The Skip Quadtree:
A Simple Dynamic Data Structure
For Multidimensional Data

David Eppstein, Michael T. Goodrich, and Jonathan Z. Sun

Univ. of California, Irvine
Donald Bren School of Information and Computer Sciences

The Problem:

Organize a set of many low-dimensional input points

Handle (approximate) range listing queries, nearest neighbor queries, etc

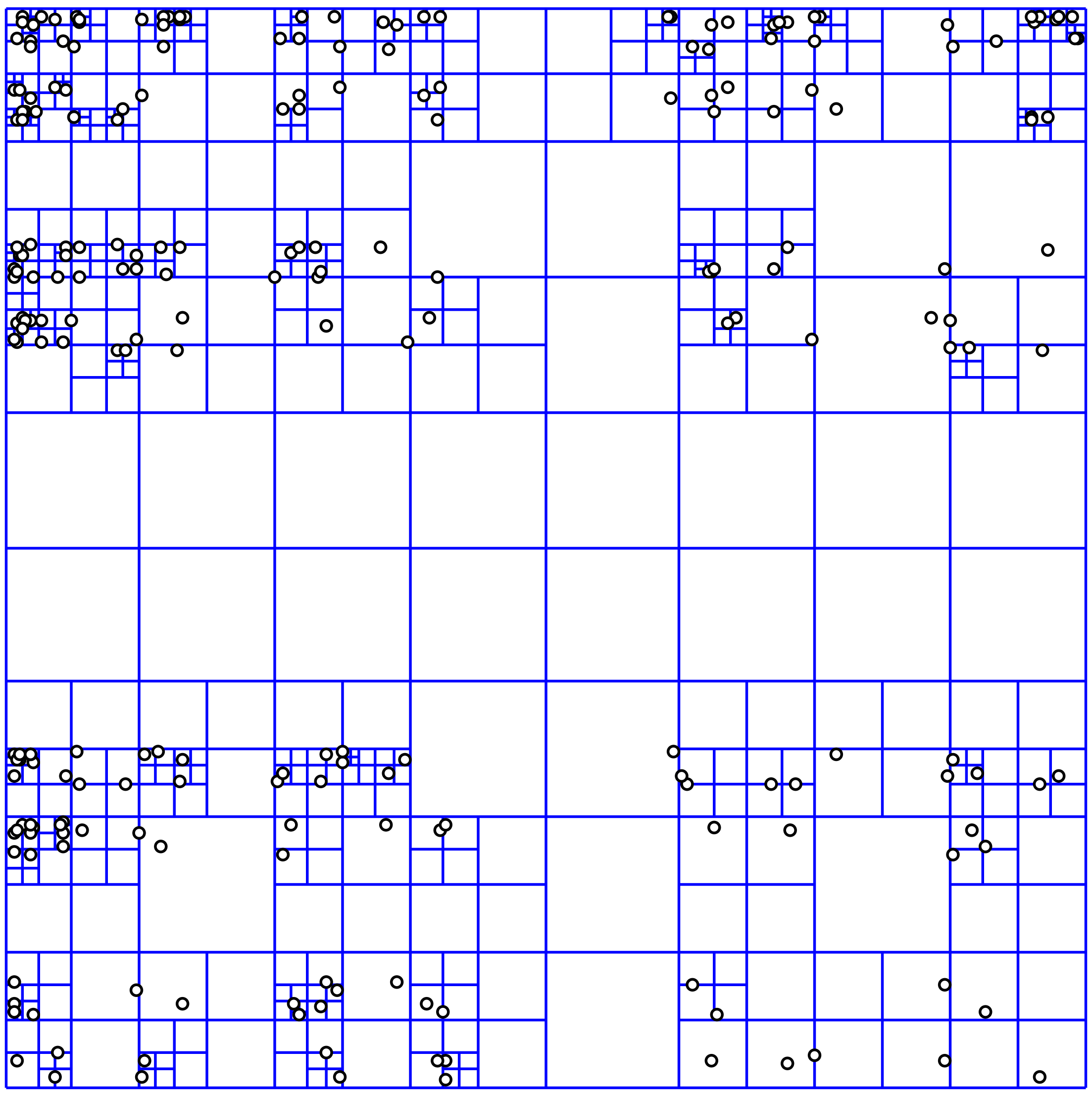
Typical solutions:

Recursively subdivide space into a hierarchy of nested convex cells
at each level, split cells by lines into smaller cells
until all leaf cells have at most one point each

Handle queries by top-down search:
if current cell is out of range, backtrack
else recursively search its children

But how to choose splits?

Quadtree



All cells are squares

To subdivide:

split into four equal squares

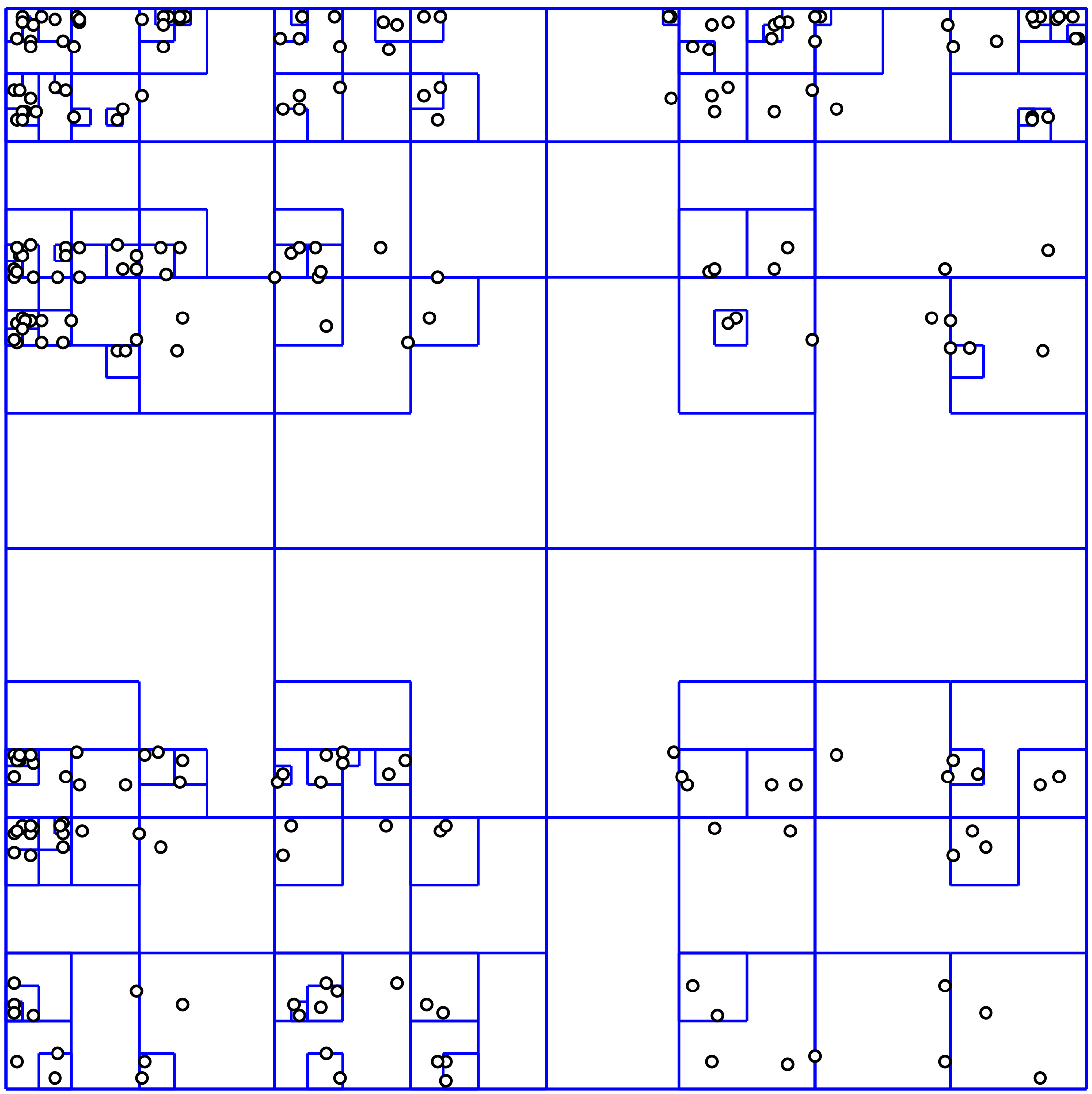
Problems:

Superlogarithmic depth

Superlinear size

No guaranteed query time
(recursion too deep)

Compressed Quadtree



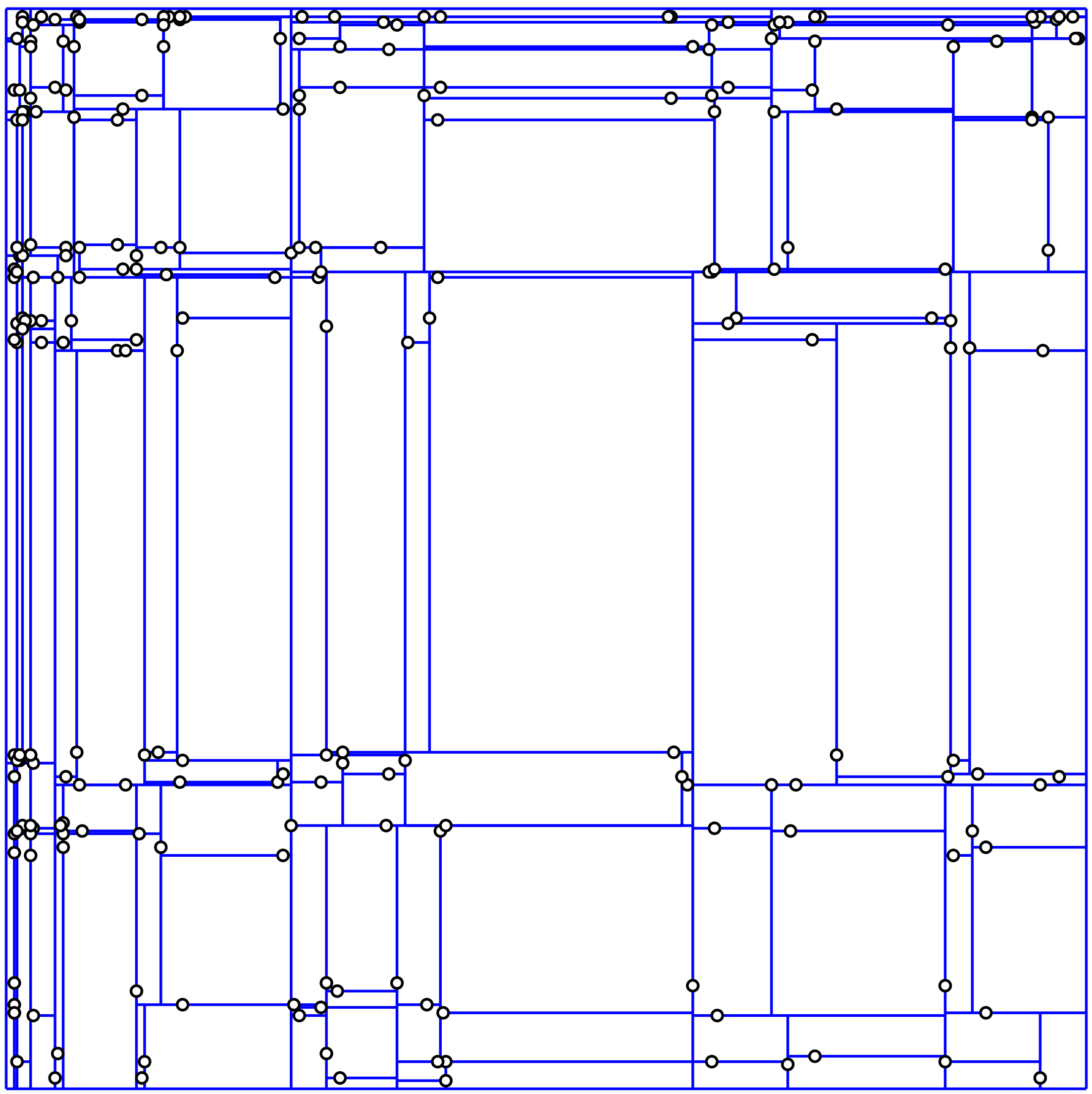
Keep only interesting squares from quadtree

Square is interesting if root or has >1 nonempty child

Problems:

- Superlogarithmic depth
- No guaranteed query time (recursion too deep)
- Unclear how to dynamize

kD-tree



All cells are rectangles

To subdivide:

split at median coordinate
alternating horizontal
and vertical

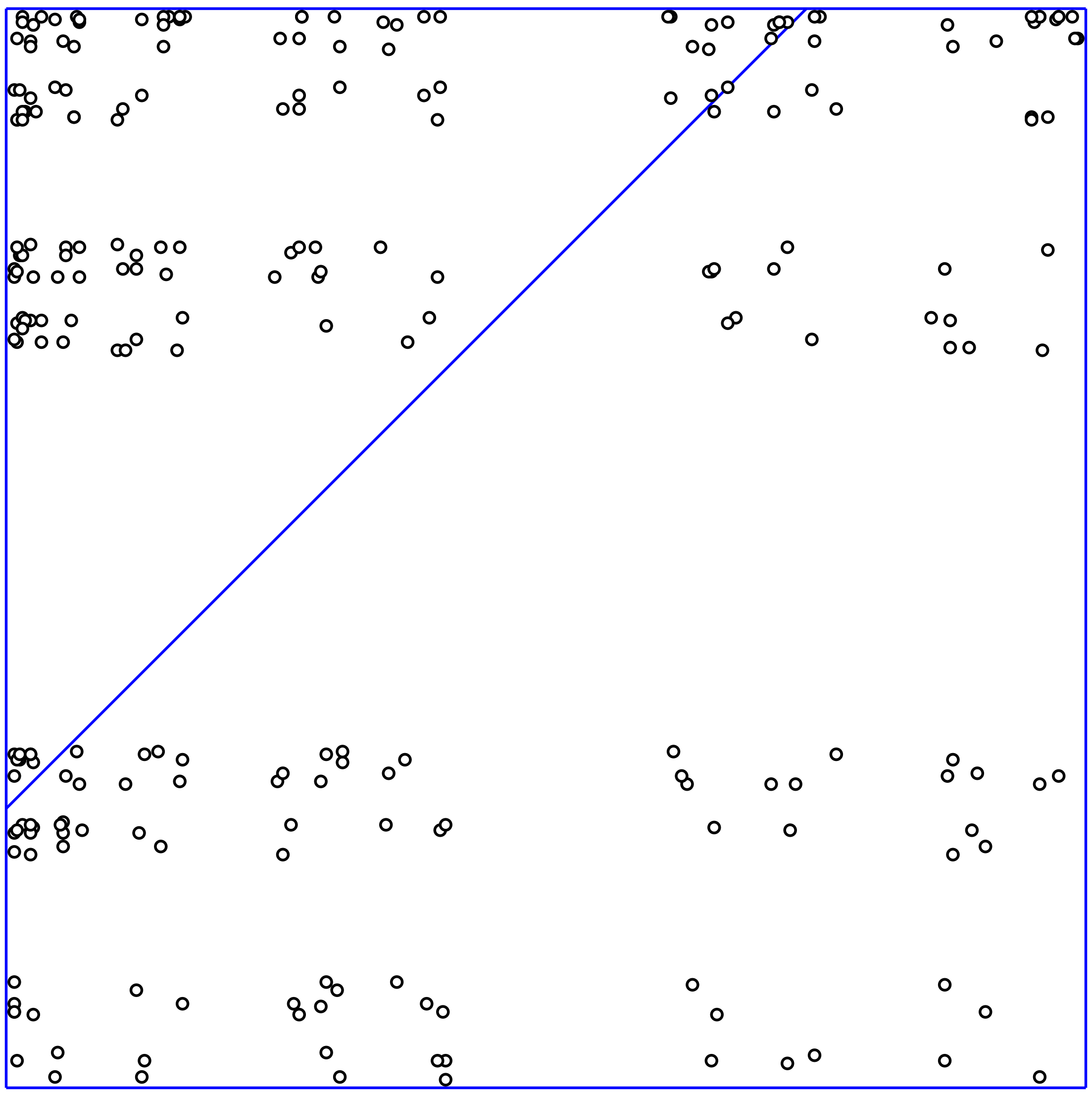
Problems:

High aspect ratio cells

No guaranteed query time
(too many cells in range)

Dynamization is amortized
(with approx median splits)

BAR-tree



All cells have ≤ 6 sides
horizontal, vertical, slope 1

Bounded aspect ratio guaranteed

To subdivide:

split at median point
choose best of 3 split slopes

Problems:

Complex implementation

Dynamization is amortized
(with approx median splits)

Skip Quadtree

Key idea:

Impose extra sampling hierarchy (analogous to skiplist) on top of compressed quadtree

Keeps the advantages as compressed quadtree...

Simple structure

Well shaped cells

...but allows logarithmic-time searches and updates

Basic version is randomized

Time bounds are high probability and expected)

But deterministic also possible (with same time bounds)

New Results

O(log n) time:

Insert or delete a point from input set

Locate query point in compressed quadtree

$O(eps^{1-d} + \log n)$ time:

(1+epsilon)-approximate fat range query

Approximation to range is decomposed into
$O(eps^{1-d})$ compressed quadtree cells

$O(eps^{1-d} (\log n + \log 1/eps))$ time

(1+epsilon)-approximate nearest neighbor query

(like spherical range query with unknown radius)

The skip quadtree

Assign a non-negative integer level to each input point
probability 2^{1-i} of being assigned level i

For each i, build a compressed quadtree Q_i of points with levels ≤ i

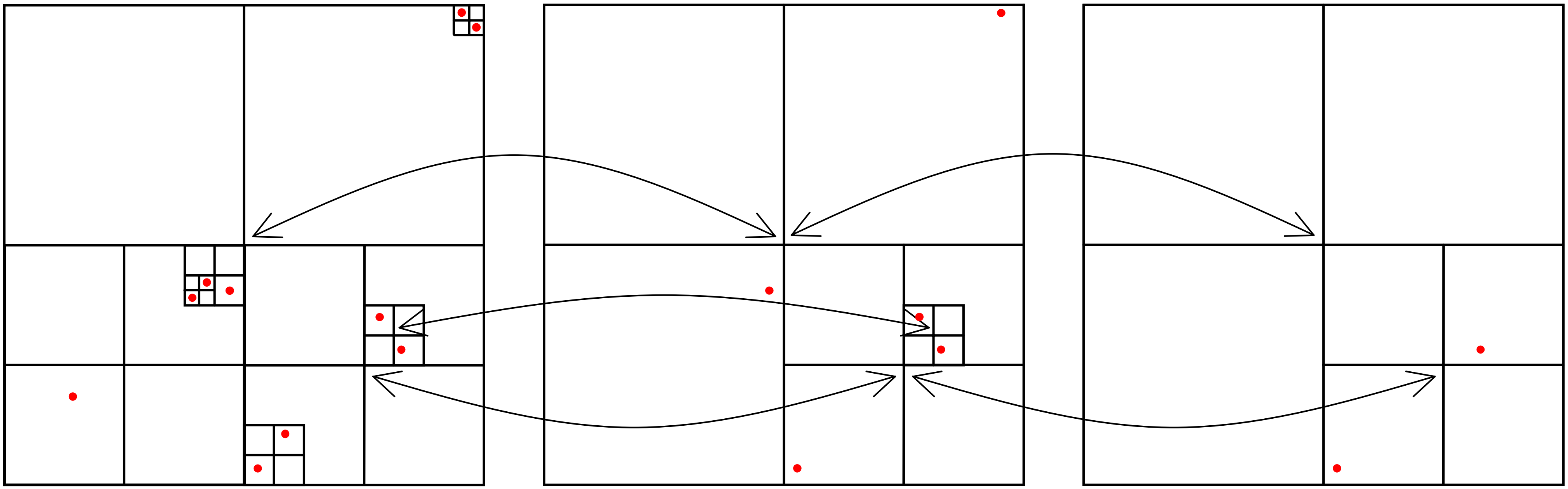
Each interesting square stores seven pointers:

next larger interesting square in Q_i (if not root)

four children (smaller squares or solitary points)

same square in Q_{i-1} (always exists unless $i = 0$)
same square in Q_{i+1} (if it exists)

The skip quadtree, visually



To locate a query point in a skip quadtree:

Start at the last nonempty level

Repeat:

if current square has a child containing query, move to it
else move to same square in next lower level

until finding smallest square containing query point in Q_0

In expectation, O(1) steps within each level
so O(log n) steps overall

To insert a new point into a skip quadtree:

Assign a level to the point

Locate the point
(finds smallest interesting square containing it in all levels)

Perform O(1) local changes in each modified level

To delete a point from a skip quadtree:

Same as insertion in reverse

To perform range queries:

Simulate standard subdivision-data-structure search in Q_0:
repeatedly replace squares by children intersecting range
until remaining squares approximately cover the range

Problem:
long chain of replacements of square by one child

Instead, use skip structure to find descendant at end of chain
like point location, O(log n) time using skip structure

To perform nearest neighbor queries:

Similar to range query

Use priority queue to keep track of which square to expand

Conclusions

New data structure combines quadtree and skiplist

All advantages of similar subdivision-based structures:

easy to implement
fast updates and queries
well shaped cells
generalizes to arbitrary dimension

Future work

Distributed version (to appear at PODC)