Upright-Quad Drawing of st-Planar Learning Spaces

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Design Principle for Specialized Graph Drawing Algorithms

If you're designing algorithms for graphs with some **special structure**

Then the output should make that structure **visually apparent**

(else why not just use a general-purpose drawing algorithm?)

Examples:

forests — do not separate plane
planar graphs — no edges cross
symmetric graphs — symmetry display
distance-hereditary graphs — delta-confluent [E-Goodrich-Meng GD05]

Learning Spaces

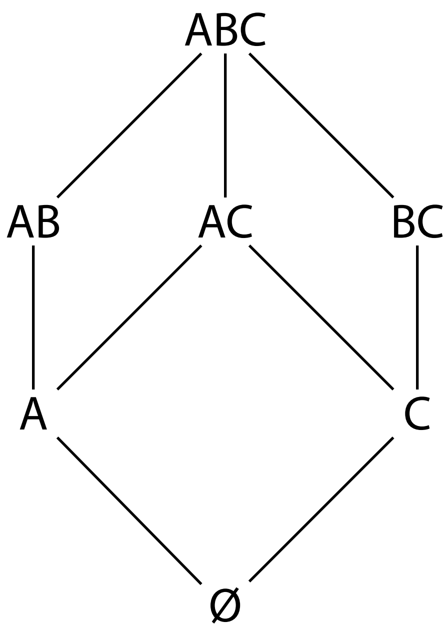
Family of sets of concepts
representing possible states of knowledge of students

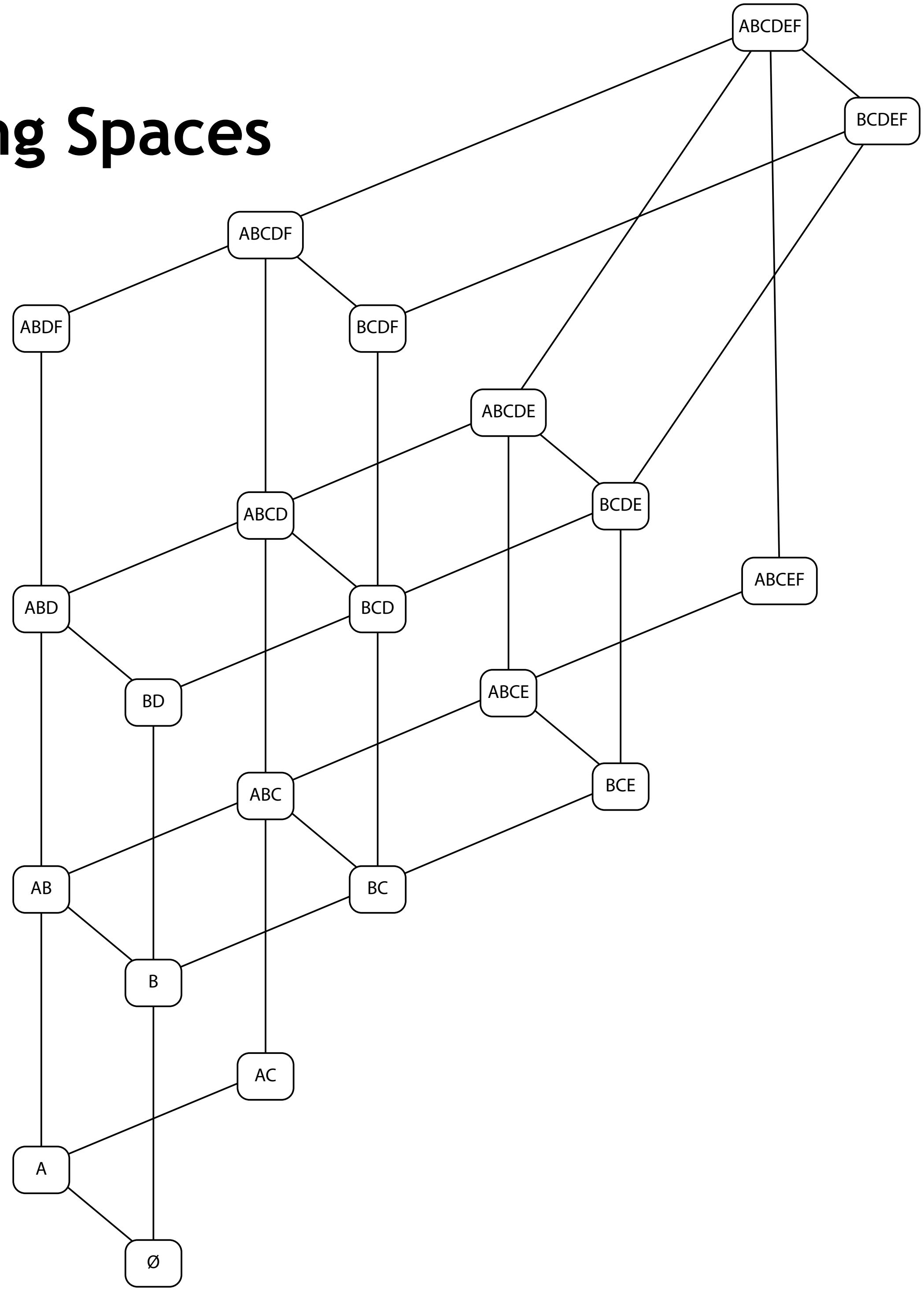
Choice of sets to include in the family
reflects prerequisite relations among the concepts

Concepts may be learned via multiple pathways:
prerequisites need not form a partial order

Used to guide knowledge assessment algorithms
for electronic learning systems

Examples of Learning Spaces





Upright-quad drawing

Axioms for Learning Spaces

Learning one concept doesn't interfere with possible learnability of any other:

$S \in \mathcal{L}$, $S \cup \{x\} \in \mathcal{L}$, and $S \cup \{y\} \in \mathcal{L} \Rightarrow S \cup \{x, y\} \in \mathcal{L}$.

Any state of knowledge can be achieved by learning a single concept at a time:

$S \in \mathcal{L} \Rightarrow$ for some $x \in S$, $S \setminus \{x\} \in \mathcal{L}$.

Axioms define a structure called an antimatroid
We focus on adjacency relation of sets $\Rightarrow$ **"learning space"**
[Math. Psych. literature vs Combinatorics literature]

Some Properties of any Learning Space

View as a directed graph with edge S to T iff T = S u {x}

must be st-oriented

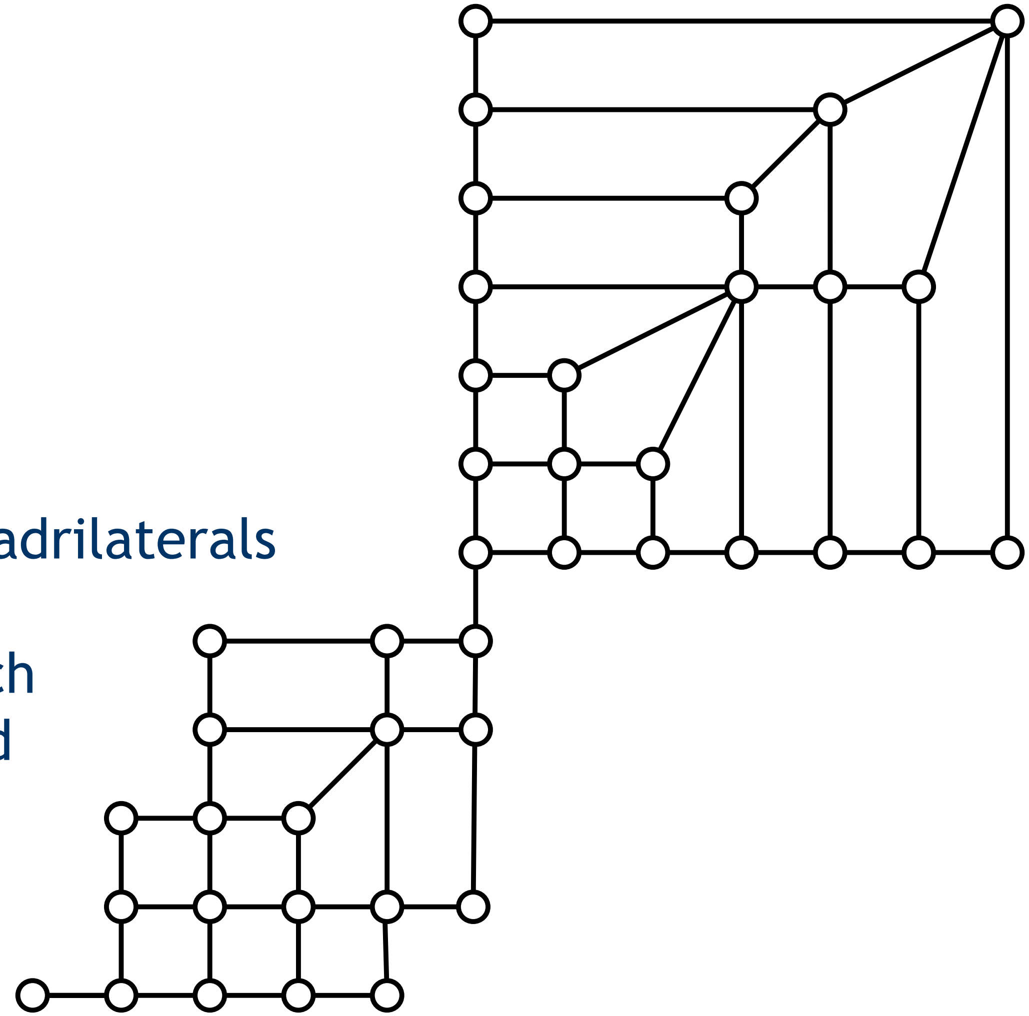
Empty set is the only source

Whole domain is the only source

Any chain of nested sets is part of a path
from the empty set to the whole domain

Any pair of edges both leaving the same vertex
can be completed to form a quadrilateral

Upright-Quad Drawing



Dominance drawing
of an st-planar graph

Internal faces are convex quadrilaterals

Bottom and left edges of each
internal face are axis-aligned

Main Result:

Graph G has an upright-quad drawing
iff G represents an st-planar learning space

Outline of Proof

Constructing learning spaces from arrangements of quadrants

Constructing upright-quad drawings from arrangements of quadrants

Constructing arrangements of quadrants from upright-quad drawings

Constructing upright-quad drawings from learning spaces

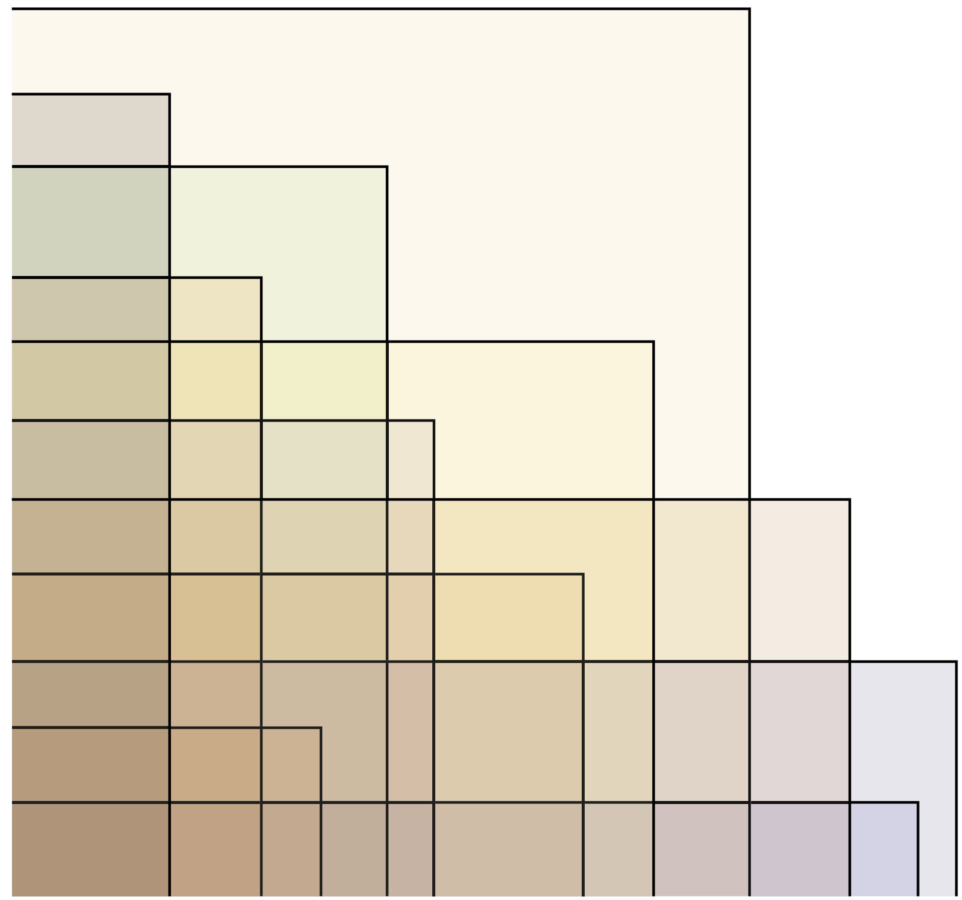
Learning Spaces from Quadrant Arrangements

Translate copies of
the negative quadrant

General position:
no two translates
share a boundary line

Correspond region to set of
quadrants below or left of it

Sets form a learning space
union = coordinate max
accessibility = move down



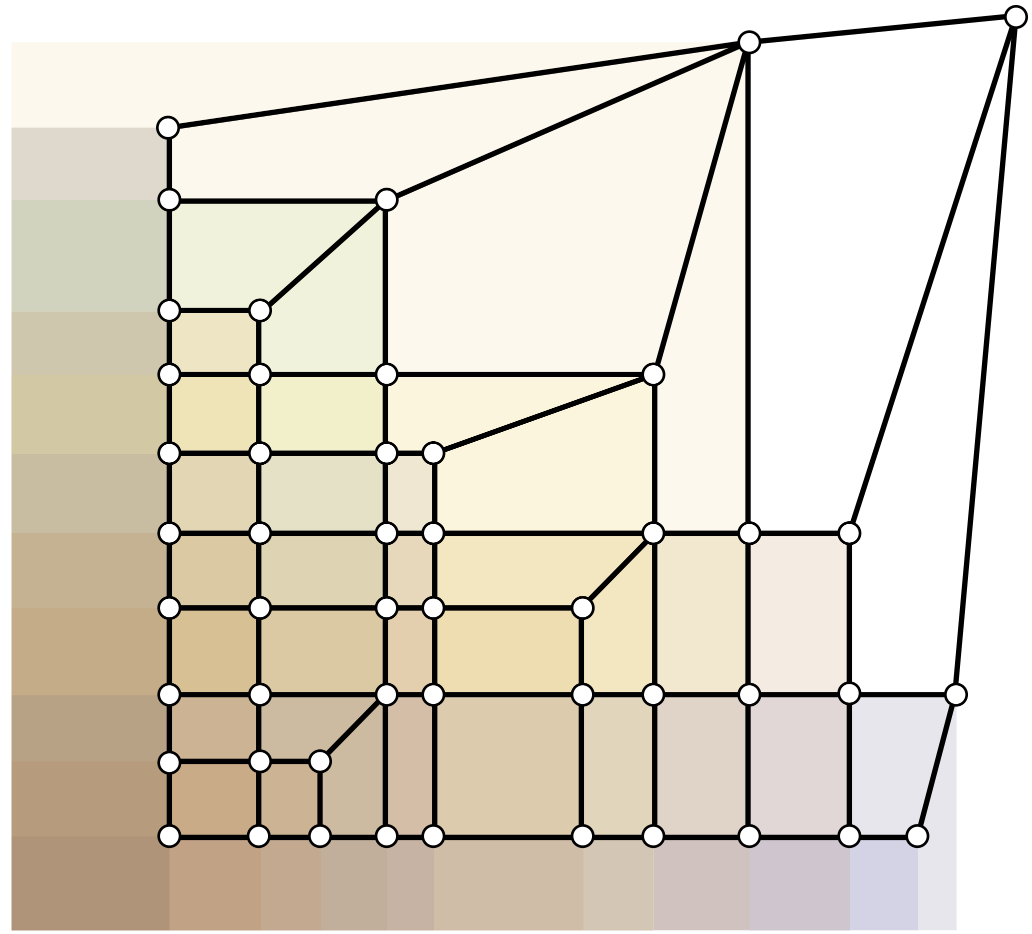
Upright-Quad Drawings from Quadrant Arrangements

Represent each region by its maximal point

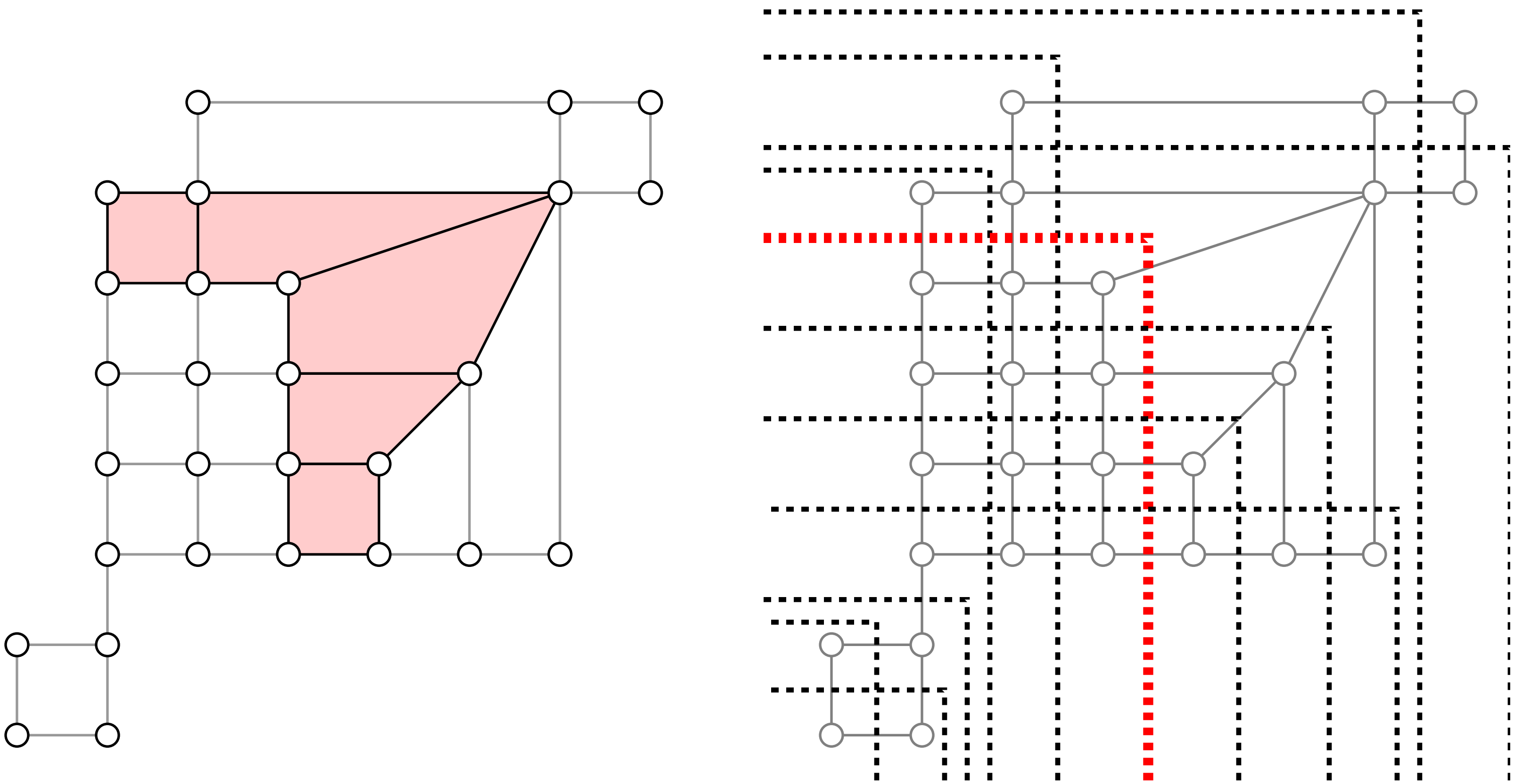
(for the unbounded region, choose a point dominating all other chosen points)

Connect points representing adjacent regions

Result is a dominance drawing with all faces convex quadrilaterals, bottom & left axis-aligned



Arrangements of Quadrants from Upright-Quad Drawings



Define **zone** = set of cells sharing bottom-top, left-right, or top-right edges

Each face belongs to two zones

Cover by quadrant containing all lower left vertices (w/tie break rules)

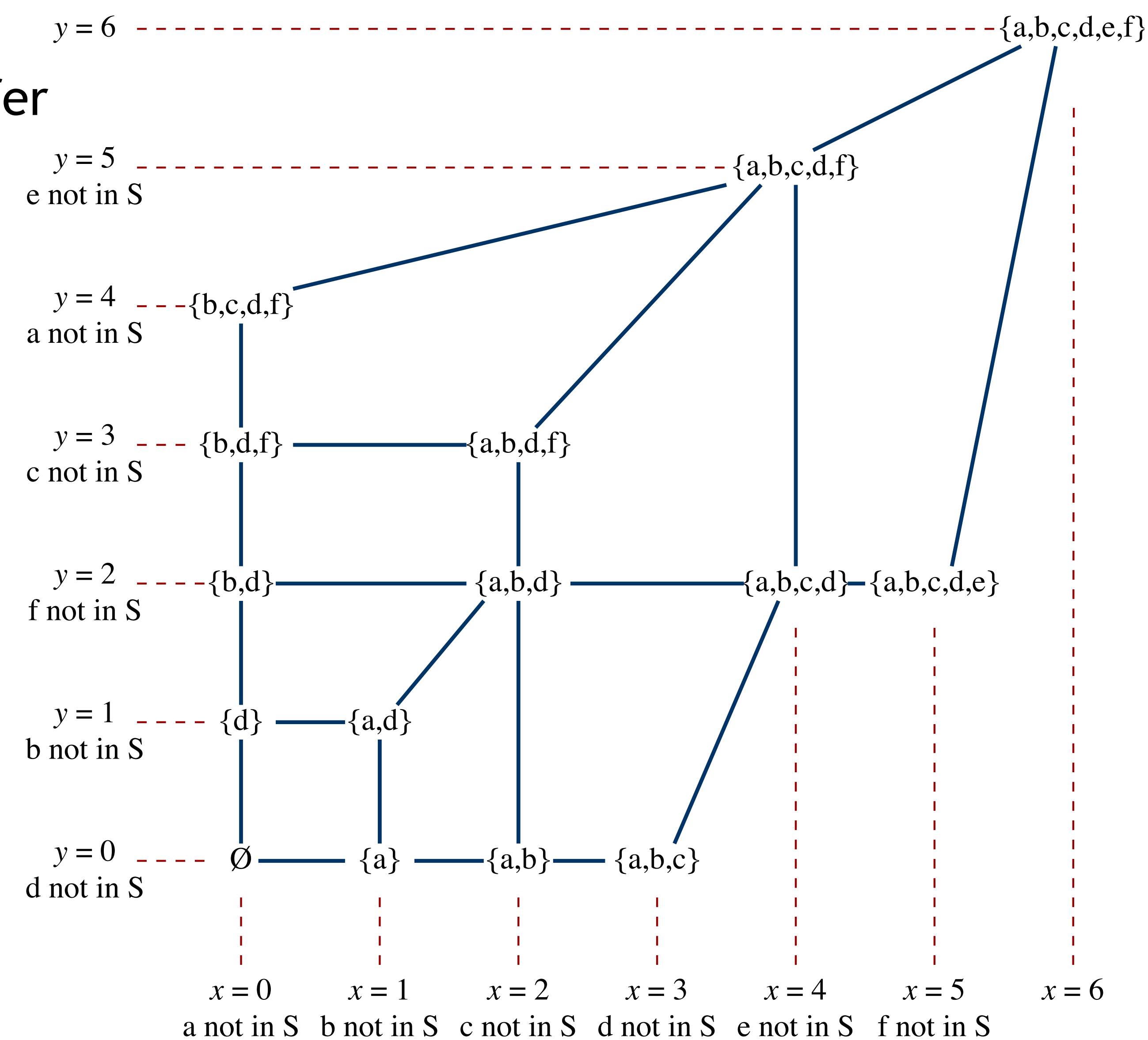
Upright-Quad Drawings from st-Planar Learning Spaces

Form sequence of elements by which consecutive sets differ on rightmost source-sink path

x-coordinate of set S = position of first element that does not belong to S

y-coordinates similar but using leftmost path

(Then compact drawing by assigning consecutive positions equal coordinates maintaining dominance order)



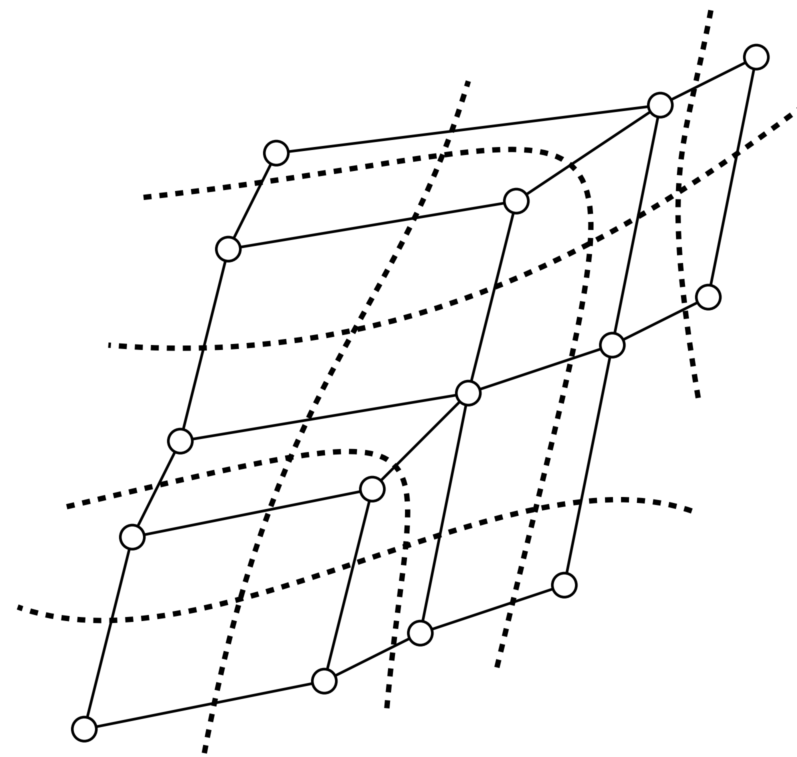
Why does it work?

Lemma: any st-planar learning space has quadrilateral faces

Define **zone** = sequence of faces connected through opposite edges

Planar dual to drawing forms family of curves (weak pseudoline arrangement) with combinatorial properties similar to quadrant arrangement

Use these properties to prove that coordinate assignment draws each face as upright quad



Conclusions

Combinatorial equivalences between three concepts:

- st-planar learning spaces
- upright-quad drawings
- arrangements of quadrants

Drawing algorithm making learning space structure visually apparent

Future work

Draw nonplanar learning spaces by projecting higher dimensional embeddings?

Convex dimension: fewest sequences
s.t. learning space = family of unions of prefixes

Results here: convex dimension is 2 iff order dimension is 2

Use minimum excluded element to embed spaces with higher convex dimension?

Reduce visual complexity by avoiding drawing all sets in learning space
e.g. omit sets when forced to exist by union property of learning space

Can such drawings lead to understandable planar drawings
for nonplanar learning spaces?