ICS 162 – Spring 2001 – Final Exam

Name:

Student ID:

1:

2:

3:

4:

5:

Total:

1. (7 points)

Draw a nondeterministic finite automaton over the alphabet $\Sigma = \{0, 1\}$ that recognizes the language $(11 + 110 + 011)^*$.

2. (9 points)

For any string s of zeros and ones, let $\omega(s)$ denote the number of ones after the last zero of s. E.g., $\omega(\epsilon) = 0$, $\omega(010111) = 3$. Let $\alpha(s)$ denote the number of occurrences of 01 in s, so e.g. $\alpha(\epsilon) = 0$, $\alpha(010111) = 2$. Let L be the language $\{s \mid \alpha(s) \leq \omega(s)\}$.

(a) List two strings in L, and two strings in the complement of L.
Clearly mark which strings are in L and which are in the complement of L.

(b) Show that L can be pumped: there exists a number p such that, whenever s is a string in L with $|s| \geq p$, s can be partitioned $s = xyz$ with y nonempty, $|xy| \leq p$ and all strings xy^iz remaining in L. What is your choice of p, and how do you choose xyz for a given s?

(c) Does part (b) imply that L is a regular language? Why or why not?

3. (12 points)

True or false:

(a) Any language that can be recognized by a nondeterministic finite automaton can be recognized by a deterministic finite automaton.

(b) Any language that can be recognized by an nondeterministic pushdown automaton can be recognized by a deterministic pushdown automaton.

(c) Any language that can be recognized by a nondeterministic Turing machine can be recognized by a deterministic Turing machine.

(d) Any language can be recognized by a nondeterministic Turing machine.

(e) If L_1 is NP-complete, and there exists a polynomial-time function f such that $f(x)$ is in L_1 if and only if x is in L_2, then L_2 must be in NP.

(e) If L_1 is NP-complete, and there exists a polynomial-time function f such that $f(x)$ is in L_1 if and only if x is in L_2, then L_2 must be NP-hard.

(f) If L_1 is NP-complete, and there exists a polynomial-time function f such that x is in L_1 if and only if $f(x)$ is in L_2, then L_2 must be in NP.

(g) If L_1 is NP-complete, and there exists a polynomial-time function f such that x is in L_1 if and only if $f(x)$ is in L_2, then L_2 must be NP-hard.

(h) If L is a regular language, the complement of L must also be regular.

(i) If L is a context-free language, the complement of L must also be context free.

(j) If L is a decidable language, the complement of L must also be decidable.

(k) If L is an NP-complete language, the complement of L must also be NP-complete

4. (7 points)

Write down a Chomsky normal form grammar for the language $\{0^i 1^i \mid i \geq 1\}$.

Recall that the requirements for Chomsky normal form are:

- Only the start symbol can have a rule $S \to \epsilon$
- The start symbol is not on the right hand side of any rule
- The right hand side of any rule consists either of a single terminal or of two nonterminals

5. (5 points)

In this course we discussed three important complexity classes P, NP, and PSPACE. Why didn't we discuss a fourth class NPSPACE?