ICS 162 – Fall 2003 – Midterm

Name: **Solution Key**

Student ID:

1: **30**

2: **20**

3: **15**

4: **15**

5: **20**

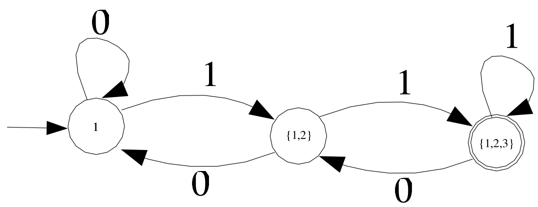
6: **20**

Total: **100** plus **20** extra credit

1. (30 points)

a. (20 points) Find a deterministic finite automaton that recognizes the same languages as the one recognized by the following nondeterministic finite automaton.

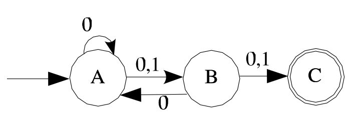
NFA omitted; solution DFA:



b. (10 points) If a nondeterministic finite automaton has three states, what is the largest number of states that might be needed when it is converted to a deterministic finite automaton?

$2^n = 2^3 = 8$

One such NFA, recognizing $(0 + 10)^*(0 + 1)(0 + 1)$:



2. (20 points)

Find a regular expression describing the set of strings of zeros and ones in which every zero is followed by two consecutive ones. For instance, 10111011 is in this language, but 0011 is not.

Two possible solutions:

$(1^*011)^*1^*$

or,

$(1 + 011)^*$

3. (15 points)

For each of the following three grammars, state whether or not the grammar is in Chomsky Normal Form. If it is not, explain why not.

$$\begin{array}{rcl} S & \rightarrow & \epsilon \mid XY \\ X & \rightarrow & 0 \mid YS \\ Y & \rightarrow & 1 \mid XX \end{array}$$

No; the start rule S may not appear on the right side of any rules. It appears on the right side of the rule for X.

$$\begin{array}{rcl} S & \rightarrow & AB \mid AA \mid BB \\ A & \rightarrow & 0 \mid BA \\ B & \rightarrow & 1 \end{array}$$

Yes.

$$\begin{array}{rcl} S & \rightarrow & 0P \mid 1Q \mid PQ \\ P & \rightarrow & 1 \mid QP \\ Q & \rightarrow & 0 \mid PQ \end{array}$$

No; the right hand side of every rule must either be a single terminal, or two variables. On the contrary, the rules for S have a combination of a terminal and a variable.

4. (15 points)

Show that the following grammar is ambiguous:

$$S \to 0\,S\,1 \quad | \quad 0\,1\,S \quad | \quad \epsilon$$

The string 01 may be generated two ways:

$S \to 0S1 \to 0(\epsilon)1 = 01$
or
$S \to 01S \to 01(\epsilon) = 01$

Since the grammar can generate a string in two distinct ways, it is ambiguous.

5. (20 points)

Let $L = \{ww \,|\, w \in 0^*10^*\}$. That is, L consists of strings of zeros and ones in which the first and second halves of the string are equal to each other, and each half contains a single one and any number of zeros. Use the pumping lemma to show that L is not regular.

Assume that L is regular. Then the pumping lemma guarantees that any string $s \in L$, at least as long as the pumping length p, may be split into three parts, $s = xyz$, such that:

1. $\forall i \geq 0, xy^iz \in L$
2. $|y| > 0$
3. $|xy| \leq p$

We will examine all possible choices of the substring y. Suppose that y contains a 1 symbol. Any given string $s = xyz \in L$ must contain two 1 symbols; hence, any xy^iz for $i > 1$ will contain at least three 1 symbols, and cannot possibly be in the language L. This violates clause 1 of the pumping lemma, so y cannot contain any 1 symbols.

Now suppose y is composed entirely of 0 symbols. Note that any string in L may have 0s in three places: before the first 1, between the two 1s, and after the second 1. If y is chosen to be before the first 1, then it is possible to pump y until there are more 0s before the first 1 than between the 1s. Such a string cannot be split into two equal parts, so clause 1 does not hold for all values of i.

A similar situation arises if y is chosen to be after the second 1. Sufficient repititions of y will result in a string that has too many trailing 0s to be split into two even parts. By similar reasoning, if y is chosen to be located between the two 1s, then it will always be possible to pump y until the resulting string has too many 0s in the middle to be split into two equal parts.

So any choice of y that includes a 1 symbol leads to a contradiction of the pumping lemma. Setting y to be empty violates clause 2. The only other alternative is to define y to be one or more 0 symbols. But in this case pumping y will eventually result in a string that is not in the language, no matter which cluster of 0s y contributes to. So every possible way of forming y leads to a contradiction of the pumping lemma. Hence, our initial assumption that L is regular must be false. So we may conclude that L is not regular.

6. (Extra credit: 20 points)

Let $L = \{ww \mid w \in 0^*10^*\}$ be the same language as described in problem 5. Find a context free grammar for L. You do not need to use Chomsky Normal Form for your grammar.

Hint: the expression ww shows how to split strings in L into two equal parts.Instead, split the strings into two parts that might not be equal.

$S \to XX$
$X \to 1|0X0$

You may use this page (or the back of the other pages) as scratch paper.