Cache Optimization for Embedded Processor Cores: An Analytical Approach

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Embedded microprocessor cores are increasingly being used in embedded and mobile devices. The software running on these embedded microprocessor cores is often a priori known; thus, there is an opportunity for customizing the cache subsystem for improved performance. In this work, we propose an efficient algorithm to directly compute cache parameters satisfying desired performance criteria. Our approach avoids simulation and exhaustive exploration, and, instead, relies on an exact algorithmic approach. We demonstrate the feasibility of our algorithm by applying it to a large number of embedded system benchmarks.

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General Terms: Algorithm, Design

Additional Key Words and Phrases: Cache optimization, core-based design, design space exploration, system-on-a-chip

1. INTRODUCTION

The growing demand for embedded computing platforms, mobile systems, general-purpose handheld devices, and dedicated servers coupled with shrinking time-to-market windows are leading to new core based system-on-a-chip (SOC) architectures [International Technology Roadmap for Semiconductors; Kozyrakis and Patterson 1998; Vahid and Givargis 1999]. Specifically, microprocessor cores (a.k.a., embedded processors) are playing an increasing role in such systems design [Malik et al. 2000; Petrov and Orailoglu 2001; Suzuki et al. 1998]. This is primarily due to the fact that microprocessors are easy to program using well-evolved programming languages and compiler tool chains, provide high degree of functional flexibility, allow for short product design cycles, and ultimately result in low engineering and unit costs. However, due to continued increase in complexity of these systems and devices, the performance of such embedded processors is becoming a vital design concern.

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The use of data and instruction caches has been a major factor in improving processing speed of todays microprocessors. Generally, a well-tuned cache hierarchy and organization would eliminate the time overhead of fetching instruction and data words from the main memory, which, in most cases, reside off chip and require power costly communication over the system bus that crosses chip boundaries.

Particularly, in embedded, mobile, and handheld systems, optimizing of the processor cache hierarchy has received a lot of attention from the research community [Petrov and Orailoglu 2001; Su and Despain 1995; Balasubramonian et al. 2000]. This is in part due to the large performance gained by tuning caches to the application set of these systems. The kinds of cache parameters explored by researchers include deciding the size of a cache line (a.k.a., cache block), selecting the degree of associativity, adjusting the total cache size, and selecting appropriate control policies such as write-back and replacement procedures. These techniques, typically, improve cache performance, in terms of miss reduction, at the expense of silicon area, clock latency, or energy.

The remainder of this article is organized as follows: In Section 2, we summarized related work. In Section 3, we outline our technical approach and introduce our data structures and algorithm. In Section 4, we present our experiments and show our results. In Section 5, we outline our future work. In Section 6, we conclude with some final remarks

2. PREVIOUS WORK

Traditionally, a design-simulate-analyze methodology is used to achieve optimal cache performance [Li and Henkel 1998; Shiue and Chakrabarti 1999; Wilton and Jouppi 1996; Sato 2000]. In one approach, all possible cache configurations are exhaustively simulated, using a cache simulator, to find the optimal solution. When the design space is too large, an iterative heuristic is used instead. Here, to bootstrap the process, arbitrary cache parameters are selected, the cache subsystem is simulated using a cache simulator, cache parameters are tuned based on performance results, and the process is repeated until an acceptable design is obtained.

Central to the design-simulate-analyze methodology is the ability to quickly simulate the cache. Specifically, cache simulation takes as input a trace of memory references generated by the application. In some of the efforts, speedup is achieved by stripping the original trace to a provably identical (from a performance point of view) but shorter trace [Wu and Wolf 1999; Lajolo et al. 1999]. In some of the other efforts, one-pass techniques are used in which numerous cache configurations are evaluated simultaneously during a single simulation run [Kirovski et al. 1998; Mattson et al. 1970]. While these techniques reduce the time taken to obtain cache performance metrics for a given cache configuration, they do not solve the problem of design space exploration in general. This is primarily due to the fact that the cache design space is too large. Figure 1(a) depicts the traditional approach to cache design space exploration. Our approach uses an analytical model of the cache combined with an algorithm to directly and efficiently compute a cache configuration meeting designers

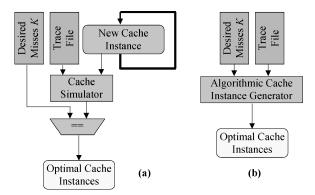


Fig. 1. Design space exploration of caches: (a) traditional approach, (b) proposed approach.

performance constraints. Figure 1(b) depicts our proposed analytical approach to cache design space exploration. In our approach, we consider a design space that is formed by varying cache size, degree of associativity and the block size. In addition to the time-ordered trace file, our algorithm takes as input the design constraint in the form of the number of desired cache misses. The output of the algorithm is a set of cache instances that meet the constraint.

The strength of our analytical approach is the ability to quickly search an exponentially large solution space to obtain optimal cache design instances. Our analytical approach is limited by the capacity of the host main memory to accommodate the number of unique references in the application trace. Since in embedded software the number of unique references is usually far less than the actual trace size, our approach is extremely practical. Specifically, we recognize that in most embedded systems, one can identify one or more small computational cores (e.g., signal processing engines, control loops, database query, etc.), whose memory reference trace is sufficiently small to be processed by our algorithms. Here, the design challenge is to enable these computational cores to execute on limited processor/memory resources, meet stringent performance (i.e., execution time) constraints, and achieve optimal design objectives (e.g., area, power, etc.). Our analytical approach is ideal for these types of constraint driven design.

Another distinguishing characteristic of our approach is the fact that it treats performance (i.e., execution time) as a design constraint and other metrics (e.g., area, power, etc.) as design objectives. Specifically, our algorithm generates each and every cache design instance (always a very small set) that meets the performance constraint. Subsequent analysis of this small set of design instances, using appropriate models (e.g., power and area models, etc.) can be used to obtain a final design instance. In a large class of applications (e.g., multimedia, control systems, target recognition, etc.), performance of software is a hard design constraint. In other words, timeliness of task completion defines, in part, the correctness of the application. Our approach is directly applicable in design of such applications.

In other related work, researchers have defined cache inclusion properties in order to link miss/hit rates of one cache organization in terms of

another [Hill 1987; Baer and Wang 1998]. Specifically, in Hill [1987], the authors use algorithms forest-simulation (to simulate direct-mapped caches) and all-associativity-simulation (to simulate set associative caches) by relying on inclusion, a property that all larger caches contain a superset of the data in smaller caches. In other words, inclusion property is used to avoid multiple passes of simulation. However, in the broader search (i.e., one that includes associativity) the algorithm is slightly less efficient than a one pass simulation as the inclusion property does not always hold. In Baer and Wang [1998], the authors have defined cache inclusion properties to determine cache coherency policies in a multiprocessor system utilizing multiple levels of cache hierarchy. Here, down a chain of caches in a hierarchy, cache inclusion properties are used to determine where and when a data write by a particular processor will invalidate cache entries. We note that cache inclusion based analysis techniques are limited to a subspace of all possible cache combinations in terms of size and associativity.

Brehob and Enbody [1996] and Harper et al. [1999] have proposed analytical models for computing cache performance. Brehob and Enbody [1996] have introduced a model based on stack distance for measure of locality. They have proposed a stack-distance based quantitative model for measuring cache performance. Harper et al. [1999] introduced novel analytical models for cache behavior. Their models, applicable to numerical codes consisting mostly of array operations, predict the overall hit/miss ratio of set-associative caches through an extensive hierarchy of cache reuse and interference effects, including numerous forms of temporal and spatial locality.

Our approach differs from previous approaches in three important aspects. First, the difference between the approach presented in this article and previous techniques, including the cache-inclusion-based techniques, is that our approach is one of analytical design space exploration while previous work focuses on performance estimation. Specifically, full simulation, partial simulation, or cache inclusion techniques compute a miss/hit value given a benchmark and a cache configuration, while in our approach, we arrive at a set of cache configurations that meet a target miss/hit rate (i.e., finding cache instances with certain performance characteristics). Secondly, our algorithm explores an exponential design space in polynomial time. Exploration of an exponential set of cache configurations using simulation, even using a fast cache estimator, as in the case of cache inclusion, can be impractical based on the time it could take. Finally, while the design space can be pruned by heuristics like hill-climbing and simulated annealing to obtain near-optimal results, our approach produces exact results. Thus, our approach is optimal, considering all possible configurations and producing the ones that meet the desired design constraints.

3. TECHNICAL APPROACH

3.1 Overview

In the following approach, we consider a design space that is obtained by varying cache depth D, the degree of associativity A and the block size B. Cache depth

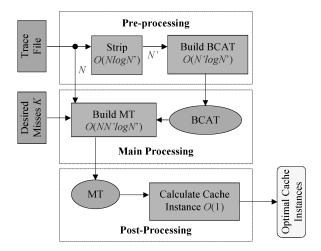


Fig. 2. Block diagram of proposed algorithm.

D gives the number of rows in the cache. In other words, $\log_2(D)$ gives the bit width of the index portion of the memory address. Degree of associativity A is the amount of storage available to accommodate data/instruction words mapping to the same cache row (a.k.a., cache block). Block size B is the number of words that can be stored in a single cache line. Our objective is to obtain a set of optimal cache triples (D, A, B) for a given number K of desired cache misses. Note that by using the cache depth D, degree of associativity A, and block size B, we obtain the total cache size by computing $D \times A \times B$. Also, note that the K desired caches misses are assumed to be those beyond the cold misses, as cold misses cannot be avoided. Finally, we have assumed fixed least recently used (LRU) replacement and write-back policies. The LRU replacement policy is the most common and often optimal choice [Intel Pentium IV; Motorola MPC500; Motorola MPC5200; Motorola MPC823]. Write-back policies affect the memory traffic between the cache and the main memory and have no effect on the traffic between the CPU and the cache. As such, choice of write back policies is not going to have an effect on our algorithm.

Our approach can be divided into three phases, the preprocessing phase, the main processing phase and the post-processing phase. During the preprocessing phase, we read the time-ordered trace file and construct a binary tree data structure, called the Binary Cache Allocation Tree BCAT. In the main processing phase, we compute the Miss Table MT during a depth-first traversal of the BCAT. In the post-processing phase, we generate the optimal cache triples (D, A, B), which are guaranteed to result in a miss rate of less than K. A block diagram of our analytical approach is shown in Figure 2. We next describe in detail the three phases of our algorithm and the associated data structures.

3.2 Preprocessing Phase

Recall that a time-ordered original trace of N instruction/data memory references $(R_1 \cdots R_N)$ is obtained after simulating the target application on a

Table I. The Original Trace

R_i	B_3	B_2	B_1	B_0
R_1	1	0	1	1
R_2	1	1	0	0
R_3	0	1	1	0
R_4	0	0	1	1
R_5	1	0	1	1
R_6	0	1	0	0
R_7	1	1	0	0
R_8	0	0	1	1
R_9	1	0	1	1
R_{10}	0	1	1	0

Table II. The Stripped Trace

B_3	B_2	B_1	B_0
1	0	1	1
1	1	0	0
0	1	1	0
0	0	1	1
0	1	0	0
	1 1 0 0	1 0 1 1 0 1 0 0	$\begin{array}{cccccccccccccccccccccccccccccccccccc$

processor whose cache is being optimized. We reduce this trace into a unique set of N' unique references $(U_1\cdots U_{N'})$, where $N'\leq N$. Every reference R_i is the kth $(k\in\{1\cdots N\})$ occurrence of a unique reference U_j . As part of a running example, consider the trace shown in Table I and the stripped version shown in Table II.

In this example, the trace contains N=10 4-bit references $(R_1\cdots R_{10})$. Of those, there are N'=5 unique references $(U_1\cdots U_5)$. Thus reference R_2 is the 1st occurrence of unique reference U_2 , and R_7 is the 2nd occurrence of U_2 . Similarly, references R_1 , R_5 and R_9 are the 1st, 2nd and 3rd occurrences respectively of U_1 . Next we describe the BCAT data structure.

A BCAT data structure fully captures how references are mapped onto a cache of any possible organization. Prior to computing the BCAT data structure, we transform the unique set into an array of zero/one sets. The array of zero/one sets contains a pair of sets for each address bit. Specifically, for index bit B_i , we compute a pair of sets called zero Z_i and one O_i . The set Z_i contains all U_k that have a bit value of 0 at B_i . Likewise, the set O_i contains all U_k that have a bit value of 1 at B_i . For the running example, shown in Table I, the zero/one sets are given in Table III.

Next, the zero/one sets are used to construct the BCAT tree. We use these sets because the set intersection operation nicely defines how references are allocated to each cache location. For an example, in a cache of depth 4 (i.e., 4 rows), using B_0 and B_1 as the index bits, we can compute the following cross intersections:

$$L_{00} = Z_0 \cap Z_1 = \{2, 5\}$$

 $L_{01} = Z_0 \cap O_1 = \{3\}$
 $L_{10} = O_0 \cap Z_1 = \{\}$
 $L_{11} = O_0 \cap O_1 = \{1, 4\}$

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Table III. The Zero/One Sets

	Z	0
$\overline{B_0}$	$\{U_2,U_3,U_5\}$	$\{U_1, U_4\}$
B_1	$\{U_2,U_5\}$	$\{U_1,U_3,U_4\}$
B_2	$\{U_1, U_4\}$	$\{U_2,U_3,U_5\}$
B_3	$\{U_3,U_4,U_5\}$	$\{U_1,U_2\}$

```
\begin{split} \textbf{Input:} \ & \textbf{U}_0, \textbf{U}_1 \cdots \textbf{U}_{N'-1} \ (\textbf{unique set}) \\ \textbf{Output:} \ & \textbf{BCAT} \\ \text{for each } i \in [\texttt{M}-1\cdots0] \ \text{do} \ / \! / \text{assume M-bit references} \\ & \textbf{Z}_i := \textbf{O}_i := \emptyset \\ & \text{for each } \textbf{U}_j \ \text{do} \\ & \text{if } i^{th} \ \text{bit of } \textbf{U}_j \ \text{is 0 then} \\ & \textbf{Z}_i := \textbf{Z}_i \cup \{\textbf{U}_j\} \\ & \text{else} \\ & \textbf{O}_i := \textbf{O}_i \cup \{\textbf{U}_j\} \\ & \textbf{BCAT.root} \Leftarrow \textbf{Z}_0 \cup \textbf{O}_0 \\ & \textbf{BCAT} := \textbf{Recursive-Build-BCAT}(\textbf{BCAT.root}, \textbf{Z}, \textbf{O}, \textbf{1}) \end{split}
```

Fig. 3. Algorithm 1: Builds the BCAT data structure.

```
\label{eq:local_continuity} \begin{split} &\textbf{Input:} \ BCAT, \ \eta \ (\text{tree node}), \ Z/O \ (\text{sets}), \ L \ (\text{tree level}) \\ &\textbf{Output:} \ BCAT \\ &\text{if} \ | \ \eta \ | \ge 2 \ \text{then} \\ & \ \eta. \\ &\text{left} \Leftarrow \eta \cap Z_L \\ &\text{BCAT} := \text{Recursive-Build-BCAT}(\eta. \\ &\text{left}, \ Z, \ O, \ L+1) \\ & \ \eta. \\ &\text{right} \Leftarrow \eta \cap O_L \\ &\text{BCAT} := \text{Recursive-Build-BCAT}(\eta. \\ &\text{right}, \ Z, \ O, \ L+1) \end{split}
```

Fig. 4. Algorithm 2: Builds (recursively) the BCAT data structure.

Here sets L_{00} , L_{01} , L_{10} , and L_{11} contain the reference identifiers mapped onto the 4 cache slots. Likewise, for a cache of depth 8, using an additional index bit B_2 , we cross intersect each of these 4 sets with Z_2 and O_2 to obtain the 8 new sets and so on. The new sets form the nodes of our binary tree. We stop growing the tree further down when we reach a set with cardinality less than 2. Algorithm 1 (Figure 3) and Algorithm 2 (Figure 4) recursively build a BCAT data structure as described here. The complete BCAT data structure of the running example is shown in Figure 5.

Associated with each node, we maintain a trace, called the Relevant Trace Set RTS. The RTS of a node is a subset of the RTS of its parent node containing only the references mapped onto the current node. For the root, RTS is the original trace. For other nodes, RTS is created dynamically during the main processing phase. (See Algorithm 7.)

3.3 Main Processing Phase

In the main processing phase, we build up the Miss Table MT data structure by processing each node η , as it is encountered in a depth first traversal of the BCAT tree.

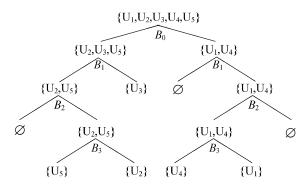


Fig. 5. BCAT data structure corresponding to application trace shown in Table I.

(Assoc., Block)	(1, 1)	(2, 1)	(3, 1)	(4, 1)	(5, 1)	
\Rightarrow						
Level						
0	5	4	4	2	0	
1	5	2	0	0	0	
2	4	0	0	0	0	
3	4	0	0	0	0	
4	0	0	0	0	0	

Table IV. The MT Data Structure

The MT data structure maintains, for each level L of the BCAT, the number of misses for every associativity being considered, that is, A=1 to $A=A_{\rm max}$ and for a range of block size $B_{\rm min}$ to $B_{\rm max}$. The block size is bounded by the product of $A_{\rm max}$ and the maximum depth $D_{\rm max}$ of the BCAT. Note that each level of the tree corresponds to a particular cache depth $D=2^L$. For example, level one of the tree (root being level zero) corresponds to a cache of depth two. Also, the maximum associativity at a given level, which results in no misses, can be calculated by setting A to the maximum cardinality of all nodes in the BCAT at that level. An entry $MT_{L,A,B}$ gives the number of misses at level L (i.e., depth $D=2^L$) for associativity A and block size B. For example, $MT_{3,2,4}=15$ means a cache of depth $D=2^3=8$ with associativity A=2 and B=4 will result in 15 misses. The complete MT data structure for our running example is shown in Table IV. For simplicity, we just illustrate the case in which the cache line size is equal to the word size, i.e., B=1.

The MT data structure is built using Algorithm 3 (Figure 6).

Processing of each node involves traversing the RTS and for each of its element, updating the MT data structure (explained later), creating the Count Vector CV (explained next) and creating the children RTS (explained earlier) for the children nodes, as shown in Algorithm 4 (Figure 7).

The CV data structure is an array of integer counters $(C_1 \cdots C_N)$ corresponding to the unique references $(U_1 \cdots U_N)$. For every R_i in the original trace, and for all unique references U_j , the value i-CV[U_j]-1 gives a count of the unique

```
Input: T (original trace), K (desired misses)
Input: BCAT, B (block size)
Output: MT

MT := \emptyset; BCAT.root.RTS = T
for each node \eta \in BCAT (depth first) do

(MT, \eta) := Process-Node(MT, \eta, K, B)
```

Fig. 6. Algorithm 3: Builds the MT data structure.

```
Input: MT, \eta (tree node), K (desired misses)

Output: MT, \eta (tree node)

CV[1 \cdots N] = 0
for each reference R_i in \eta.RTS do

U_j := \text{Unique-Reference-Type}(R_i)

MT := \text{Update-MT}(CV[U_j], K, MT, \eta. \text{level}, R_i, B)

CV := \text{Update-CV}(CV, U_j, R_i)
\eta := \text{Create-Children-RTS}(\eta, R_i)
```

Fig. 7. Algorithm 4: Processes a tree node while building the MT data structure.

references that have occurred since the last occurrence of U_j in the original trace.

The CV can be populated by using the sequence number i of a reference R_i in the original trace. Specifically, if a given reference R_i is an occurrence of a unique reference U_j , then $CV[U_j]$ is set to i. However, at this point, there could be U_k $(k \in \{1 \cdots N'\}, k \neq j)$ unique references whose last occurrence was before the last occurrence of U_j . To prevent U_j being counted more than once for each of those U_k unique references, we increment $CV[U_k]$ by 1 for all k. To illustrate, when unique reference "0110" (j=3) is encountered for the first time in original trace, CV[3] is set to the then current i (=3). At i=8, reference "0011" is encountered for the 2nd time since the last occurrence of "0110". Hence CV[3] is incremented by 1 (to 4). Similarly, at i=9, CV[3] is again incremented (to 5) due to repetition of "1011" (Table V, Table VI).

The final issue is to identify all the k unique references for which $CV[U_k]$ need to be incremented when a unique reference U_j occurs in the original trace. From our earlier observations, it is obvious that U_k should have occurred before the last occurrence of U_j . Thus, all k for which $CV[U_k]$ is less than the previous value of $CV[U_j]$ are the references that occurred before the last occurrence of U_j .

Incorporating block size is important in the analysis as it contributes to calculating memory access time which is an important metric in evaluating cache performance. Blocks belonging to the same cache line are simultaneously fetched from the main memory and replaced from the cache subsystem, and are never in conflict with each other. For a cache line with block size B, the B references being mapped onto different blocks of the same cache line will differ only in the lower $\log_2(B)$ bits. By assigning the same numeric identifier to each of these B references, we maintain a single position in the CV and avoid them being counted as different references. This eliminates all conflicts they have with each other but accounts for all conflicts that any of the B references could have with other references. A simple way of achieving this is to

Table V. CV, After Processing i = 7

ID	CV[ID]
1	5
2	7
3	3
4	4
5	6

Table VI. CV, After Processing i = 9

ID	CV[ID]
1	9
2	7
3	5
4	8
5	6

ignore the lower $\log_2(B)$ bits of the bit patterns while stripping the original trace.

Algorithm 5 (Figure 8) updates the CV data structure as described above.

The MT is updated for every reference R_i in the RTS. Specifically, the objective is to identify all the associativities for which R_i will be a miss. If R_i is the kth occurrence of unique reference U_j , then the number of unique references between the (k-1)th occurrence and kth occurrence of U_i is given by the value $i - CV[U_i] - 1$. This value provides the upper bound on the degree of associativity, for which kth occurrence of unique reference U_i will result in a miss. To illustrate, let us look at Table VI. If the next reference (i = 10) is an occurrence of $U_i = 1$, then number of unique references between the last and the current occurrence of U_i is equal to 10-9-1=0. This is indeed the case as the immediately preceding reference was an occurrence of $U_i = 1$. However, if instead the next reference is an occurrence of $U_i = 5$, then the number of unique references is equal to 10-6-1=3. This is again true as unique references 1, 4 and 2 have occurred since the last occurrence of 5. Note that a miss count is associated with each degree of associativity A under consideration (i.e., 1, $2 \cdots A_{\text{max}}$) and block size B. We stop to consider a particular degree of associativity A and block size B when its miss count goes beyond the desired number of desired misses K, as shown in Algorithm 6 (Figure 9).

Finally, to build the RTS of the children, we follow the steps outlined in Algorithm 7 (Figure 10).

3.4 Post-Processing Phase

During the last phase of the algorithm, we read the MT data structure and output a set of triples consisting of cache depth, associativity and block size that satisfy the desired performance in terms of the number of cache misses, as shown in Algorithm 8.

In Algorithm 8 (Figure 11), for depths (number of rows) equal to 1, 2, 4, etc., we print the optimal caches having the smallest degree of associativity and block size to guarantee no more misses than the desired value K.

```
Input: CV, U_j, R_i

Output: CV

for U_k \in CV, k \neq j do

if (CV[U_k] < CV[U_j])

CV[U_k] = CV[U_k] + 1

CV[U_i] = i
```

Fig. 8. Algorithm 5: Updates the CV data structure.

```
Input: CV[U_j], K (desired misses), R_i, B Input: MT, L (tree level)

Output: MT

a_{max} = i - CV[U_j] - 1;
for A \in [1 \cdots a_{max}] do

if (MT[L][A][B] ! = -1) && (MT[L][A][B] > K)

MT[L][A][B] := -1 and break

MT[L][A][B] := MT[L][A][B] + 1
```

Fig. 9. Algorithm 6: Updates the MT data structure.

The number of optimal cache instances is no less than the minimum cache depth for which associativity is equal to 1 and is no more than the width of the bus. This small set of cache instances can be further pruned by using appropriate power, timing and/or area models to yield one final cache instance that satisfies all design constraints.

3.5 Time Complexity

For time complexity analysis, we use the size of the trace N and the number of unique references N' as the input parameters. We note that in most cases, N' is much smaller than N. Moreover, $\log_2(N')$ is bounded by the width of the memory references (i.e., processor data-path), which is typically 32 or 64. We have shown the time complexity of each part of the algorithm in Figure 2, as explained next.

The time taken to strip the trace amounts to sorting the references and thus is $O(N \times \log_2(N'))$.

The time taken to build the BCAT data structure is $O(N' \times \log_2(N'))$. At the root, we processes one node by looking at the N' unique references at a cost of $O(1 \times N')$, at level one, we process two nodes by looking at N'/2 unique references at a cost of $O(2 \times N'/2)$, at level two, we process four nodes by looking at N'/4 unique references at a cost of $O(4 \times N'/4)$, etc. In general, at each level of the tree, the computation is bounded by O(N'). Since the number of nodes in the tree is O(N') it follows that the depth of the is $O(\log_2(N'))$. Combining these, we obtain $O(N'\log_2(N'))$.

The time taken to build the MT data structure at each level of the BCAT is $O(N \times N')$ which is dominated by the computation involved in building the CVs of each node in BCAT. At the root, we process each reference (O(N)) by setting its value in the CV and including it in the RTS of either the left or the right child. In addition, the entire length of CV needs to be traversed to account for repetition which takes O(N') time. The overall time at this level is thus $O(N \times N')$.

```
\label{eq:continuity} \begin{split} &\textbf{Input: } \eta \text{ (tree node), } R_i \\ &\textbf{Output: } \eta \text{ (tree node)} \\ &\text{ if } R_i \in \eta. \text{left-child then} \\ &\eta. \text{left-child.RTS} := \eta. \text{left-child.RTS} \cup R_i \\ &\text{else} \\ &\eta. \text{right-child.RTS} := \eta. \text{right-child.RTS} \cup R_i \end{split}
```

Fig. 10. Algorithm 7: Generates children RTS data structures.

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\label{eq:local_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_continuous_cont
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Fig. 11. Algorithm 8: Outputs cache instances.

At the next level, we process two nodes for which we compute the CV data structure (taking $O(2 \times N'/2)$) followed by the RTS of its children for each reference $(O(2 \times N/2))$, taking $O(N \times N')$, and so on for the remaining levels. In general, at each level of the tree, the computation is bounded by $O(N \times N')$. Since the number of nodes in the tree is O(N') it follows that the depth of the is $O(\log_2(N'))$. Combining these, we obtain $O(N \times N' \times \log_2(N'))$ for the entire BCAT.

Finally, the post-processing phase of the algorithm takes constant time to output the cache instances.

Overall, the presented technique takes $O(N \times N' \times \log_2(N'))$ step to execute. In the best-case scenario, all references are repetitions of a single reference, (i.e., N'=1). Thus, the best case time complexity of our algorithm becomes O(N). In the worst-case scenario, all references are unique (i.e., N'=N). Thus, the worst-case time complexity of our algorithm becomes $O(N^2 \times \log_2(N))$. In our experiments, the average case is closer to the best-case scenario, as the size of a memory trace is usually very large with respect to the working set of the corresponding application.

3.6 Space Complexity

The original trace of size N is processed by reading one reference R_i at a time. As such, there is no need to store the trace in the main memory and hence this requires O(1) space.

Processing of a reference R_i , involves building the BCAT, updating the CV and updating the MT.

The BCAT, as might be recalled, stores for each level of the tree, the number of unique references N', that are mapped onto different nodes. Thus, at any

given level, the worst-case memory requirement is O(N'). If W is the bit-width of the bus, then the number of levels in the tree is no more than W. As such, the worst-case memory requirement is $O(W \times N')$. However, as explained in the next section, the BCAT is traversed in a depth-first fashion and no more than one node per level is stored in the main memory at any given time. As such, the worst-case memory requirements is actually O(N').

The CV is an array of size N', and has thus a space complexity of O(N').

Finally, the MT data structure has one entry for each possible cache depth. If W is the bit-width of the bus, then this data structure requires a space of O(W).

Since N' is considerably lesser than N and since W is typically equal to 32, the space complexity of our algorithm is dominated by O(N').

3.7 Final Remarks

The data structure and algorithms described above are presented in a manner to illustrate the logic and intuition behind our analytical cache optimization technique. Here, we comment on issues to be considered in an actual implementation (such as the one used to obtain the results in our experiments).

Stripping of a trace can be improved substantially by using a hash-table structure to keep track of unique reference. Moreover, the building of the MRCT data structure can be performed during the stripping of the trace with no additional added time complexity if a hash-table is used.

The extensive use of sets in our technique is due to the fact that sets are efficient to represent, store, and manipulate on a computer system using bit vectors. In addition, the use of sets allows for execution of the algorithm on a cluster of machines by utilizing a distributed set library, enabling the processing of very large trace files.

The implementation of Algorithm 1 and Algorithm 7 can be combined. Specifically, the BCAT does not need to be calculated in its entirety. Instead, a depth first traversal of the tree can be performed to reduce memory usage. Further, the data structures associated with each node can be deleted, after the node has been processed.

The advantage of our approach lies in the fact that the entire design space, obtained by varying the cache depth and associativity, can be exhaustively explored in $\mathrm{O}(N\times N'\times \log_2(N'))$ time. Simulation of a single cache configuration requires $\mathrm{O}(N)$ time. As such, our approach is better suited for an exhaustive exploration of the complete design space. However, if the design space is restricted by the limited values commonly used today for the cache parameters, then traditional simulation could be equally efficient for exploration.

4. EXPERIMENTS

For our experiments, we have used 16 typical embedded system applications that are part of the PowerStone [Vahid and Givargis 1999] and the Media-Bench [Lee et al. 1997] benchmark applications. The applications include a JPEG decoder called *jpeg*, a modem decoder called *v42*, a Unix compression

	m . 1		m:
	Total	Unique	Time
Benchmark	Refs. N	Refs. N'	(sec)
adpcm	18431	381	2.7
bent	456	162	0.11
blit	4088	2027	6.879
compress	58250	8906	466.87
crc	2826	603	0.43
des	20162	2241	19.268
engine	211106	225	10.786
fir	5608	146	0.39
g3fax	229512	3781	221.098
jpeg	1311693	39302	100576
pocsag	13467	515	1.582
qurt	503	84	0.07
ucbqsort	61939	1144	17.516
v42	649168	23942	15628
toast	884700	1548	3942.92
G723	1016262	173	281.77

Table VII. Data Trace Statistics

utility called compress, a CRC checksum algorithm called crc, an encryption algorithm called des, an engine controller called engine, an FIR filter called fir, a group three fax decoder called g3fax, a sorting algorithm called ucbqsort, an image rendering algorithm called blit, a POCSAG communication protocol for paging applications called pocsag, an implementation of the European GSM 06.10 provisional standard for full-rate speech transcoding called toast, an implementation of the International Telegraph and Telephone Consultative Committee (CCITT) G.723 voice compression called G723, and a few other embedded applications.

We first compiled and executed the benchmark applications on a MIPS R3000 simulator. Our processor simulator is instrumented to output instruction/data memory reference traces. The size of the traces N, the number of unique references N', and the execution time of our algorithm are reported for data and instruction traces in Table VII and Table VIII, respectively.

We have ran these traces through our analytical algorithm for various values of desired number of cache misses K. Specifically, we have set K to one of 1%, 2%, 3%, and 4% cache misses. For brevity, in Tables IX—XVIII, we have presented the optimal cache instances of the data and instruction traces for only five of the benchmarks, namely des (Table IX, Table X), jpeg (Table XI, Table XII), g3fax (Table XIII, Table XIV), toast (Table XV, Table XVI), and g723 (Table XVII, Table XVIII). The correctness of the proposed approach has been verified by subsequent cache simulation.

In these tables, the inner entries are the degree of associativity A and block size B necessary to ensure the desired number of cache misses. For example, if 4% cache misses are allowed for the data trace of des, a eleven-way set associative cache of block size 4 and depth 512 would suffice.

Our algorithm ran on a Pentium IV processor running at 2.8 GHz with 512 MB of memory. The time taken to produce results for data/instruction traces is shown in the last columns of Table VII and Table VIII.

Table VIII. Instruction Trace Statistics

	Total	Unique	Time
Benchmark	Refs. N	Refs. N'	(sec)
adpcm	63255	611	12.689
bent	1337	115	0.12
blit	22244	149	0.781
compress	137832	731	23.044
crc	37084	176	1.653
des	121648	570	22.954
engine	409936	244	34.47
fir	15645	327	1.60
g3fax	1127387	220	67.73
jpeg	4594120	623	693.876
pocsag	47840	560	5.988
qurt	1044	179	0.151
ucbqsort	219710	321	17.165
v42	2441985	656	389.856
toast	3526594	7208	212501
G723	4153844	71202	14575.8

Table IX. Optimal Instruction Cache Instances for des

Cache	(Degree of Associativity)							
Depth		(Desired Cache Misses K as a Percentage						
D	1%	2%	3%	4%				
2	(541, 2)	(541, 2)	(541, 2)	(541, 2)				
4	(271, 2)	(271, 2)	(271, 2)	(271, 2)				
8	(136, 2)	(136, 2)	(136, 4)	(136, 2)				
16	(69, 2)	(69, 2)	(68, 2)	(68, 2)				
32	(35, 2)	(34, 2)	(34, 2)	(34, 2)				
64	(18, 2)	(18, 2)	(18, 2)	(18, 2)				
128	(10, 2)	(10, 2)	(10, 2)	(9, 2)				
256	(5, 2)	(5, 2)	(5, 2)	(5, 2)				
512	(3, 2)	(3, 2)	(3, 2)	(3, 2)				
1024	(2, 2)	(2, 2)	(2, 2)	(2, 2)				
2048	(1, 2)	(1, 2)	(1, 2)	(1, 2)				

Table X. Optimal Data Cache Instances for des

Cache	(Degree of Associativity)					
Depth	(Desired)	Cache Misse	es K as a Per	centage		
D	1%	2%	3%	4%		
2	(1396, 2)	(1300, 2)	(1236, 2)	(1179, 2)		
4	(1191, 2)	(1124, 2)	(1070, 2)	(1035, 2)		
8	(1098, 2)	(1043, 2)	(1006, 2)	(972, 2)		
16	(352, 4)	(326, 4)	(309, 4)	(296, 4)		
32	(176, 4)	(163, 4)	(155, 4)	(148, 4)		
64	(88, 4)	(82, 4)	(78, 4)	(75, 4)		
128	(45, 4)	(42, 4)	(39, 4)	(38, 4)		
256	(23, 4)	(21, 4)	(20, 4)	(19, 4)		
512	(12, 4)	(12, 4)	(12, 4)	(11, 4)		
1024	(9, 4)	(9, 4)	(9, 2)	(7, 2)		
2048	(5, 2)	(5, 2)	(5, 2)	(4, 2)		
4096	(3, 2)	(3, 2)	(3, 2)	(2, 2)		
8192	(2, 2)	(2, 2)	(2, 2)	(1, 2)		
16384	(1, 2)	(1, 2)	(1, 2)	(1, 2)		

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Table XI. Optimal Instruction Cache Instances for jpeg

	_						
Cache	(Degree of Associativity)						
Depth	(Desired	Cache Misses	s K as a Percei	ntage			
D	1%	2%	3%	4%			
2	(246, 2)	(246, 2)	(246, 2)	(227, 2)			
4	(124, 2)	(122, 2)	(122, 2)	(114, 2)			
8	(62, 2)	(61, 2)	(61, 2)	(58, 2)			
16	(31, 2)	(31, 2)	(30, 2)	(30, 2)			
32	(17, 2)	(15, 2)	(15, 2)	(15, 2)			
64	(8, 2)	(8, 2)	(8, 2)	(8, 2)			
128	(4, 2)	(4, 2)	(4, 2)	(4, 2)			
256	(2, 2)	(2, 2)	(2, 2)	(2, 2)			
512	(1, 2)	(1, 2)	(1, 2)	(1, 2)			

Table XII. Optimal Data Cache Instances for jpeg

	1	(T) 0.1					
Cache	(Degree of Associativity)						
Depth	(Desired Cache Misses K as a Percentage						
D	1%	2%	3%	4%			
2	(38628, 2)	(38615, 2)	(38574, 2)	(38493, 2)			
4	(38461, 2)	(38461, 2)	(38461, 2)	(38430, 2)			
8	(19307, 2)	(19245, 2)	(19245, 2)	(19216, 2)			
16	(9654, 2)	(9623, 2)	(9622, 2)	(9616, 2)			
32	(4828, 2)	(4812, 2)	(4811, 2)	(4809, 16)			
64	(2414, 2)	(2406, 2)	(2405, 2)	(2405, 2)			
128	(1207, 2)	(1203, 2)	(1203, 2)	(1202, 2)			
256	(603, 2)	(602, 2)	(601, 2)	(601, 2)			
512	(302, 2)	(301, 2)	(301, 2)	(301, 2)			
1024	(151, 2)	(151, 2)	(150, 2)	(150, 2)			
2048	(76, 2)	(75, 2)	(75, 2)	(75, 2)			
4096	(38, 2)	(38, 2)	(38, 2)	(38, 2)			
8192	(19, 2)	(19, 2)	(19, 2)	(19, 2)			
16384	(10, 2)	(10, 2)	(10, 2)	(10, 2)			
32768	(5, 2)	(5, 2)	(5, 2)	(5, 2)			
65536	(3, 2)	(3, 2)	(3, 2)	(3, 2)			
131072	(2, 2)	(2, 2)	(1, 2)	(1, 2)			
262144	(1, 2)	(1, 2)	_	_			

Table XIII. Optimal Instruction Cache Instances for g3fax

Cache	(Degree of Associativity)				
Depth	(Desired	Cache Misses	K as a Percen	tage	
D	1%	2%	3%	4%	
2	(87, 2)	(87, 2)	(87, 2)	(83, 2)	
4	(44, 2)	(43, 2)	(43, 2)	(41, 2)	
8	(24, 2)	(22, 2)	(21, 2)	(21, 2)	
16	(13, 2)	(11, 2)	(11, 2)	(10, 2)	
32	(7, 2)	(7, 2)	(6, 2)	(6, 2)	
64	(4, 2)	(4, 2)	(3, 2)	(3, 2)	
128	(3, 2)	(2, 2)	(2, 2)	(2, 2)	
256	(2, 2)	(2, 2)	(2, 2)	(2, 2)	
512	(1, 2)	(1, 2)	(1, 2)	(1, 2)	

Table XIV. Optimal Data Cache Instances for g3fax

Cache	(Degree of Associativity)			
Depth	(Desired)	Cache Misse	s K as a Per	centage
D	1%	2%	3%	4%
2	(1927, 2)	(1920, 2)	(1912, 2)	(1902, 2)
4	(965, 2)	(961, 2)	(955, 2)	(950, 2)
8	(485, 2)	(481, 2)	(478, 2)	(476, 2)
16	(244, 2)	(242, 2)	(240, 2)	(239, 2)
32	(124, 2)	(122, 2)	(121, 2)	(120, 2)
64	(64, 2)	(63, 2)	(62, 2)	(61, 2)
128	(35, 2)	(33, 2)	(32, 2)	(31, 2)
256	(19, 2)	(17, 2)	(17, 2)	(16, 2)
512	(10, 2)	(9, 2)	(8, 2)	(8, 2)
1024	(5, 2)	(5, 2)	(5, 2)	(4, 2)
2048	(3, 2)	(3, 2)	(3, 2)	(2, 2)
4096	(2, 2)	(2, 2)	(2, 2)	(1, 2)
8192	(2, 2)	(1, 2)	(1, 2)	_
16384	(1, 2)	_	_	

Table XV. Optimal Data Cache Instances for toast

Cache	(Degree of Associativity)			
Depth	(Desired Cache Misses K as a Percentage			
D	1%	2%	3%	4%
2	(5924, 2)	(2283, 2)	(2246, 2)	(446, 2)
4	(5924, 2)	(2283, 2)	(2246, 2)	(446, 2)
8	(5924, 2)	(2283, 2)	(2246, 2)	(446, 2)
16	(5924, 2)	(2283, 2)	(2246, 2)	(446, 2)
32	(5924, 2)	(2283, 2)	(2246, 2)	(446, 2)
64	(5924, 2)	(2283, 2)	(2246, 2)	(446, 2)
128	(5924, 2)	(2283, 2)	(2246, 2)	(446, 2)
256	(5924, 2)	(2283, 2)	(2246, 2)	(446, 2)
512	(5924, 2)	(2283, 2)	(2246, 2)	(446, 2)
1024	(5924, 2)	(2283, 2)	(2246, 2)	(446, 2)
2048	(5924, 2)	(2283, 2)	(2246, 2)	(446, 2)
4096	(5924, 2)	(2283, 2)	(2246, 2)	(446, 2)
8192	(5924, 2)	(2283, 2)	(2246, 2)	(446, 2)
16384	(5924, 2)	(2283, 2)	(2246, 2)	(446, 2)
32768	(5924, 2)	(2283, 2)	(2246, 2)	(446, 2)
65536	(2858, 2)	(1203, 2)	(1065, 2)	(256, 2)
131072	(1447, 8)	(615, 2)	(514, 2)	(256, 2)
262144	(742, 8)	(360, 8)	(256, 16)	(256, 2)
524288	(379, 8)	(313, 4)	(256, 8)	(256, 2)
1048576	(256, 8)	(256, 4)	(256, 8)	(256, 2)
2097152	(256, 4)	(256, 2)	(256, 4)	(256, 2)
4194304	(256, 4)	(256, 2)	(256, 4)	(256, 2)
8388608	(256, 2)	(256, 2)	(256, 2)	(256, 2)
16777216	(128, 2)	(128, 2)	(128, 2)	(128, 2)
33554432	(64, 8)	(64, 2)	(64, 2)	(64, 2)
67108864	(32, 8)	(32, 8)	(32, 8)	(32, 8)
134217728	(16, 8)	(16, 8)	(16, 8)	(16, 8)
268435456	(8, 8)	(8, 8)	(8, 8)	(8, 8)
536870912	(4, 8)	(4, 8)	(4, 8)	(4, 8)
1073741824	(2, 8)	(2, 8)	(2, 8)	(2, 8)
2147483648	(1, 8)	(1, 8)	(1, 8)	(1, 8)

Table XVI. Optimal Instruction Cache Instances for toast

Cache	(Degree of Associativity)				
\mathbf{Depth}	(Desired Cache Misses K as a Percentage				
D	1%	2%	3%	4%	
2	(1143, 2)	(703, 2)	(386, 2)	(294, 2)	
4	(1143, 2)	(703, 2)	(386, 2)	(294, 2)	
8	(1143, 2)	(703, 2)	(386, 2)	(294, 2)	
16	(1143, 2)	(703, 2)	(386, 2)	(294, 2)	
32	(1143, 2)	(703, 2)	(386, 2)	(294, 2)	
64	(1143, 2)	(703, 2)	(386, 2)	(294, 2)	
128	(1143, 2)	(703, 2)	(386, 2)	(294, 2)	
256	(573, 2)	(343, 4)	(219, 2)	(174, 2)	
512	(573, 2)	(343, 4)	(219, 2)	(174, 2)	
1024	(573, 2)	(343, 2)	(219, 2)	(174, 2)	
2048	(573, 2)	(343, 2)	(219, 2)	(174, 2)	
4096	(573, 2)	(343, 2)	(219, 2)	(174, 2)	
8192	(573, 2)	(343, 2)	(219, 2)	(174, 2)	
16384	(573, 2)	(343, 2)	(219, 2)	(174, 2)	
32768	(573, 2)	(343, 2)	(219, 2)	(174, 2)	
65536	(278, 2)	(170, 4)	(128, 2)	(104, 2)	
131072	(162, 2)	(112, 4)	(98, 2)	(75, 2)	
262144	(129, 8)	(108, 2)	(93, 2)	(71, 2)	
524288	(128, 4)	(108, 2)	(89, 2)	(61, 2)	
1048576	(128, 4)	(108, 2)	(89, 2)	(61, 2)	
2097152	(128, 2)	(108, 2)	(89, 2)	(61, 2)	
4194304	(128, 2)	(108, 2)	(89, 2)	(61, 2)	
8388608	(127, 2)	(108, 2)	(88, 2)	(60, 2)	
16777216	(108, 2)	(94, 2)	(77, 2)	(59, 2)	
33554432	(54, 2)	(51, 2)	(41, 2)	(33, 2)	
67108864	(27, 8)	(26, 8)	(21, 2)	(16, 8)	
134217728	(14, 8)	(13, 8)	(10, 16)	(8, 8)	
268435456	(7, 8)	(6, 8)	(5, 8)	(4, 8)	
536870912	(4, 8)	(3, 8)	(3, 8)	(2, 8)	
1073741824	(2, 8)	(2, 8)	(2, 8)	(1, 8)	
2147483648	(1, 8)	(1, 8)	(1, 8)		

In Figure 12, we have plotted the average measured time taken to produce results along with the analytical time complexity computed as $N \times N' \times \log_2(N')$ on a logarithmic scale. As expected, the plots are consistent, with the actual time taken asymptotically equal to the estimated time complexity.

As has been mentioned earlier, our approach is exact and in that respect, it is much faster compared to other exact methods like simulation. Our approach explores a design space S that is exponential in size (where size is the size of the address space.) In other words, given a 32-bit address space, S contains cache instances obtained by varying depth and associativity in every possible way and block size from 2 to 32. Using existing cache simulation approaches in an exhaustive search algorithm will not terminate in reasonable amount of time. In order for us to provide comparison results, we limited the design space (cache depth D from 64 to 1024, associativity A from 1 to 8 and block size B from 2 to 32 in powers of 2) and obtained the results for three benchmark programs(des, g3fax, blit), shown in Table XIX. Our approach still outperformed existing approaches.

Table XVII. Optimal Data Cache Instances for G723

Cache (Degree of Associativity)					
	(Degree of Associativity)				
Depth D	(Desired Cache Misses K as a Percentage 1% 2% 3% 4%				
2	(837, 2)	(824, 2)	(815, 2)	(809, 2)	
4	(837, 2)	(824, 2)	(815, 2)	(809, 2)	
8	(837, 2)	(824, 2)	(815, 2)	(809, 2)	
16	(837, 2)	(824, 2)	(815, 2)	(809, 2)	
32	(837, 2)	(824, 2)	(815, 2)	(809, 2)	
64	(837, 2)	(824, 2)	(815, 2)	(809, 2)	
128	(837, 2)	(824, 2)	(815, 2)	(809, 2)	
256	(837, 2)	(824, 2)	(815, 2)	(809, 2)	
512	(837, 2)	(824, 2)	(815, 2)	(809, 2)	
1024	(837, 2)	(824, 2)	(815, 2)	(809, 2)	
2048	(837, 2)	(824, 2)	(815, 2)	(809, 2)	
4096	(837, 2)	(824, 2)	(815, 2)	(809, 2)	
8192	(837, 2)	(824, 2)	(815, 2)	(809, 2)	
16384	(837, 2)	(824, 2)	(815, 2)	(809, 2)	
32768	(837, 2)	(824, 2)	(815, 2)	(809, 2)	
65536	(429, 2)	(415, 2)	(409, 2)	(409, 2)	
131072	(272, 4)	(262, 2)	(258, 4)	(259, 2)	
262144	(210, 4)	(196, 4)	(193, 4)	(191, 4)	
524288	(210, 2)	(196, 2)	(193, 2)	(191, 2)	
1048576	(210, 2)	(196, 2)	(193, 2)	(191, 2)	
2097152	(210, 2)	(196, 2)	(193, 2)	(191, 2)	
4194304	(210, 2)	(196, 2)	(193, 2)	(191, 2)	
8388608	(210, 2)	(196, 2)	(193, 2)	(191, 2)	
16777216	(105, 2)	(98, 2)	(96, 2)	(95, 2)	
33554432	(53, 2)	(50, 2)	(48, 2)	(48, 2)	
67108864	(26, 8)	(25, 8)	(25, 2)	(24, 8)	
134217728	(14, 8)	(13, 8)	(12, 8)	(12, 8)	
268435456	(7, 8)	(7, 8)	(7, 8)	(6, 8)	
536870912	(4, 8)	(4, 8)	(4, 8)	(3, 8)	
1073741824	(2, 8)	(2, 8)	(2, 8)	(2, 8)	
2147483648	(1, 8)	(1, 8)	(1, 8)	(1, 8)	

5. FUTURE WORK

The focus of our current work is on core-based system-on-chip design, where the processor core, main memory controller core etc are assumed to be fixed. As such, we explore the design space parameterized by associativity, block size and cache size. In order to provide an analytical framework that encompasses more design parameters, in the future we intend to incorporate such design parameters as different replacement policies, multilevel caches, and bus architecture effects.

6. CONCLUSION

We have proposed an efficient algorithm to directly compute cache parameters satisfying desired performance criteria. The proposed approach avoids simulation and exhaustive exploration. Here, we consider a design space that is formed by varying cache size, degree of associativity and block size. For a given

Table XVIII. Optimal Instruction Cache Instances for G723

Cache	(Degree of Associativity)			
Depth	(Desired Cache Misses K as a Percentage			
$\hat{ ext{D}}$	1%	2%	3%	4%
2	(103, 2)	(100, 2)	(99, 2)	(98, 2)
4	(103, 2)	(100, 2)	(99, 2)	(98, 2)
8	(103, 2)	(100, 2)	(99, 2)	(98, 2)
16	(103, 2)	(100, 2)	(99, 2)	(98, 2)
32	(103, 2)	(100, 2)	(99, 2)	(98, 2)
64	(103, 2)	(100, 2)	(99, 2)	(98, 2)
128	(103, 2)	(100, 2)	(99, 2)	(98, 2)
256	(77, 2)	(76, 2)	(75, 2)	(75, 2)
512	(77, 2)	(76, 2)	(75, 2)	(75, 2)
1024	(77, 2)	(76, 2)	(75, 2)	(75, 2)
2048	(77, 2)	(76, 2)	(75, 2)	(75, 2)
4096	(77, 2)	(76, 2)	(75, 2)	(75, 2)
8192	(77, 2)	(76, 2)	(75, 2)	(75, 2)
16384	(77, 2)	(76, 2)	(75, 2)	(75, 2)
32768	(77, 2)	(76, 2)	(75, 2)	(75, 2)
65536	(65, 2)	(64, 2)	(63, 2)	(63, 2)
131072	(65, 2)	(64, 2)	(63, 2)	(63, 2)
262144	(65, 4)	(64, 2)	(63, 2)	(63, 4)
524288	(65, 4)	(64, 4)	(63, 2)	(63, 4)
1048576	(65, 4)	(64, 4)	(63, 4)	(63, 4)
2097152	(65, 4)	(64, 2)	(63, 4)	(63, 2)
4194304	(65, 2)	(64, 2)	(63, 2)	(63, 2)
8388608	(65, 2)	(64, 2)	(63, 2)	(63, 2)
16777216	(64, 2)	(64, 2)	(63, 2)	(63, 2)
33554432	(45, 2)	(45, 2)	(45, 2)	(43, 2)
67108864	(24, 8)	(23, 8)	(21, 8)	(21, 2)
134217728	(12, 8)	(12, 8)	(11, 8)	(11, 8)
268435456	(7, 8)	(6, 8)	(6, 8)	(5, 8)
536870912	(3, 8)	(3, 8)	(3, 8)	(3, 8)
1073741824	(2, 8)	(2, 8)	(2, 8)	(2, 8)
2147483648	(1, 8)	(18, 8)	(1, 8)	(1, 8)

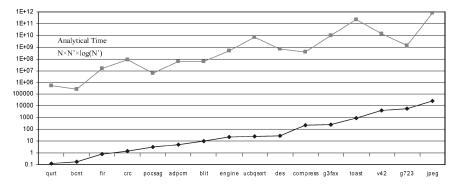


Fig. 12. Analytical time complexity vs. actual run times.

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Table XIX. Comparison with Simulation

	Analytical	_
	Approach	Simulation
	(sec)	(sec)
des	0.02	100
g3fax	2	900
blit	0.004	20

memory reference trace, our algorithm takes as input the design constraint in the form of the number of desired cache misses and outputs a set of optimal cache instances that meet the constraint. The feasibility of the proposed approach has been verified experimentally using the PowerStone and Mediabench benchmarks.

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