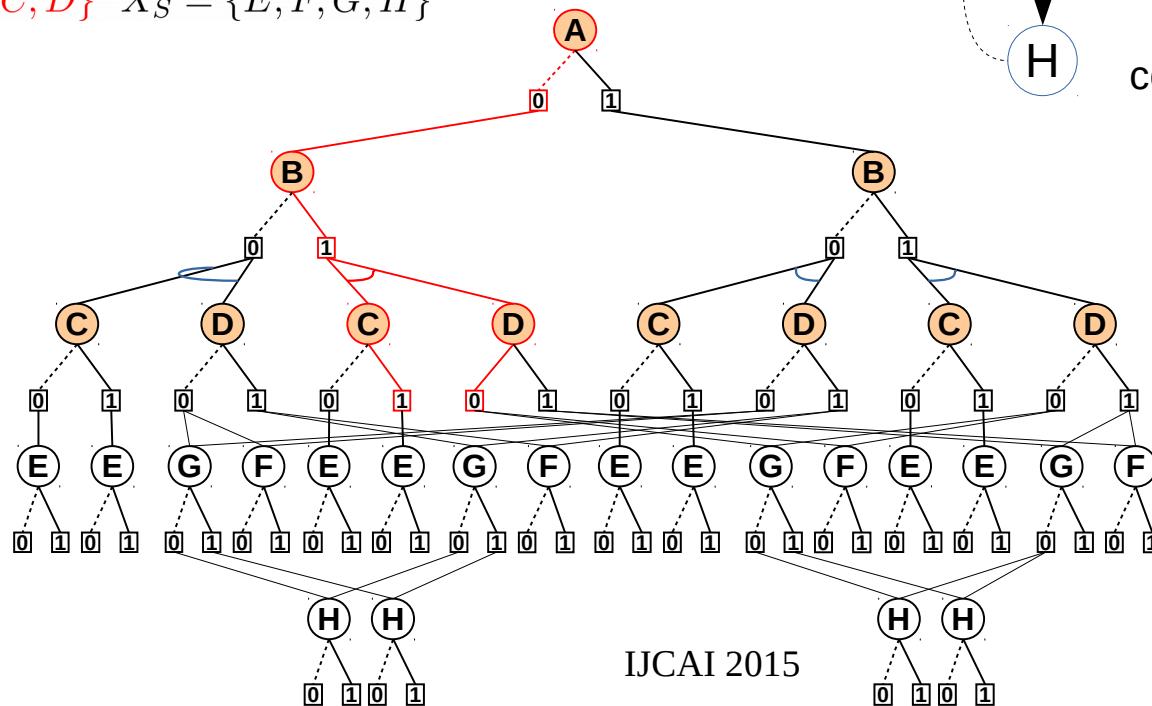
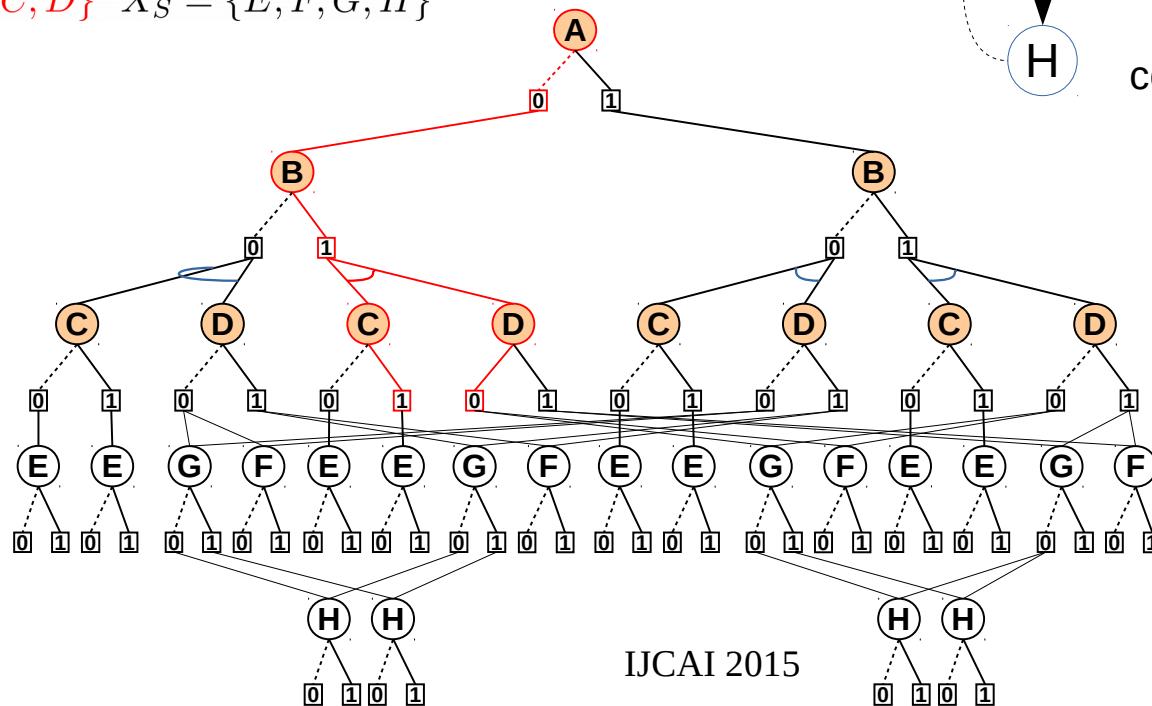
# Marginal MAP



$$x_M^* = \operatorname{argmax}_{x_M} \sum_{x_S} \prod_{\alpha} f_{\alpha}(x)$$

(NP<sup>PP</sup>-complete)

$$X_M = \{A, B, C, D\} \quad X_S = \{E, F, G, H\}$$



- Node types**
  - OR (MAP): max
  - OR (SUM): sum
  - AND: multiplication
- Arc weights**
  - derived from input  $\mathbf{F}$
- Problem decomposition over MAP variables

# AND/OR Search for Marginal MAP

- **New advances**
  - AND/OR Branch and Bound
  - Best-First and recursive best-first AND/OR Search
  - Anytime depth-first and best-first search
    - [Marinescu, Dechter, Ihler 2014,2015]; [Lee, Marinescu, Dechter, Ihler, 2016]
      - Best-performing exact and anytime Marginal MAP solvers
      - Heuristics based on Weighted Mini-Buckets (WMB)
        - WMB-MM: single pass with cost-shifting by moment matching
        - WMB-JG: iterative updates by message passing along the join-graph

# Searching for M Best Solutions

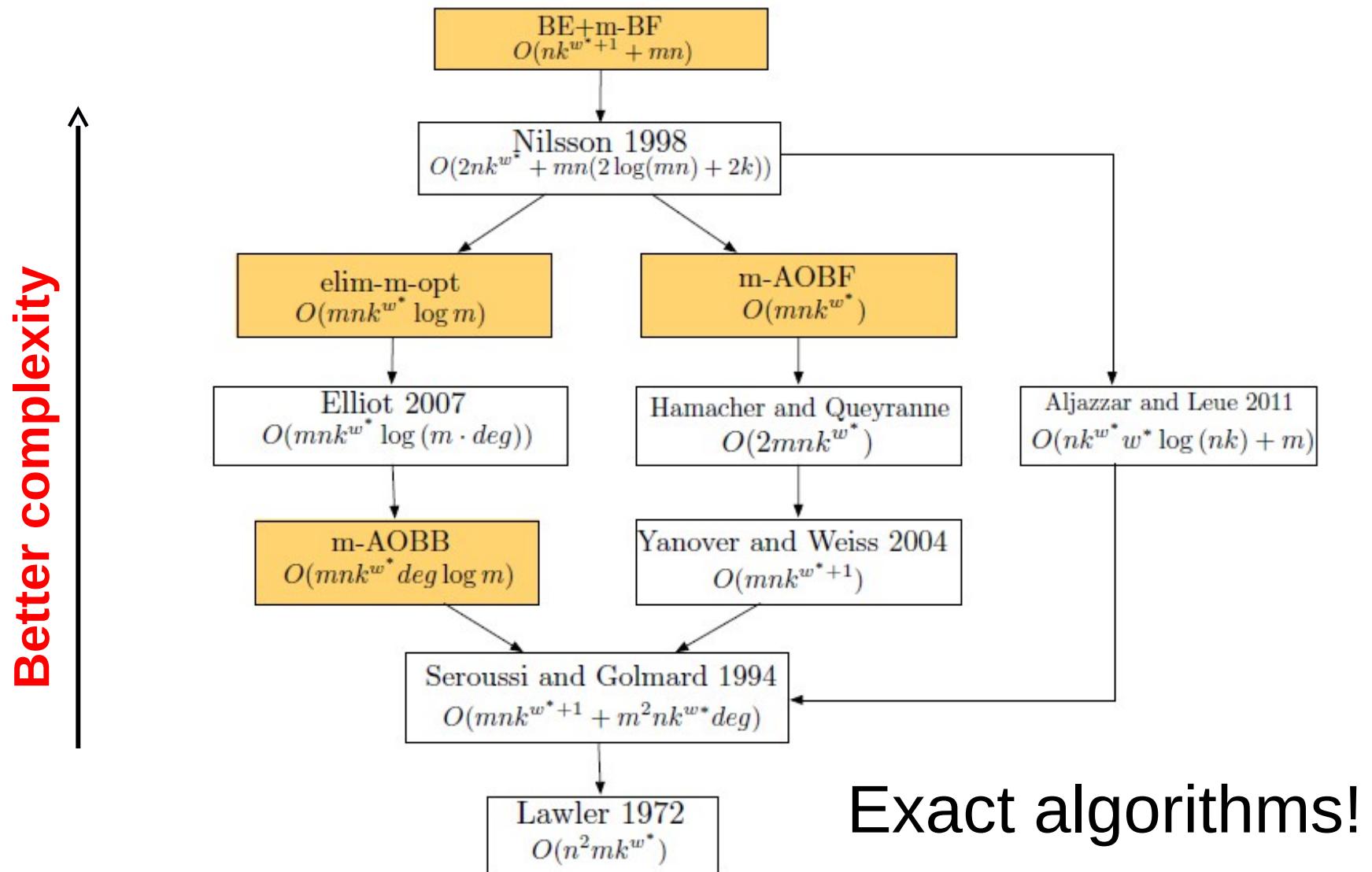
- New inference and search based algorithms for the task of finding the m best solutions
  - Search: m-A\*, m-BB
  - Inference: elim-m-opt, BE+m-BF
- Extended m-A\* and m-BB to AND/OR search spaces for graphical models
  - Algorithms: m-AOBB and m-AOBF
- Competitive and often superior to alternative (approximate) approaches based on LP relaxations

[Fromer and Globerson, 2009], [Batra, 2012]

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[Dechter, Flerova and Marinescu; 2012]

# Searching for M Best Solutions

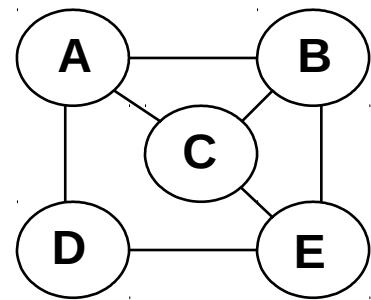


# Hybrid of Variable Elimination and Search

- Tradeoff space and time

# Search Basic Step: Conditioning

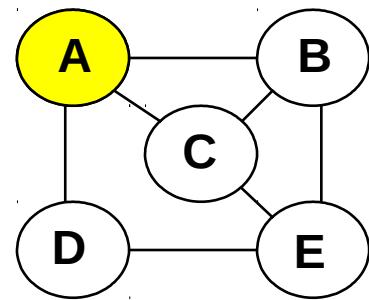
Variable Branching by Conditioning



# Search Basic Step: Conditioning

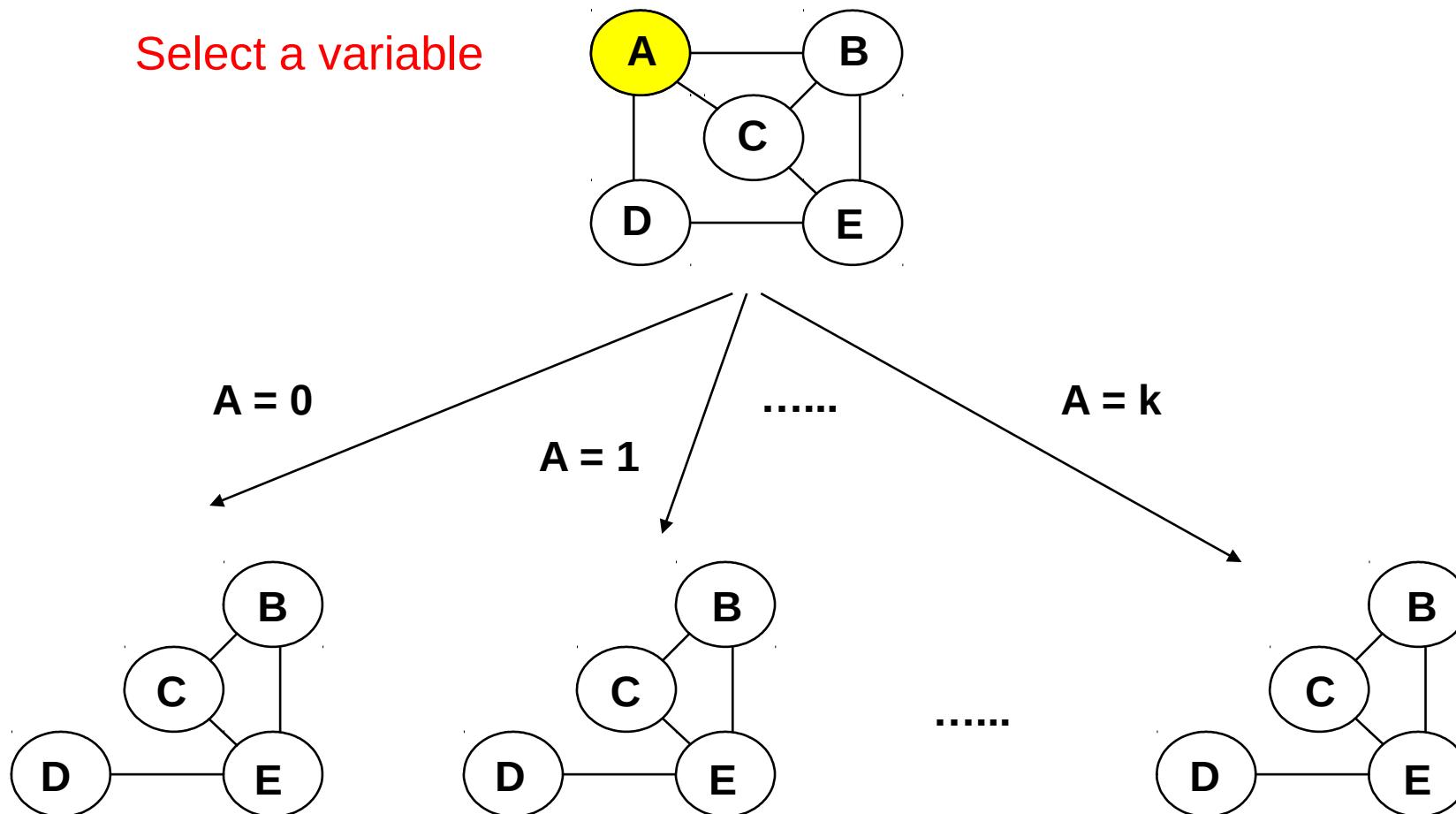
Variable Branching by Conditioning

Select a variable



# Search Basic Step: Conditioning

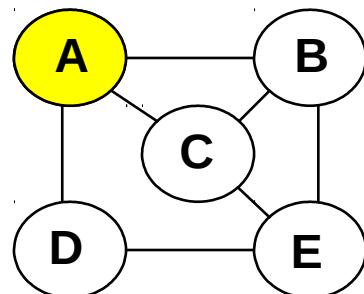
Variable Branching by Conditioning



# Search Basic Step: Conditioning

Variable Branching by Conditioning

Select a variable



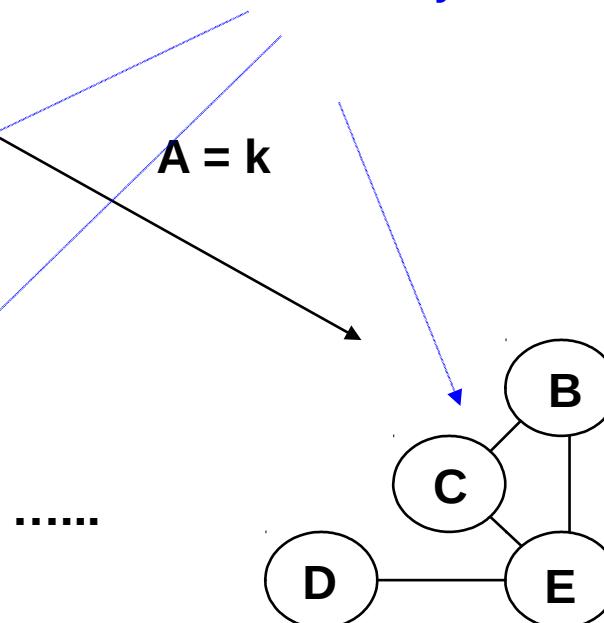
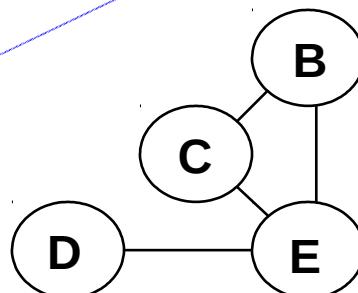
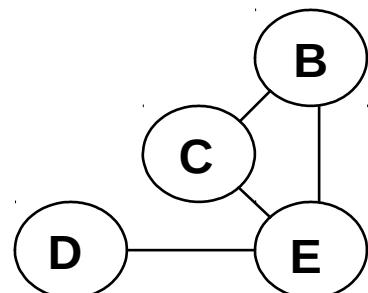
**General principle:**

Condition until tractable  
Solve each sub-problem  
efficiently

$A = 0$

$A = 1$

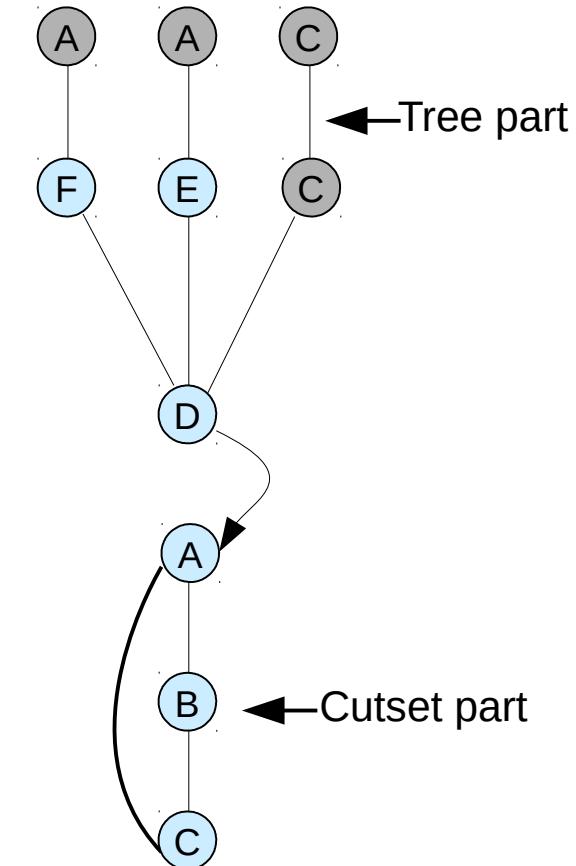
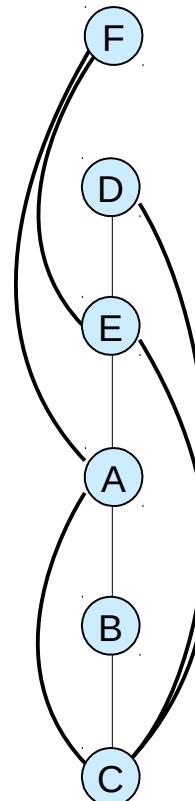
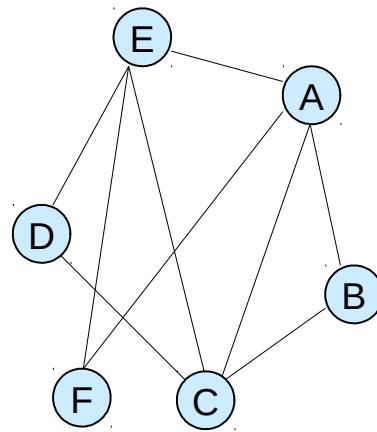
$A = k$



# The Cycle-Cutset Scheme

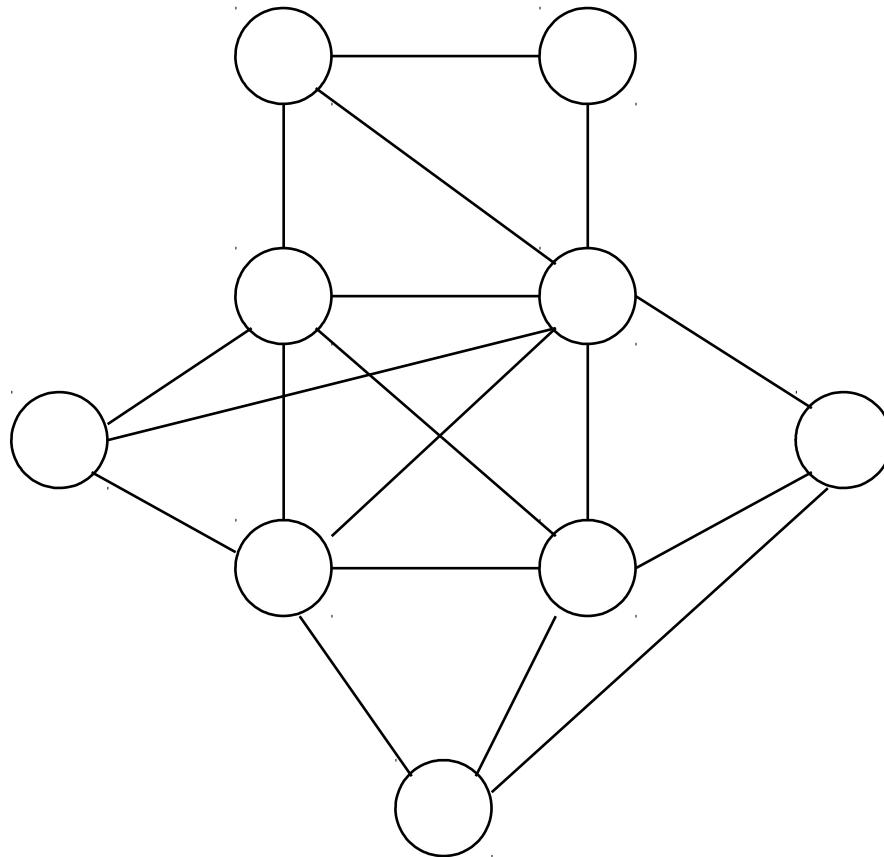
Condition until Treeness

- Cycle-cutset
- i-cutset
- C(i)-size of i-cutset

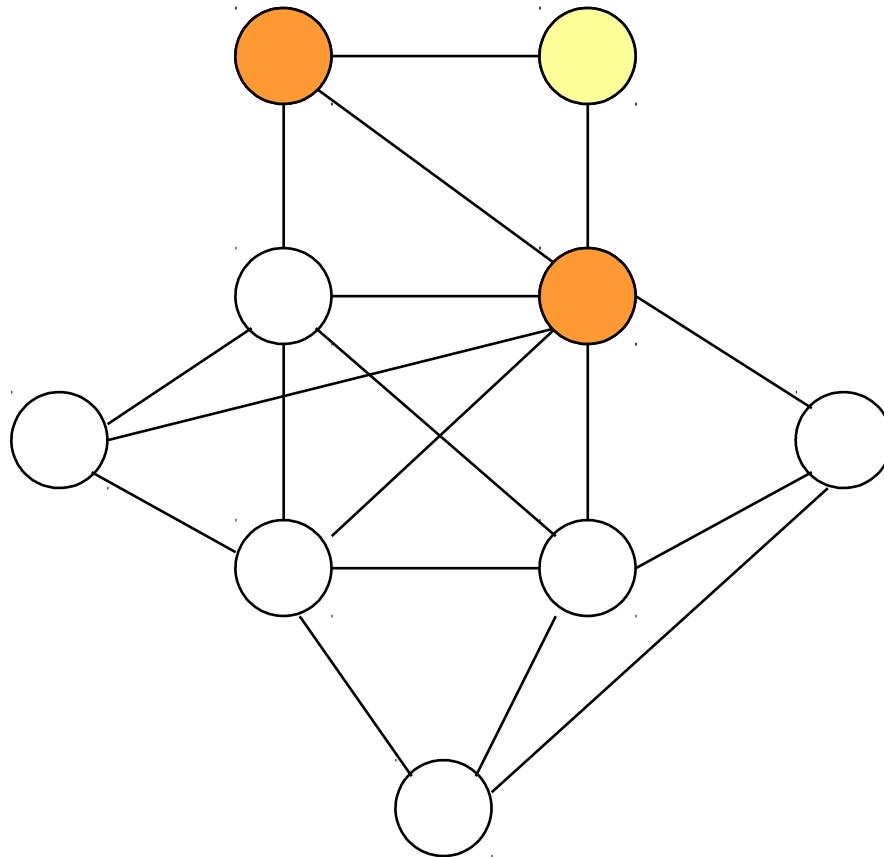


Space:  $\exp(i)$ , Time:  $O(\exp(i+c(i)))$

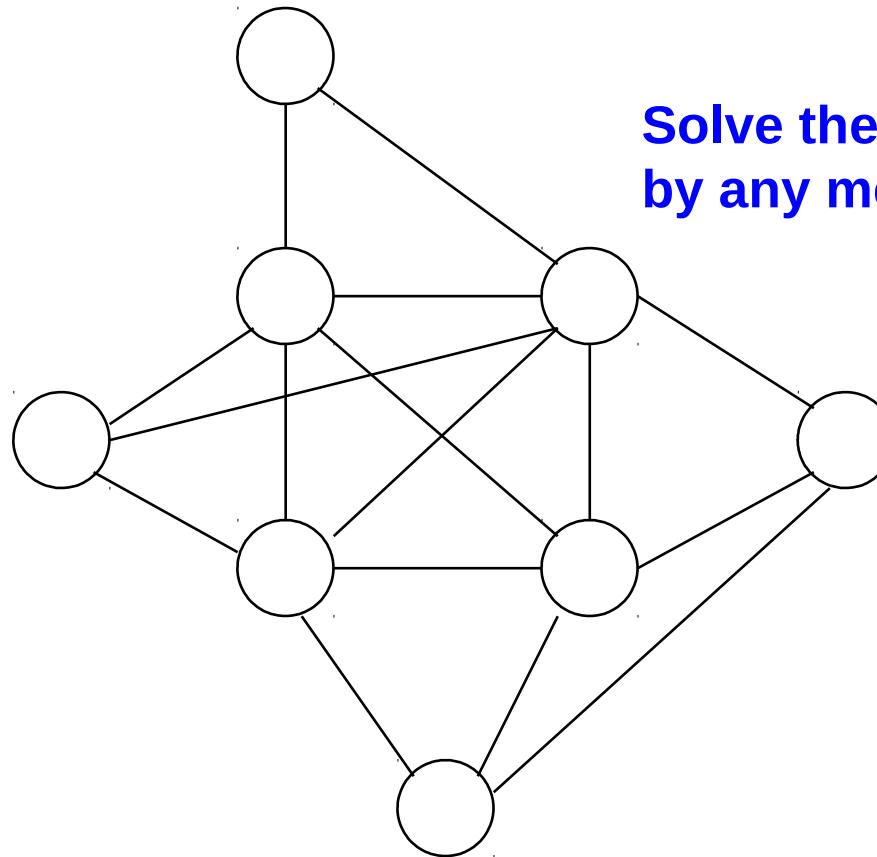
# Eliminate First



# Eliminate First



# Eliminate First

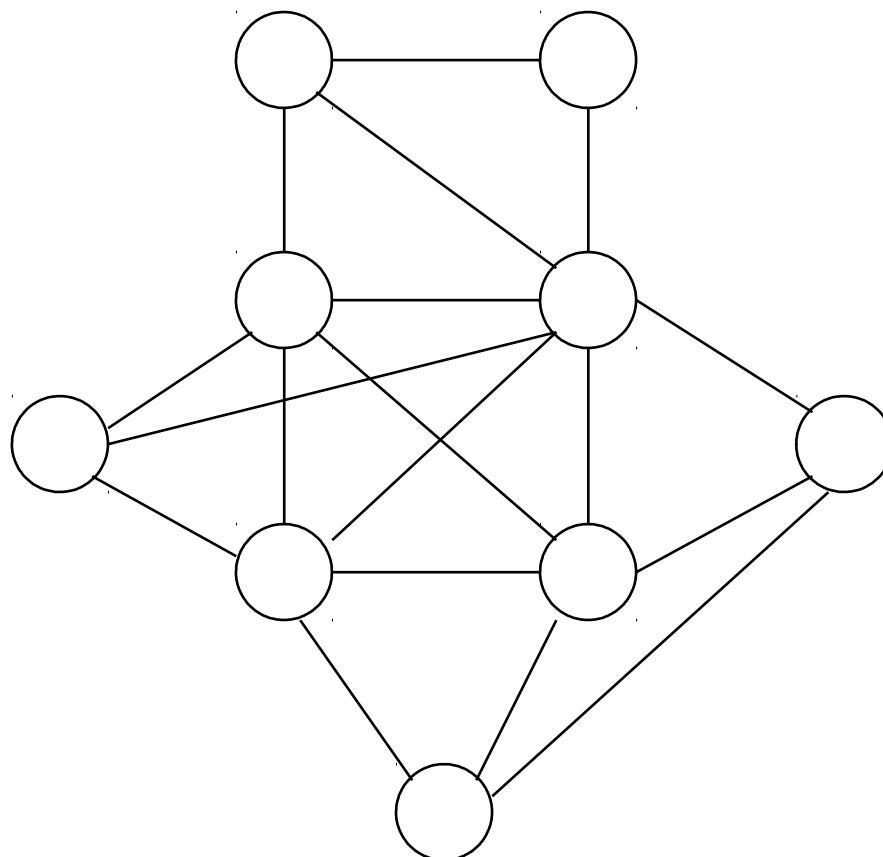


**Solve the rest of the problem  
by any means**

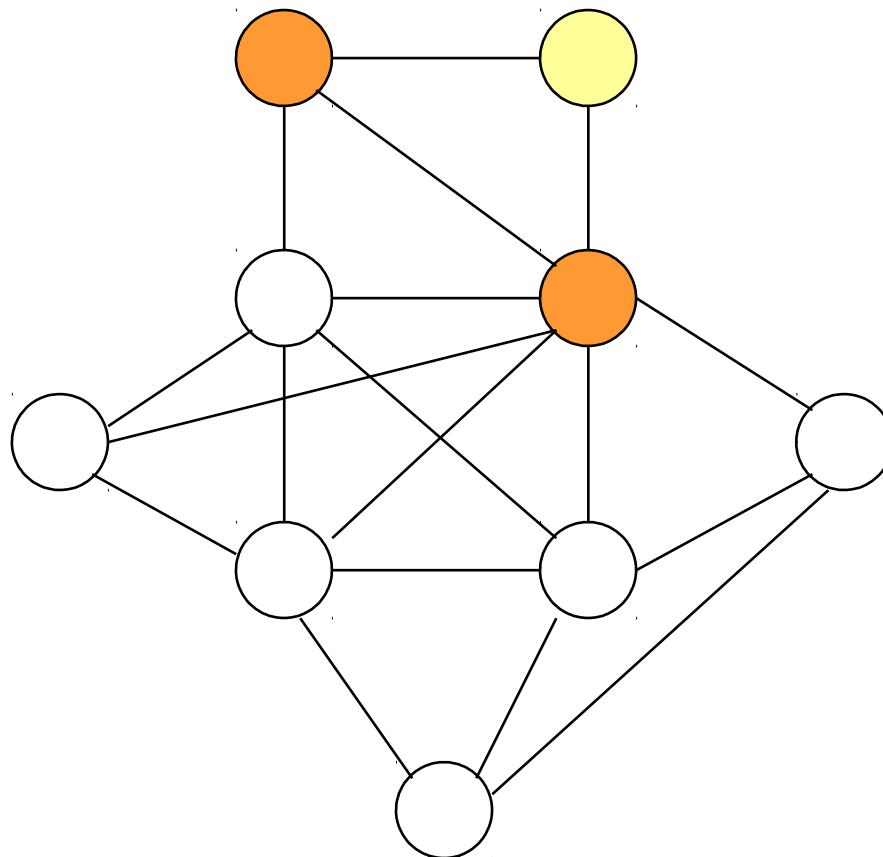
# Hybrid Variants

- **Condition, condition, condition, ...** and then only eliminate (w-cutset, cycle-cutset)
- **Eliminate, eliminate, eliminate, ...** and then only search
- **Interleave** conditioning and elimination steps (elim-cond(i), VE+C)

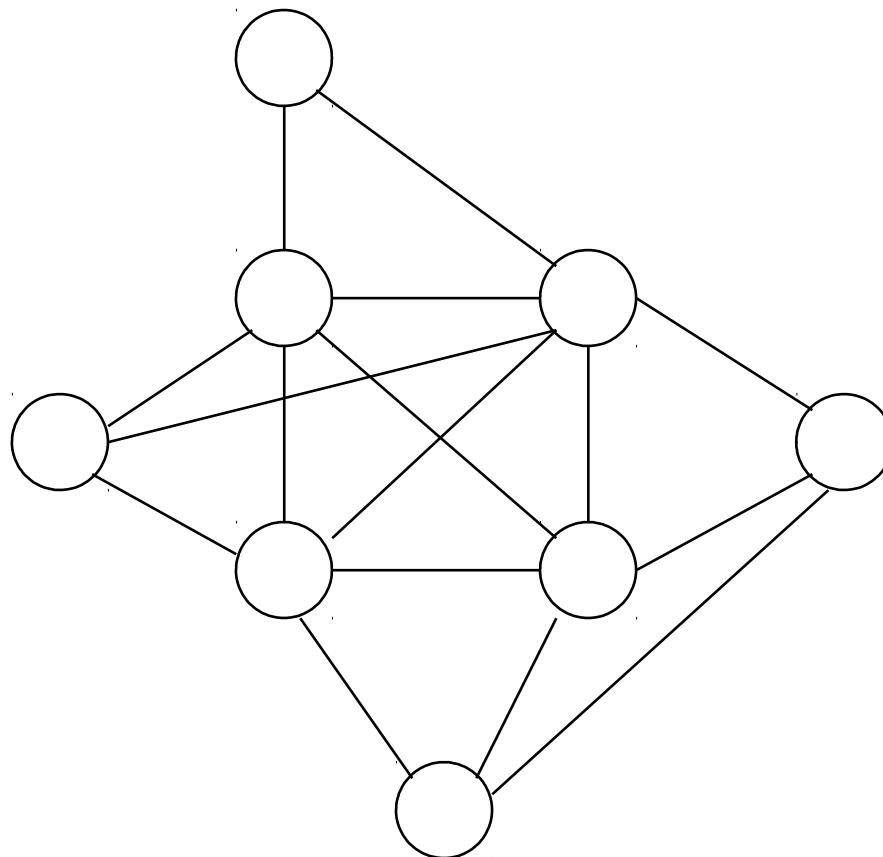
# Interleaving Conditioning and Elimination



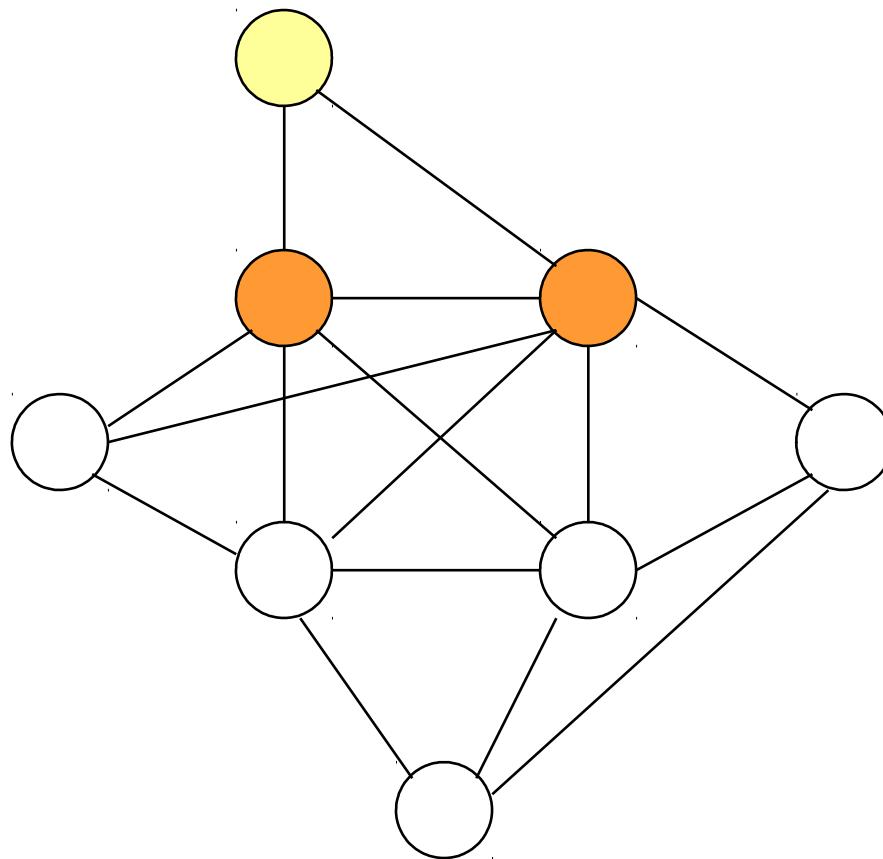
# Interleaving Conditioning and Elimination



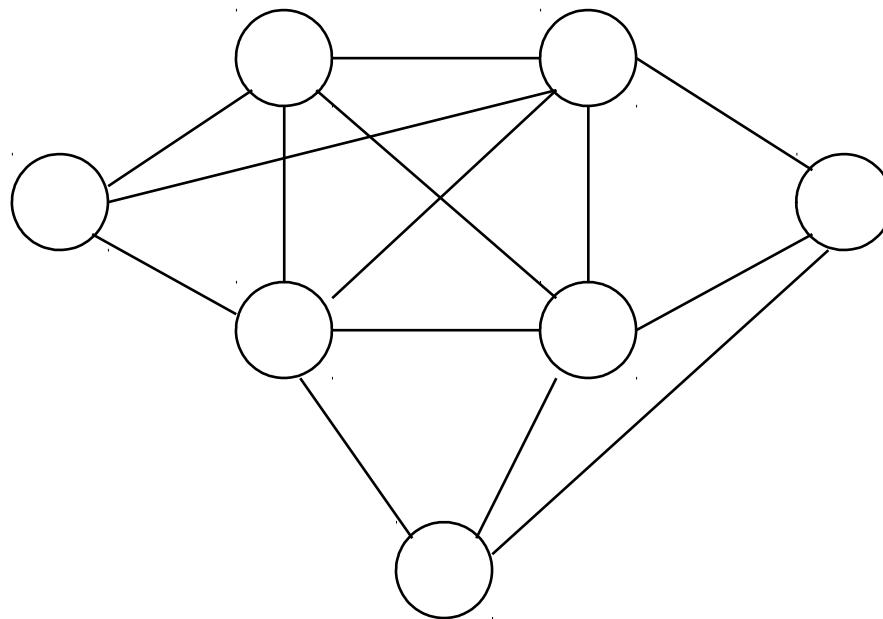
# Interleaving Conditioning and Elimination



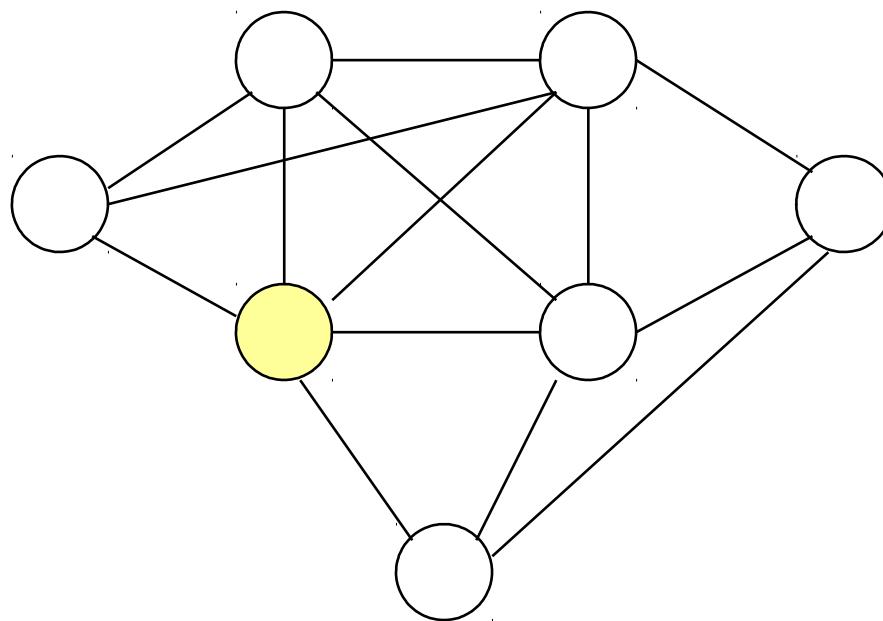
# Interleaving Conditioning and Elimination



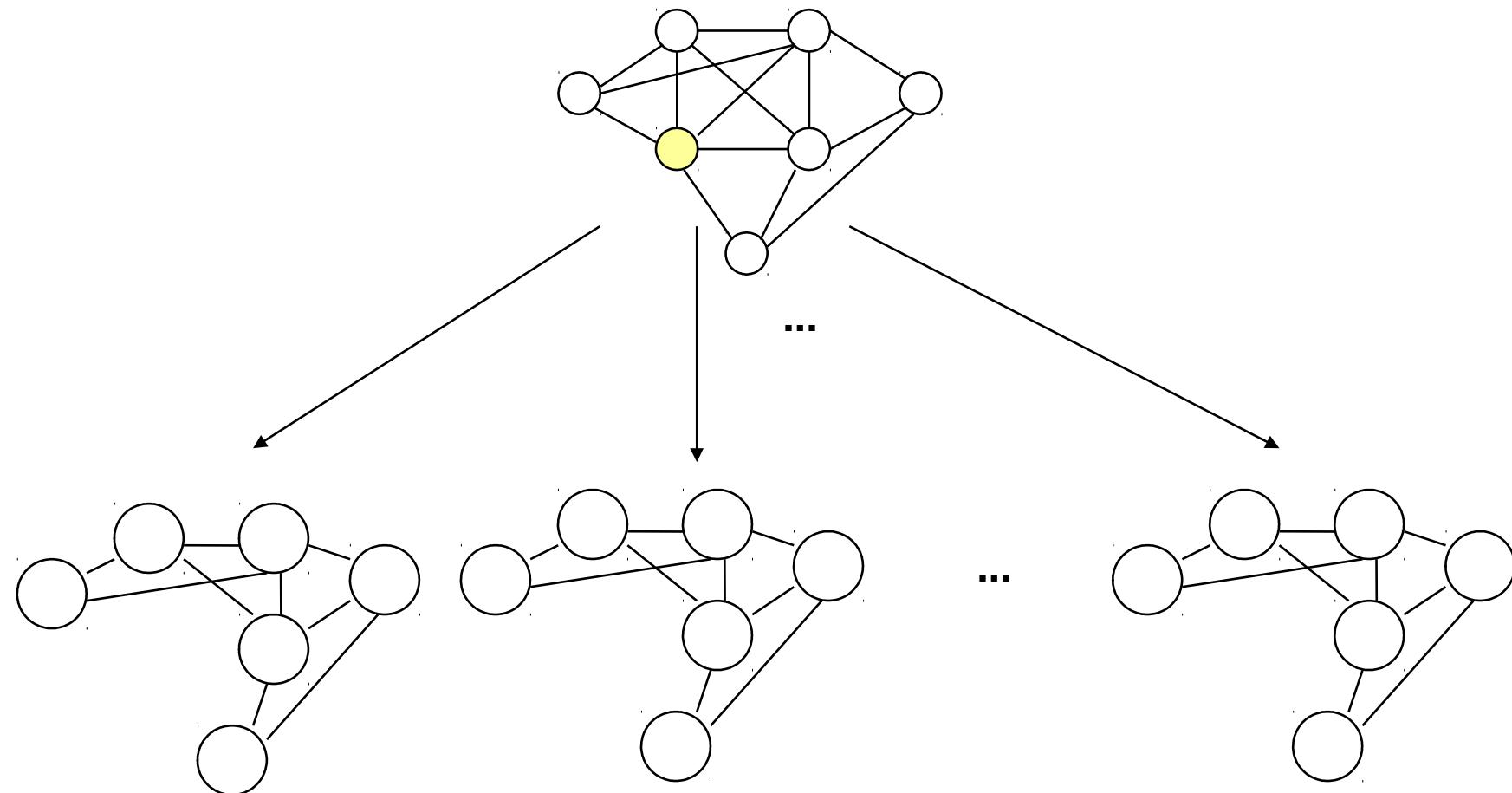
# Interleaving Conditioning and Elimination



# Interleaving Conditioning and Elimination



# Interleaving Conditioning and Elimination



# Outline

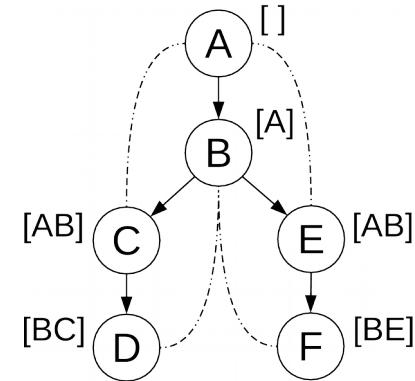
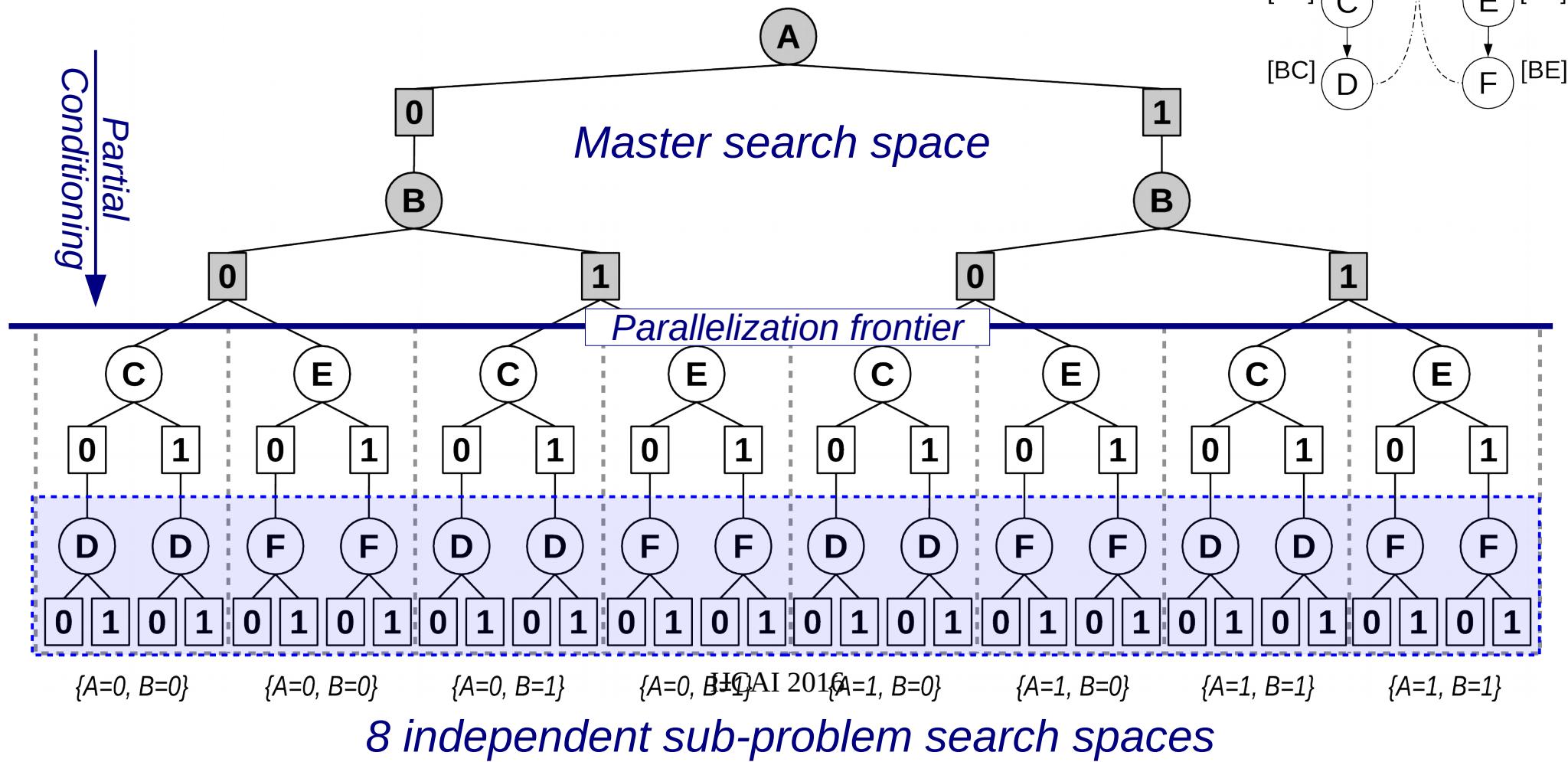
- Introduction
- Inference
- Bounds and heuristics
- AND/OR search
- **Exploiting parallelism**
  - Distributed and parallel search
- Software

# New Advances

- Parallel AOBB, first of its kind
  - Runs on computational grid
  - Extends parallel tree search paradigm
  - Two variants with different parallelization logic  
[Otten and Dechter, 2012]
- Parallel shared-memory RBFAOO
  - Parallelization of the sequential RBFAOO  
[Kishimoto, Marinescu, Botea, 2015]
- Parallel dovetailing for AOBB, RBFAOO, SPRBFAOO
  - Towards large-scale MAP/MMAP inference  
[Kishimoto, Marinescu, Botea, 2016]

# Parallel AOBB Illustrated

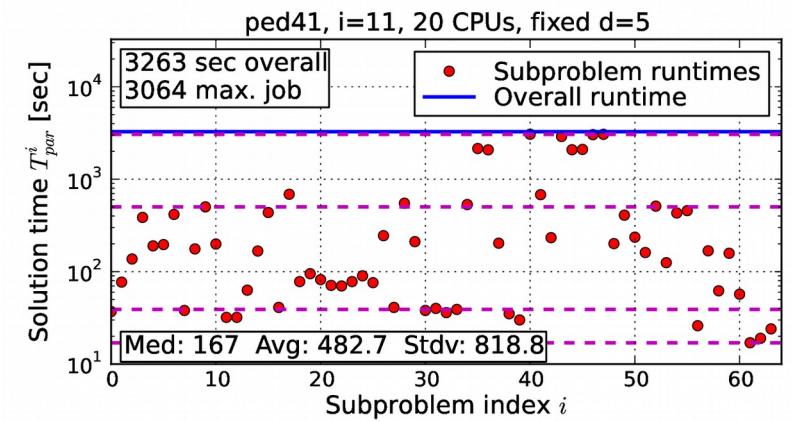
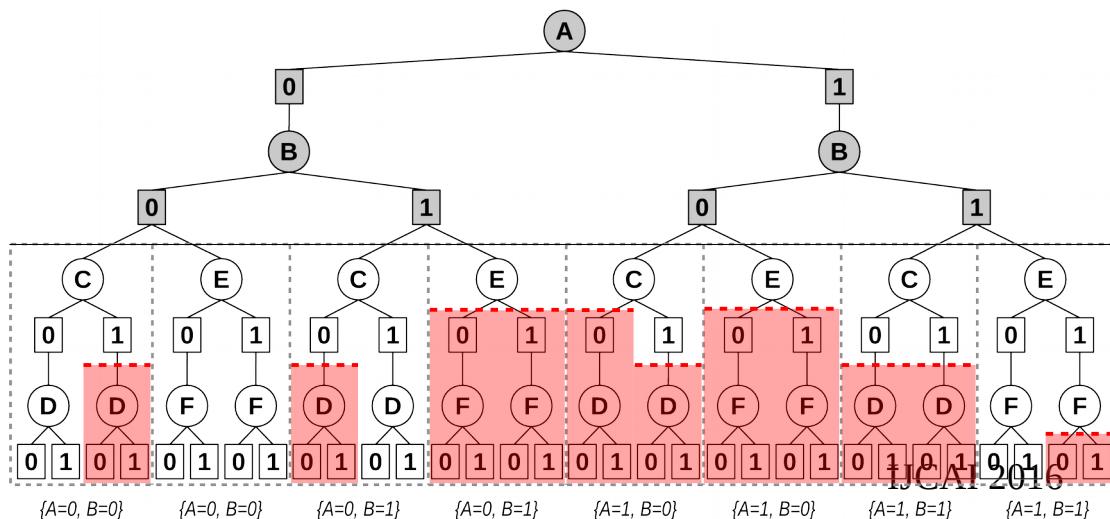
- *Master process* applies partial conditioning to obtain parallel subproblems.



8 independent sub-problem search spaces

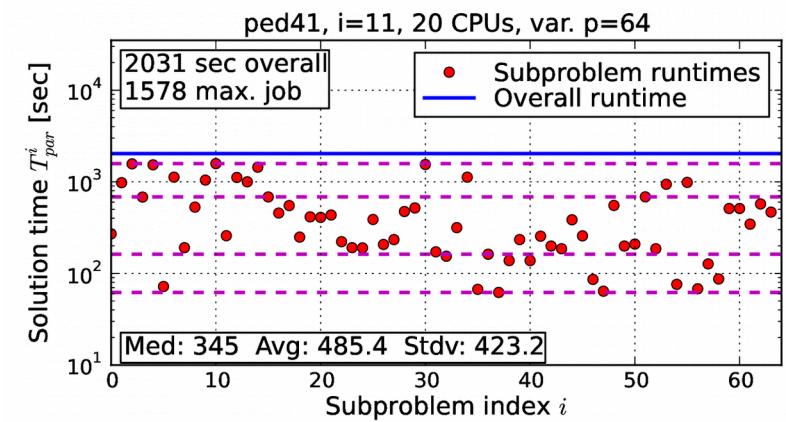
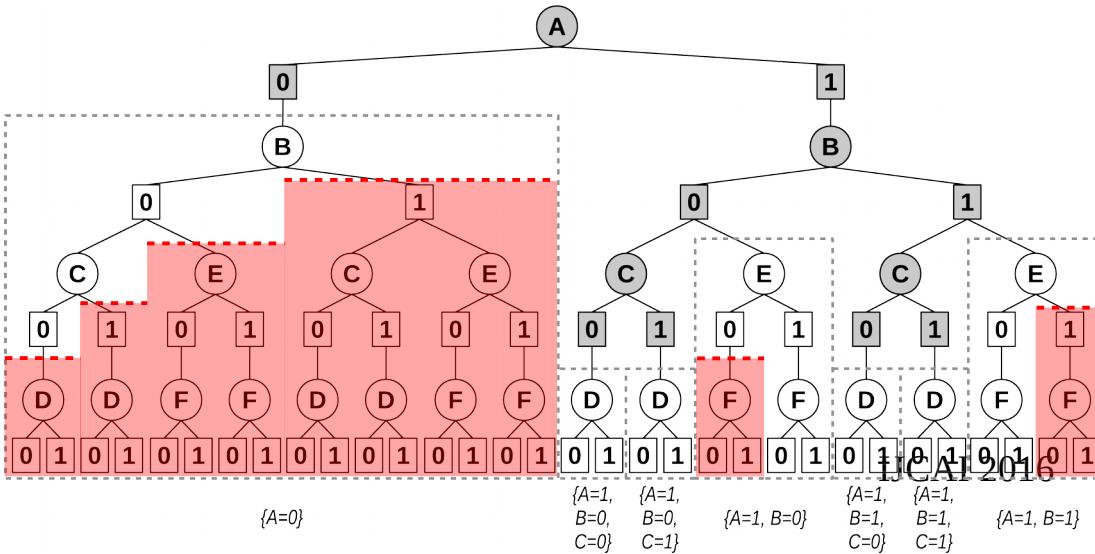
# Fixed-depth Parallel AOBB

- Algorithm receives cutoff depth  $d$  as input:
    - Expand nodes centrally until depth  $d$ .
    - At depth  $d$ , submit to grid job queue.
  - Explored sub-problem search spaces potentially very unbalanced.



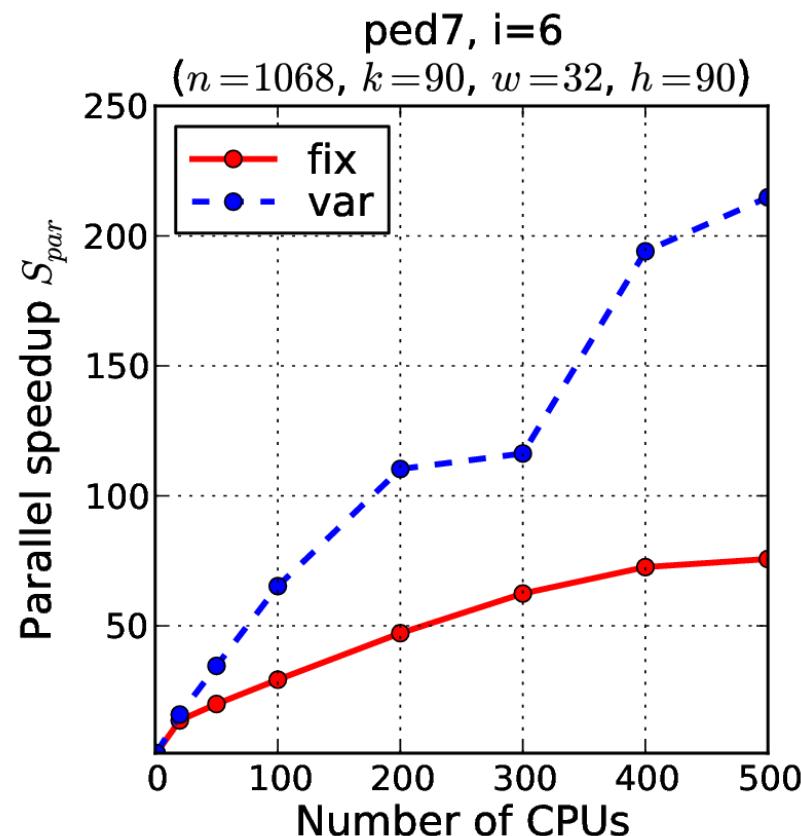
# Variable-depth Parallel AOBB

- Given sub-problem count  $p$  and estimator  $N$  :
  - Iteratively deepen frontier until size  $p$  reached:
    - Pick sub-problem  $n$  with largest estimate  $N(n)$  and split.
  - Submit sub-problems into job queue by descending complexity estimates.
- Hope to achieve better sub-problem balance.



# Parallel Scaling Summary

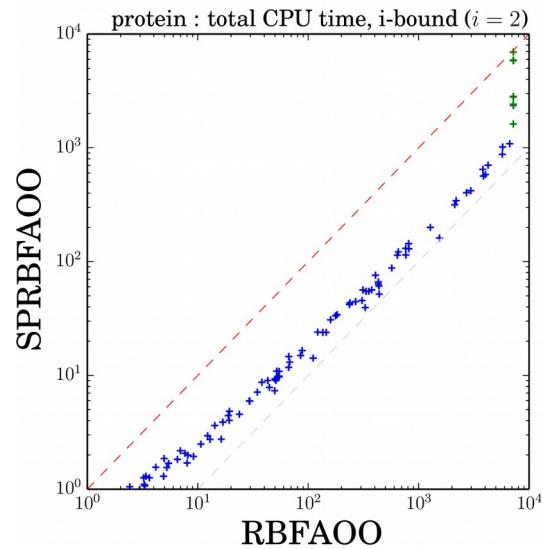
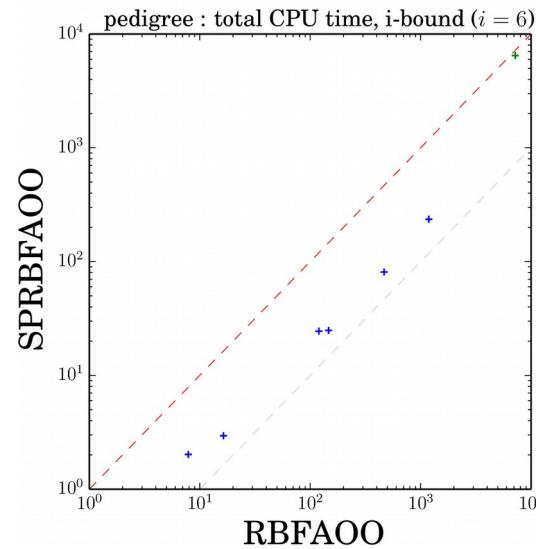
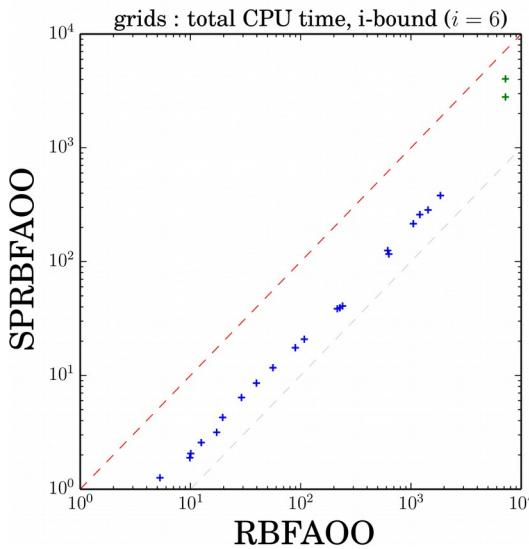
- Plot speedup against CPU count.
  - Trade off load balancing vs. overhead:
    - $\#subproblems \approx 10 \times \#CPUs$



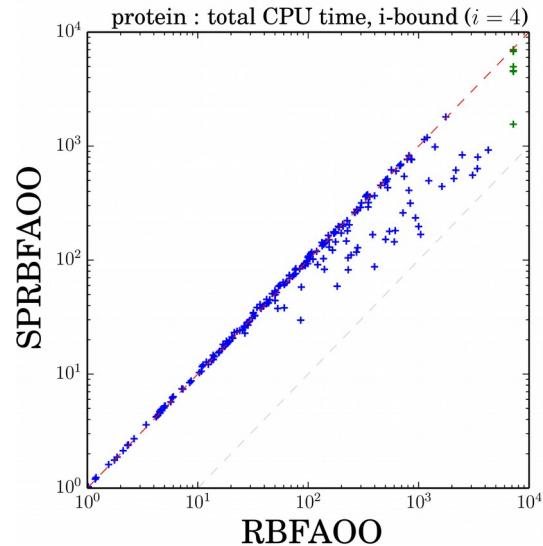
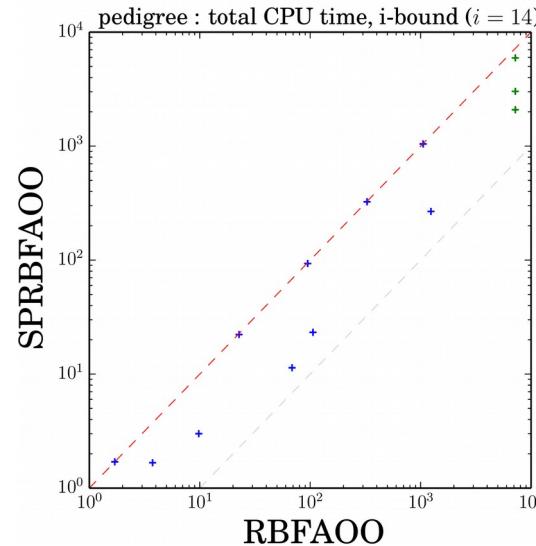
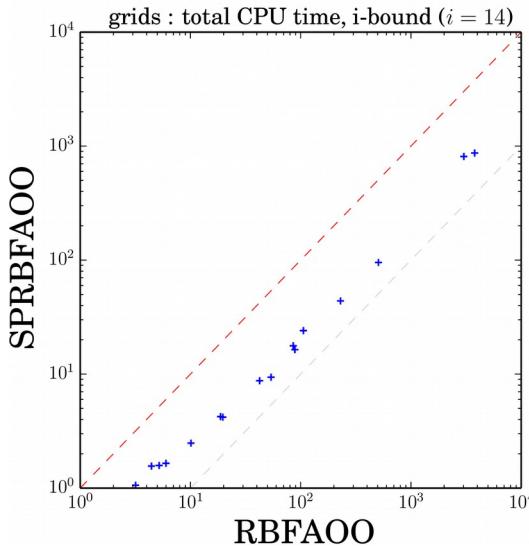
# SPRBFAOO: Parallel Shared-Memory RBFAOO

- All threads start from the root with the same search strategy and with one shared cache table
- The *virtual q-value*  $vq(n)$  for node  $n$  is used to control parallel search
  - Initially,  $vq(n)$  is set to  $q(n)$
  - When a thread examines  $n$ ,  $vq(n)$  is incremented by a small value  $\zeta$
  - When all threads finish examining  $n$ ,  $vq(n)$  is set to  $q(n)$
- An effective load balancing is obtained without any sophisticated schemes, while promising portions of the search space are examined
- The algorithm guarantees solution optimality

# Empirical Evaluation

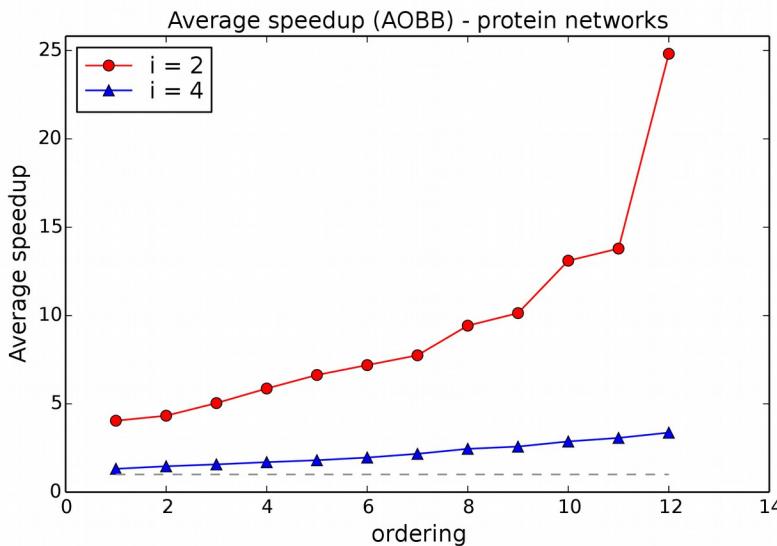


(up to 7-fold speedup with 12 threads)

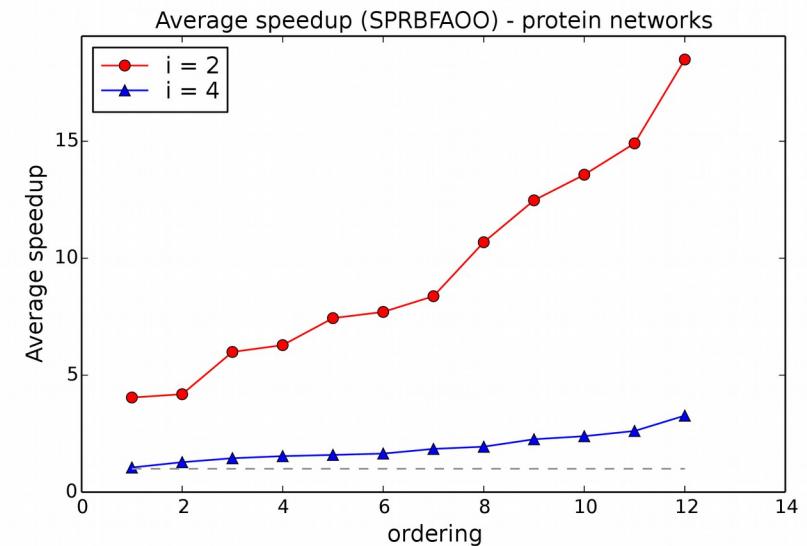


# Parallel Dovetailing

- Simple distributed scheme
  - Launch in parallel  $m$  instances of the inference algorithm (AOBB, AOBF, RBFAOO, SPRBFAOO) each one solving the same problem instance but with a different input parameter configuration
    - e.g., parameter configuration = pseudo tree (ordering)



12 cores



IJCAI 2016

12 nodes x 12 cores = 144 cores

# Outline

- **Introduction**
- **Inference**
- **Bounds and heuristics**
- **AND/OR search**
- **Exploiting parallelism**
- **Software**
  - UAI probabilistic inference competitions

# Software

- **aolib**
  - <http://graphmod.ics.uci.edu/group/Software>  
(standalone AOBB, AOBF solvers)
- **daoopt**
  - <https://github.com/lotten/daoopt>  
(distributed and standalone AOBB solver)
- **merlin**
  - <https://developer.ibm.com/open/merlin>  
(standalone WMB, AOBB, AOBF, RBFAOO solvers)  
open source, BSD license

# UAI Probabilistic Inference Competitions

- 2006



(aolib)

- 2008



(aolib)

- 2011



(daoopt)

- 2014



(daoopt)



(daoopt)



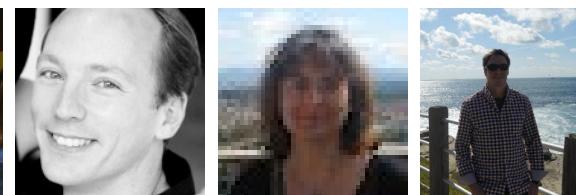
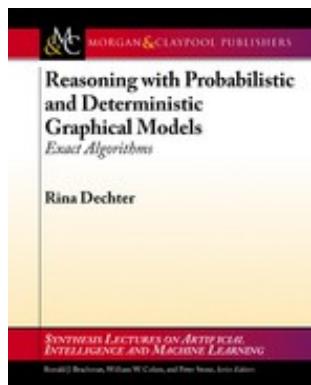
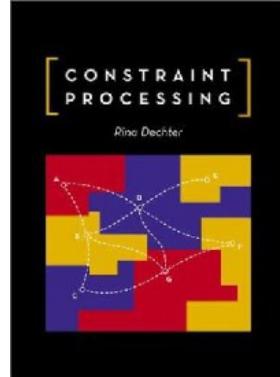
(merlin)

**MPE/MAP**

**MMAP**

# Summary

- Only a few principles
  - Inference and search should be combined
    - Time-space tradeoff
  - AND/OR search should be used
  - Caching in search should be used
  - Parallel search should be used if distributed and/or shared-memory environments are available



For publication see:  
<http://www.ics.uci.edu/~dechter/publications.html>

# Thank You

Kalev Kask  
Irina Rish  
Bozhena Bidyuk  
Robert Mateescu  
Radu Marinescu  
Vibhav Gogate  
Emma Rollon  
Lars Otten  
Natalia Flerova

